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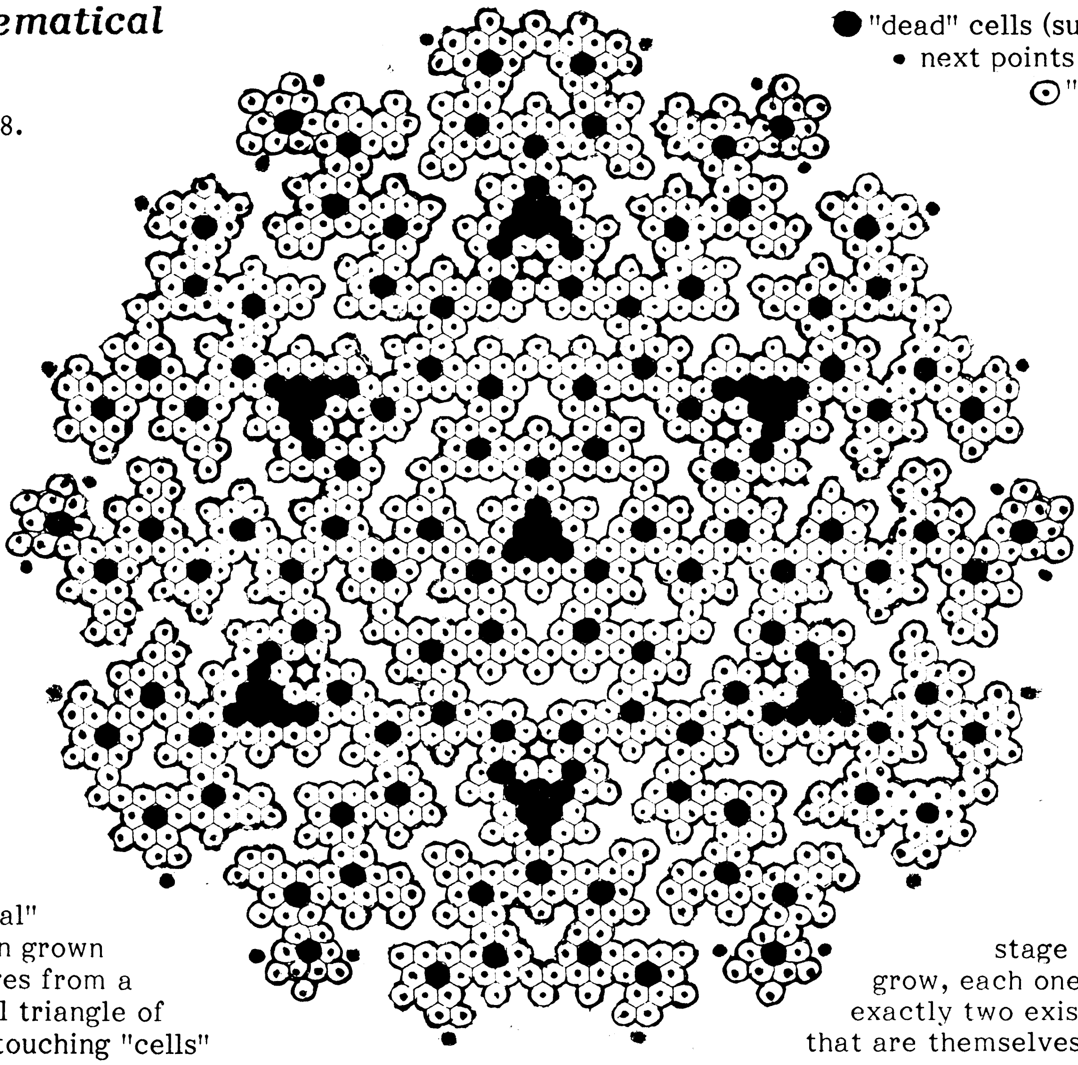
Mathematical Art

See p48.



A "coral" pattern grown in stages from a central triangle of three touching "cells"

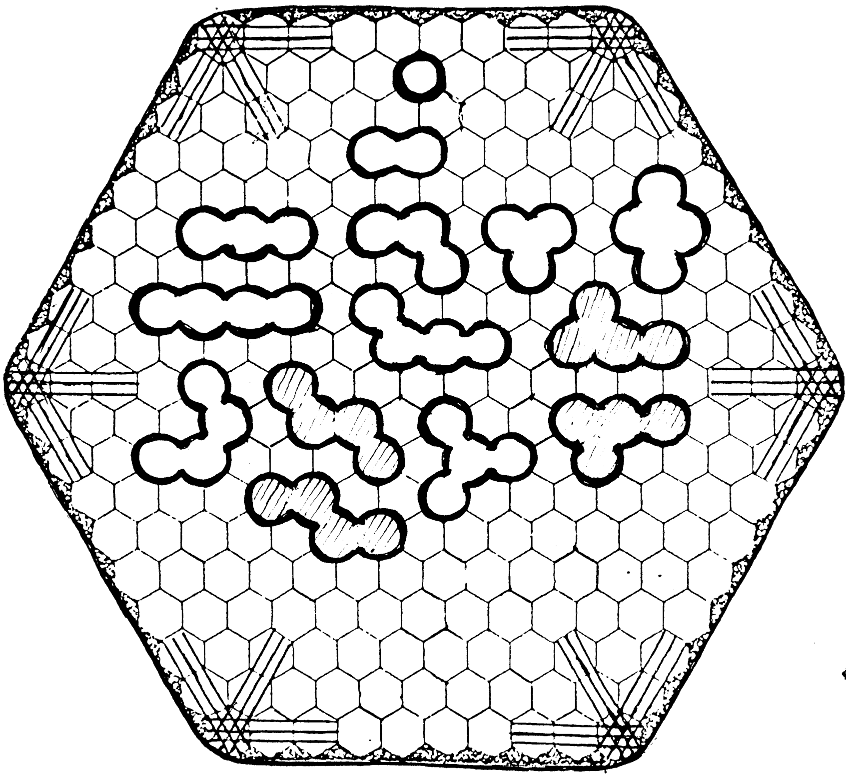
At each stage new cells grow, each one touching exactly two existing cells that are themselves touching

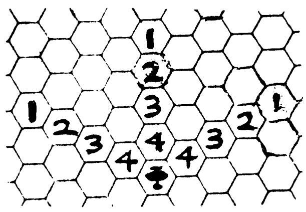
Contents

BOARD GAMES

Sopwith

Tommy Sopwith, the pioneer aviation engineer, after whom this game is named, recently attained his 100th birthday, so this article seems appropriately timed. The game is a simple but effective simulation of aerial combat set in the biplane era, and is one of the mainstays of the postal games hobby. Like Diplomacy it has the feature of simultaneous movement by all the players. The board is a large hexagonal honeycomb, representing an area of countryside, with an airfield in each corner, each with three runways. In the set as produced commercially (distributed by Quantum Games Ltd, 5 Dorking Road, Tunbridge Wells, Kent, TN1 2LN, copyright Dave Dyer 1978) the hexagon has only 9 cells per side, whereas the postal gamers prefer a more spacious board with 10 cells on each side, as illustrated. The number of cells in a hexagon of side N is 3N(N-1)+1 [cf Chessics 11, p8] so that for N=9 there are 217 cells and for N=10 there are 271.



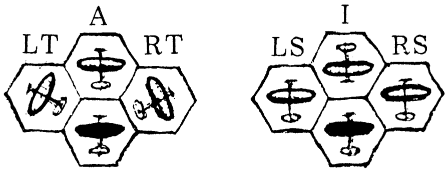


Firepower

All possible cloud shapes of 1 to 4 cells are shown

See p 45 for problems on Sopwith manoeuvres

The moves



Each plane begins with 16 units of ammunition, from which up to 3 bursts may be fired in each turn of play, when airborne, and stocks can be replenished by returning to home base. Each plane is capable of withstanding 12 units of damage, the degree of damage inflicted by a burst of fire being 4 units at close range reducing to zero at five cells distance. The planes fire only in the three forward directions. Two units of repairs can be carried out if the plane is grounded at base for a turn. A number of "clouds" are distributed randomly over the board at the start. These are in the shape of various polyhexes of 1 to 5 cells. In the postal version they may move in the direction of the wind, as specified by the games master. Damage can result from involvement with these storm clouds, but they can also be used to hide behind or within. A flyer who achieves a good score in inflicting damage points qualifies as an "Ace" who inflicts an extra point of damage than a normal Aviator thereafter.

One of the most interesting aspects of the play lies in the moves of the planes. Six types of move are permitted: Ahead (A), Left Turn (LT), Right Turn (RT), Left Slip (LS), Right Slip (RS) and Immelman (I). Only Aces can execute the Immelman reversal. Each turn consists of three moves (or can be cut short to two or one) and up to three bursts of fire. The moves are specified in advance to the games master, or in the o.t.b. version by "setting the controls" which are dial-like diagrams provided around the edges of the hexagon and concealed from the other players by screens until everyone has chosen their moves. The moves and firing are then played out, in three stages, and the damages and ammunition used assessed.

Psychological Jujitsu

By Stephen TAYLOR

The following card game that I know as "psychological jujitsu" is the only card game I know where all the players have, at all times, complete information regarding the state of the game; hence luck plays no part whatever in the outcome - or does it? [I think this constitutes quite an interesting question in itself.] One description of the game is in Games for the Super-Intelligent by J.F.Fixx (published by Frederick Muller) but he does not specify any origin for it.

Two people play and each is given a hand of thirteen cards all of one suit - clubs and spades say. The suit is immaterial and the cards rank from Ace (highest) through King, Queen, Knave, 10,9 etc down to 2. A third suit is shuffled and then placed in a fan face upwards on the table. The cards in this fan assign a numerical value to the 13 tricks in sequential order according to the scheme: picture cards and aces are worth 10 points and pip cards worth their face value (making a total value of 94 points). e.g. the fan A,7,3,Q,4,6,K,J,2,5,10,8,9 results in the first trick having a value of 10 points, the second 7 pts, the third 3 pts, etc. The two players now compete for the 13 tricks using their identical hands, subject to: (a) the objective, which is to acquire the greater number of points, and (b) the manner of play, which decrees that they shall play simultaneously to each trick! This is achieved in practice by playing the cards face downwards and revealing them only when both players havemade their choices. The higher card wins each trick and equality of the cards played results in a division of the points for that trick. All cards played are subsequently available for inspection by either player.

Well that's it; and one of the most fascinating facts about the game is that any random strategy will lose to one which plays cards whose value is proportionate to the value of the trick to which it is played, whilst any deducible strategy is trivially bettered by a corresponding counter-strategy.

Blackmarkit

This is a new card game for four players. An ordinary pack is used and each player is dealt thirteen cards. Each player in turn, beginning with the player to the left of the dealer, plays a card face up in the centre. Each card played must be a different suit and rank to the cards previously played. If a player cannot go he receives a "black mark". and indicates this by showing one of his cards and placing it face down in front of him. It will be of the same rank or suit as one of the cards already played. The chance of getting a black mark is greater the later you play in the round. To counteract this the player of the last card leads to the next round, thus the turn to lead rotates in the opposite direction to the turn to play. The player with the fewest black marks wins the hand. Scores can be kept, or the game can be played with a kitty to which each player contributes a coin - the kitty accumulates and is won by the first player to go clear, i.e. survive a hand without getting a black mark.

Royal Carpet Patience

Our playing card page will regularly include a patience. This is an old favourite one. The Royal Carpet Patience (also called Elevens) is played by shuffling the usual pack and dealing out 12 cards in a face-up 3x4 array. Any picture cards (i.e. K, Q, J) are removed from the array to the bottom of the pack and their places filled from the top. This process is continued if necessary until there are no royal cards on show. The Cards are now played out onto the array two at a time, the two cards played covering two cards that add to 11 (i.e. 10 and Ace, 9 and 2, 8 and 3, 7 and 4 or 6 and 5). If a royal card appears, no other card can be placed on top of it. The patience is won if all the twelve court cards are displayed in a 3x4 "royal carpet". [The rules are very simple, so can any reader work out the accurate odds on getting this patience out?]

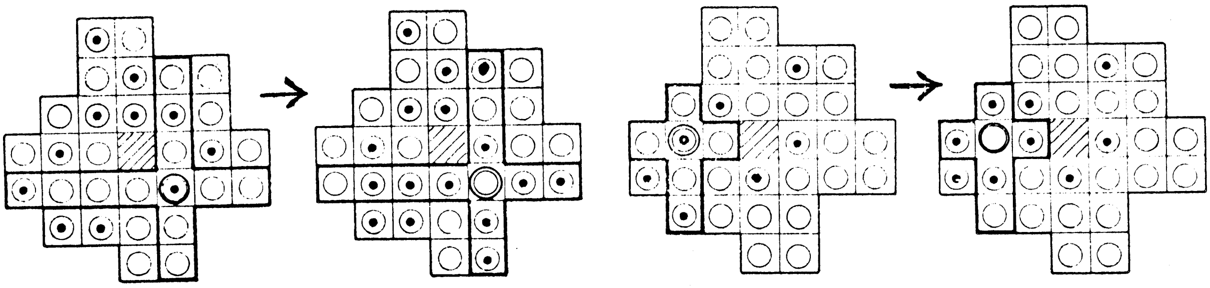
What is your favourite card game or patience? Write up a description of it so that others can try it as well. Your opinion of any of the games described would also be appreciated. Can you think of any improved variants? Can you provide some theoretical analysis?

Ringing the Changes

By Vladimir PRIBYLINEC

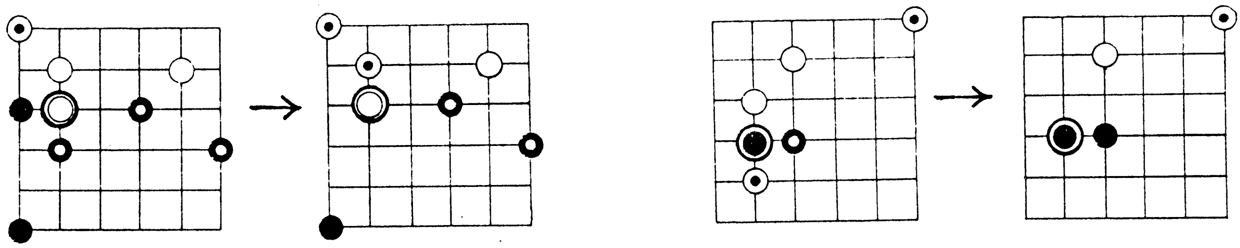
Two new games are described, for one and for two players respectively, each employing pieces that can be turned over (the white pieces have a black dot on the back and the black pieces a white dot) and a ring which when placed round a piece effects reversals or captures (in differing ways in the two games).

The Cross Game is for one player, using an irregular board with a hole, as illustrated. Initially the board is covered with pieces at random, some dot-up and some dot-down. One ring is used. Placing it round a piece results in all the pieces in the same rank and file (including the piece in the ring) being turned over. The object is to get rid of all the spots except one in the ring. Two example moves are illustrated:



The play can be speeded up by noting that (a) application of the ring at two points in the same rank results in the rank being unaltered while the two files are reversed, except where they cross the rank, and (b) applying the ring at four points at the corners of a rectangle of ranks and files results merely in changing the four ringed points, no others.

The Ring Game is for two players, using a 6x6 board. The players begin by alternately placing one piece at a time on the board - each has ten men. They may be placed either way up. Thereafter they are moved one piece at a time along a line to a free point. The only restriction is that it is not allowed to have four pieces of the same colour on four points forming a small square. On the twelfth move the players place their rings on the board. Thereafter a move can be by a piece not in the ring, or by the ring itself. The ring can be placed round any piece of your colour provided at least two pieces on the adjacent (rook-wise) points are the same way up. The piece in the ring is unaffected. Opposing pieces next to the ring are captured. Friendly pieces there are turned over. Two example moves are illustrated:



The aim is to capture the piece in the opponent's ring, or to leave him with no move. The game can also be drawn by agreement between the players.

Editor's Note: The author is looking for a manufacturer who would like to produce these (and other) games commercially. He has a puzzle called IRIS with 13 coloured dice currently manufactured in Czechoslovakia. Copyright of the games remains with the author. Any interested party should write to: Dr Vladimir Pribylinec, Marxova 460, 027 43 Nizna, Czechoslovakia.

CHESS SOLUTIONS

Chess Problems: Solutions and Comments

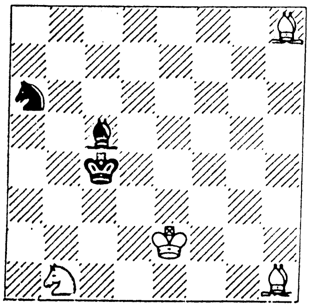
Chessics **126** McDOWELL. E. Albert reports an anticipation. See diagram below.
Chessics **184** SHANKAR RAM. The author points out that the Circe Equipollents rules do not apply to Kings. This invalidates the set play example I gave.
Chessics **198** SHANKAR RAM. Has a dual 2Kf8 after the key 1Bc3. Correction see below. In Pacific Retractors the sides play in opposition, not cooperation the author emphasises.

13. MIHALEK. 1Rg6 Ke3 2Re6 Rb5‡ and 1Kf4 Kd4 2Rg4 Rf7‡ Side[ways] echo. [R.Brain]. E.Albert reports an anticipation. See diagram below.
14. BOGDANOV & VLADIMIROV. 1Raxb5+ Ka3 2Ra5+ Sa4‡ and 1Rcxb5+ Kc3 2Rc5+ Sc4‡
15. VLADIMIROV & OSHEVNEV. 1Bb3 Rc1 2Bd1 c4‡ and 1Qb3 Ra2 2Qb2 a4‡
Nos 14 and 15 make a good pair. [N.Nettheim].
16. MIHALEK. 1Se7 Kd6 2Se4+ Ke5 3Bf6‡ and 1Bd3 Ke6 2Se4 Kf5 3Sc5‡ "House" shape, quasi-symmetry. [F.M.M.]. I found this the hardest of the first seven to solve. I didn't find much of a method for solving it - was reduced to trying many configurations. If I am right, this makes it more of a puzzle than a logical problem. But am I right? [N.N.]
17. GRUBERT. (a) 1Kd4 Kf3 2Rc4 Rg2 3c5 Rd2‡ (b) 1Qe6+ Kf4 2Kd5 Rg3 3Sc4 Rd3‡
Cooks solving both parts: 1Kd5 Rg7/8 2Qd4+ Kf5 3Qc4 Rd7/8‡ [A.W.Ingleton, and others].
18. GRUBERT. 1...Kc3 2Rf5 Kd2 3Kd4 Bd6 4Rd5 c3‡ and 1...c4+ Kc5 Kc3 3Rf6 Bc7 4rc6 d4‡
Cooks: 1Ka/b2 Kd4 2Kb/c1 Kc3 3Bg3 Rd4 4Be1‡ [T.G.Pollard and others].
19. McDOWELL. 1-4Pf1=R Pa8=R 5Rf5 Ra4‡ Copycat! [T.G.P.] Sly R-promotions insitu! [A.W.I.] Cute double R promotions [N.N.] R. Prom + Ruckkehr doubled [D.Nixon] Astonishingly the only way! [R.B.]
20. MIHALEK. 1Gg3 Ke4 2gh2 Se3 3Rf4‡ Overshadowed by No.23. [A.W.I.] Ideal mate.
21. GRUBERT. 1Rc5+ Kf4 2Rd5 Bb4 3Kd4 Se5‡ Duplex: 1Gc6 Ke3 2Sf6 Rg7 3Ge6 Rg5‡
Each pleasing, together quite remarkable! [R.B.]
22. MIHALEK. 1Be3 Ge2 2Kd4 Kd6 3Gd3 Pfxe3‡
23. MIHALEK. 1Gg8 Ke5 2Re7+ Kd6 3Gf7 Sd7 4Re6‡
Cook: 1Gg4 Ke4 2Kb3 Ke3 3Kc2 Se4 4Rf3‡ [R.B.]
24. MIHALEK. 1Rf2 Gf3 2Rg2 Gg4 3Rg3 Gh3 4Rf3 Ge2‡
Cooks: 1Rg3 Gg1 2Rg2 Gg/h3 3Rf2/g3 Gc/g4 4Rf3 Ge2‡ [T.G.P. and R.B.]

For solving ladder see page 47.
To allow more time for solving solutions will in future appear in the second issue following.

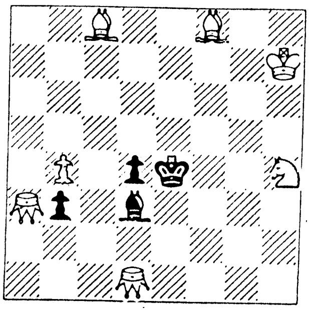
25. MIHALEK. 1Rb6 Sb7 2Ga7 Kd5 3Gc7 Sc5 4Rd6‡ A nice one. [D.N.]
26. MIHALEK. (a) 1Kb5 Gc6 2Kb6 Rb1+ 3Ka7 Gd5 4Ka8 Rb7‡ (b) 1Rf8 2Rf4 3Ge4 4Rg4 5Gf3 6Rg2‡ Imaginative pairing. [R.B.]
27. JELLISS. (a) 1Ge5 Gf1 2Ge1 Gf5 3Ge3 Gf3‡ (b) 1Gd2 Gg4 2Gg5 Gd1 3Ge3 Gf3‡
Severe move restrictions. [D.N.] Deux pas de deux. [R.B.].
28. JELLISS. (a) 1Rd6+ Kc7 2Rd5 Gc4 3Rb5 Kc8 4Ga5 Ga6‡ No one found this. The try 1Rc2 Gc7(illegal self-check) 2Gb1 Gc1 3Ra2 Kc8 4Ra1 Gxa1‡ (or 3Rh2 etc 4Rh1 etc). was too tempting. (b) 1Re6+ Kc8 2Gd7 Gd5 3Re8+ Gd8 4Rxd8+ Kc7‡ but Cooks: 1Rc4 Gf4 2Gc8 Gb4 3Rc7+ Kd6 4Rb7 Gb8‡ [A.W.I.] or 1Rb6 Gf4 2Gc8 Kd8 3Ge8 Kc7 4Rb5 Gb8‡ [G.P.J.] New and attractive fairy type. [D.N.] The idea of twinning by this method was in fact suggested to me by W.H.Reilly during the FCCC days. Sensitive Kings first appeared in France under the name of "Roi Bicolorè". Re-setting below.

126(A) Helmer TERNBLAD
Feenschach 1955
hm2 (with set)



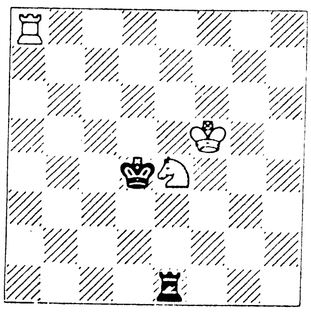
1...Bc6 2Sb4 Sd2‡
1Bb4 Sc3 2Sc5 Bd5‡

198(C) N.SHANKAR RAM
Mate in -2+1 in 4 ways
Pacific Retractor



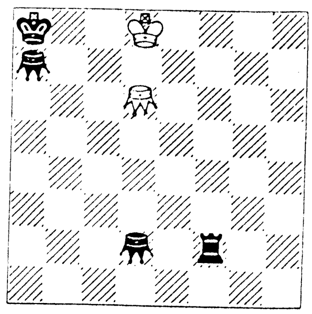
Keys 1Gc3/Gc5/Bd6/Bh6 with the same moves on the 2nd move also, foll. by Bf5‡

13(A) P.A.PETKOW
5445 Arbejder-Skak 1961
hm2 (two ways)



1Rc1 Kf4 2Rc4 Rd8‡
1Ke3 Ke5 2Re2 Ra3‡

28(C) G.P.JELLISS
hm3 (b) Sensitive Kings



(a) 1Rf4 Kc8 2Rb4 Ga3+ 3Ga5 Ga6‡
(b) 1Gd7+ Kc8 2Rf8+ Gd8 3Rxd8+ Kc7‡

CHESS VARIANTS

Parallel Time-Stream Chess

By C.M.B.TYLOR

This is an application to Chess of the science fiction idea of parallel universes. A player, before making his move, must first switch to a 'parallel time-stream' by changing one of his earlier moves. The rest of the moves in the game are then played, according to exactly defined rules, to follow as closely as possible those in the original 'time-stream'; the player then makes a normal move. (It is possible that the changes may result in the game ending in mate without this extra move being played.)

The rule for a 'parallel' move is that the piece moved is the one geometrically closest to the starting square of the original piece, and it is moved to the square geometrically closest to the original finishing square. If in either case there is more than one possibility, the piece furthest back is moved to the square furthest forward; if there is still more than one possibility, the piece furthest from the centre is moved to the square nearest to the centre. Thus if White has played 1Pd4, 2Pe4 and 3Qh5, and changes his second move to Qd3, his third move will (assuming that Black's moves do not interfere) become Bh6. This is because c1 and e1 are equally close to the starting square d1, and also equally far back, but c1 is further from the centre; g5 and h6 are equally close to the finishing square h5, but h6 (although further from the centre) is further forward. The change to 2Qd2 would lead to 3Kd1. Castling is taken as a King move; promotion is to the piece that the player had moved most recently (or, if only the King and Pawns had moved, to a Queen).

As a possible game, White opens 1Pa3 and Black replies ...Pa6. These opening moves do not really matter, as they must both be changed next time; in practice the game could be started at move 2. White changes 1Pa3 to Pe3, and plays 2Qh5. Black similarly changes 1...Pa6 to ...Pe5 and plays 2...Qf6. The best opening strategy I can think of is to go all out for Scholar's Mate! White changes 1Pe3 to Pe4, which does not affect the rest of the moves already made (...Pe5 2Qh5 Qf6) and plays 3Qxf7+. This is not really a sacrifice, as pieces captured need not stay captured.

Things now get interesting, and there are at least three traps that Black can fall into. (a) If Black changes 1...Pe5 to Pe6 and plays 3...Kxf7, White can change 3Qxf7+ into Qc5. Black's third move now becomes ...Kd8, and White can play 4Qxf8 mate. (b) If Black changes 2...Qf3 to Sf3, and again plays 3...Kxf7, White can change 3Qxf7+ into Qg5, and now after 3...Ke7 play 4Qxe5 mate. (c) If Black changes 2...Qf3 to Sh3, and plays 3...Sxh7, White can change 3Qxf7 into Bc4, and now after 3...Sg8 play 4Qxf7 mate.

Since a player needs to review the whole game before deciding on a change and a move, this chess variant is probably better played by correspondence than over the board. Four correspondence games between myself and R.M.W.Musson all lasted less than ten moves; in one, Black mated on move 6 after a mistake by White on move 7! As these games progressed, it emerged that it was actually possible to plan ahead to some extent, making a particular move because of the possible effects of changing it in a later parallel game.

Problem: What does White do after 1Pe4 Pb6 2Bc4 Bb7 3Qf3 Pe6 4Sc3 Pf5?

Caissa's Kaleidoscope

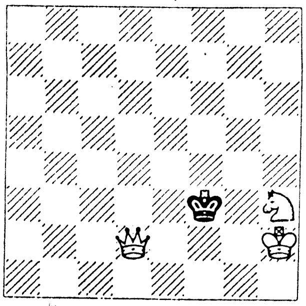
Notes on the original problems. **30** and **31** were sent by Theodor Steudel, describing them as "2 examples of the so-called 'half-home-base' problems, by my friend the Count of Whitefield, who is living most of his time in my Bavarian home. (Nevertheless by the way he shouts twice a day 'I remain British ...!') "

Serieshelpstalemate (Shp) see issue 1. Grasshopper (**42** and **43**) and Helpmate (hm) see issue 2. Problems **29** and **41** are Direct mates. The board in **41** can be any size you like! N is the number of squares along a side of the board. Problems **39** and **40** are Direct stalemates. In Reflex mates (**30**) White moves first and tries to force Black to checkmate. Either player must mate on the move if able. The Moose c2 in **47** can hop to the squares a5,b6,c6,d4,e3, over the three hurdles. The Eagle has a similar choice of two destination squares - the squares to right or left of the hurdle. The twinning in **33** is: (a) as diagram, (b)e1-d4, (c)e1-g3, (d)d5-d3, (e)b7-b3, (f)b7-b6, (g)b7-e7, (h)b7-c7, (i)b7-f6.

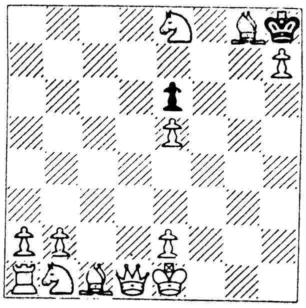
Further problems are invited - particularly Circce compositions and Leapers.

CHESS PROBLEMS

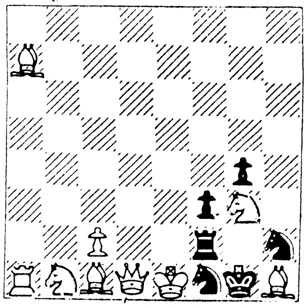
29. F.M.MIHALEK
m3, (b)h2-h5, (c)h3-b1



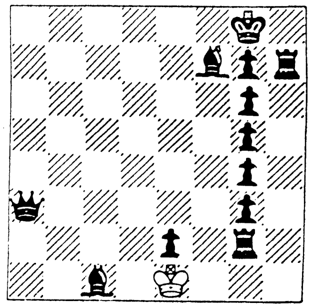
30. & **31.** The Count of WHITEFIELD
Reflex mate in 5



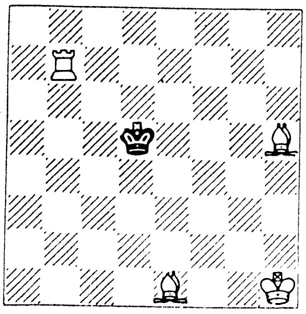
Shp 9



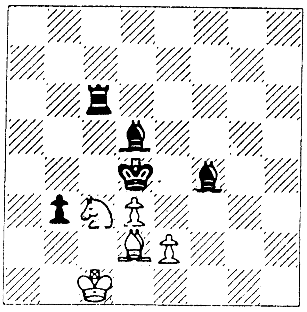
32. I.SHANAHAN
Shp 19 "Sword & Shield"



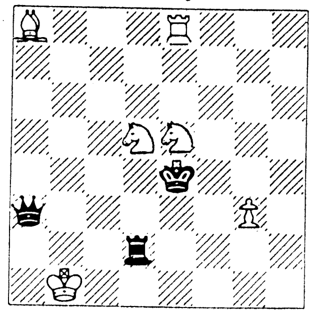
33. E.HOLLADAY
hm1½ (b) - (i) see text



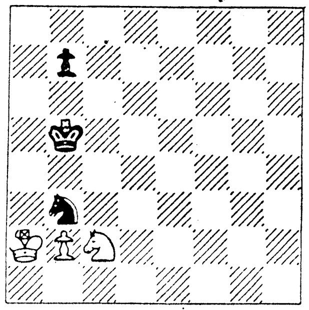
34. S.J.G.TAYLOR
hm2 (two ways)



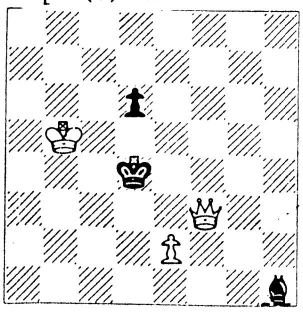
35. S.J.G.TAYLOR
hm2 (two ways)



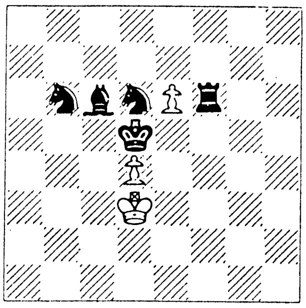
36. E.HOLLADAY
hm3½ (two ways)



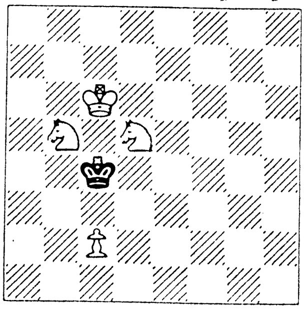
37. F.M.MIHALEK
hp2 (b) h1 - h8



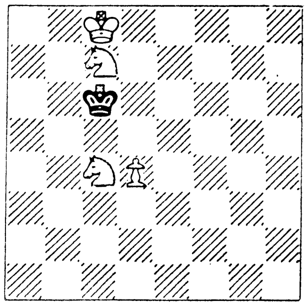
38. F.M.MIHALEK
hp3



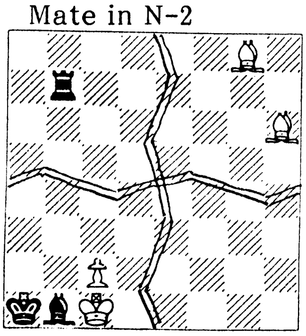
39. E.HOLLADAY
(a) p3 (b) 90° right p4



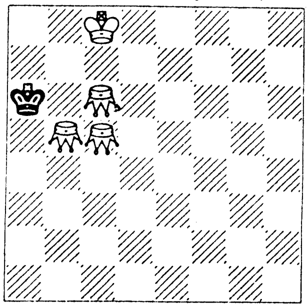
40. E.HOLLADAY
p4 (b) d4 - b4



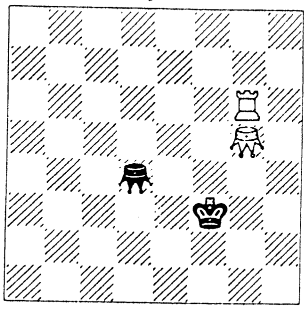
41. Peter WONG
Mate in N-2



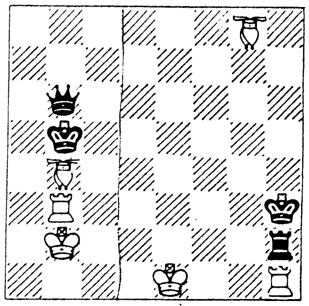
42. C.VAUGHAN
hm3 (how many ways?)



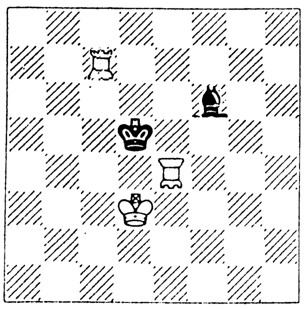
43. A.W.INGLETON
hm4(2 ways) No WK



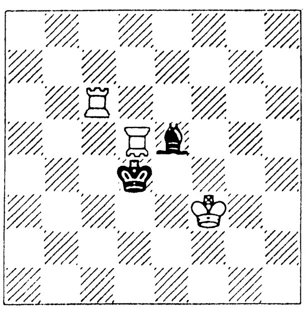
44. G.P.JELLISS
hm2 Eagle hm2½



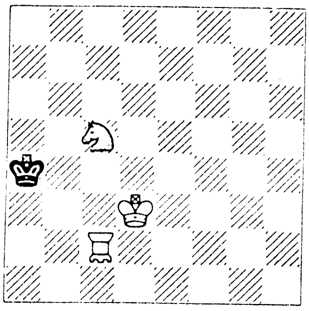
45. G.P.JELLISS
hm 2½ (2 ways) Moose



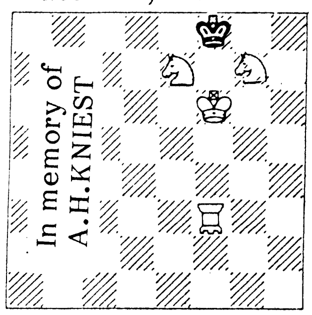
46. G.P.JELLISS
hm 2½ (2 ways) Moose



47. G.P.JELLISS
hm3 Moose, (b)c5=B



48. G.P.JELLISS
Mate in 3, Moose



Solutions to reach me by 1st May

TRANSITIONS

Sliding Block Puzzles

Problem 3: Solution in three loops of lengths 6,6,4 or 4,6,6 giving 16 moves in all, 4 ways:

From

5	0	1
4	3	2

via

1	0	2
5	4	3

2	0	3
1	5	4

or

5	0	3
4	2	1

4	0	5
2	1	3

or

3	0	1
5	4	2

1	0	2
3	5	4

or

4	0	5
3	2	1

3	0	4
2	1	5

to

2	0	4
1	3	5

Problem 4: A solution in 35 moves is: 34,5875,465784,657846,531265,8738,265472,4 but this is probably much longer than necessary. It transforms

0	1	2
3	4	5
6	7	8

to

1	6	5
8	4	0
3	2	7

Problem 5: Held over pending further work (i.e. I haven't solved it yet!).

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

A

1	12	7	14
8	13	2	11
10	3	16	5
15	6	9	4

A'

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	0

B

5	2	8	15
9	14	4	3
6	1	11	12
10	13	7	0

B'

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	0

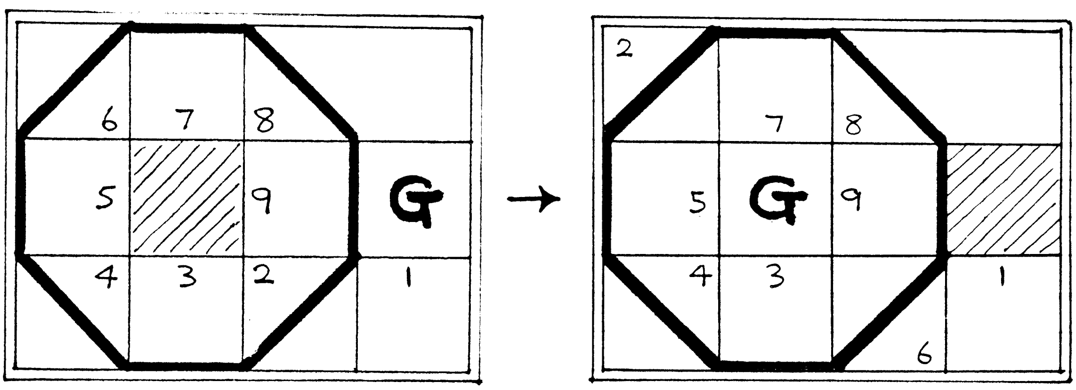
C

1	10	15	4
13	6	3	8
2	9	12	7
14	5	0	11

C'

No solution to Sam Loyd's magic square problem, mentioned last time, was given in Tit-Bits at the time, but Robert B.Ely III (ref 4) states that the magic square reached in fewest moves from the normal array **A** by removing 16 and replacing it after the moves is **A'**, but he does not say how many moves. Loyd gave the problem of transforming **B**, the 14-15 Puzzle start, to a magic square, the blank counting zero, in 1908 (ref 5). He gave a solution in 50 moves, but Sliding Piece Puzzles gives one to **B'** in only 36. The similar problem starting from **C** is given by H.E.Dudeney (ref 6) with a solution in 37 moves, which has since been improved to 35 to position **C'**. Edward Hordern writes in a letter that these improved results were found by Len Gordon of Chico, California, with the aid of a computer.

References (continued): (4) Journal of Recreational Mathematics, January 1968, Vol 1, No 1, p4. (5) Our Puzzle Magazine, July 1908, Vol 2, No 1. (6) Amusements in Mathematics, 1917, problem 403.



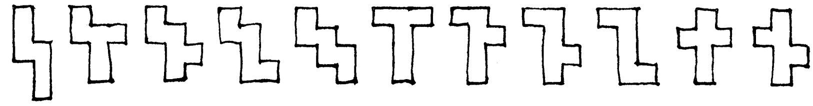
Problem 6: Get My Goat. T.W.MARLOW writes: "The excellent book Sliding Piece Puzzles by L.E.Hordern includes the "Get My Goat" puzzle diagrammed above [it is **C2**, p69 in the book, and dates back to 1914, when it was patented by J.I.Wiley]. This calls for moving the goat, G, inside the fence, by using the empty space to slide the pieces. It solves in 28 moves as follows: 56756 43296 296G1 69342 5725G 961.

Hordern refers to the apparent impossibility and the simple unexpected solution. This of course relates to the fact that pieces 2 and 6 become interchanged in the solution. However, the form of words caused me to look for a solution even more unexpected. As a consequence I managed to solve in 24 moves in a way which I think is still within the requirements of the wording of the puzzle. Can you find this solution?"

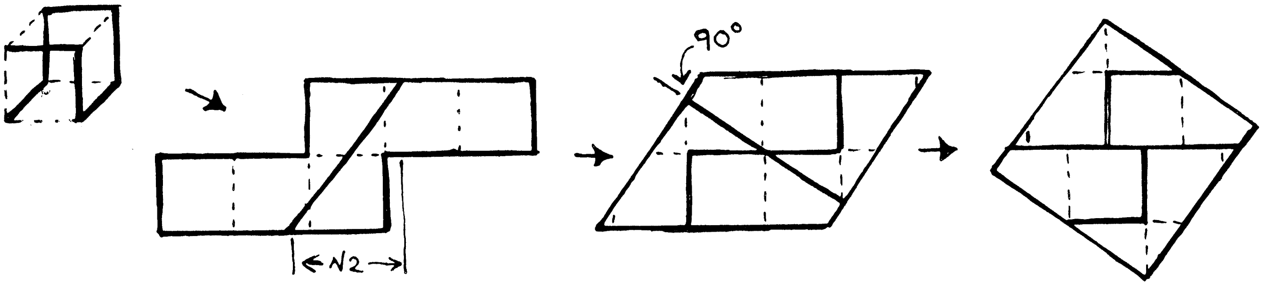
Squaring the Cube

Of course, it is simple to cut a cube into N^2 layers of equal thickness and arrange them to form an NxN square array - but the result is not really a square, but a prism on a square base. Anyway, we could take N=1, since the cube is a square prism already.

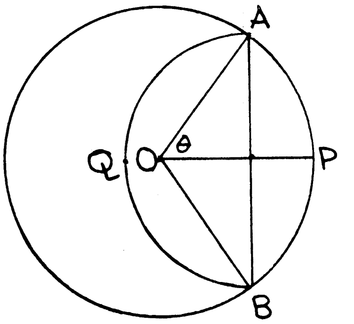
The riddle is answered by using a hollow cube, cutting it along sufficient edges to allow it to be folded flat, and then transforming the shape thus obtained into a square. There are 11 ways of flattening a cube [Stephen Ainley, Mathematical Puzzles, 1977, p65] as follows:



The first of these shapes is easily transformed to a square in four pieces, by well-known methods [e.g. Harry Lindgren, Geometric Dissections, 1963] as illustrated:



Circular Saws



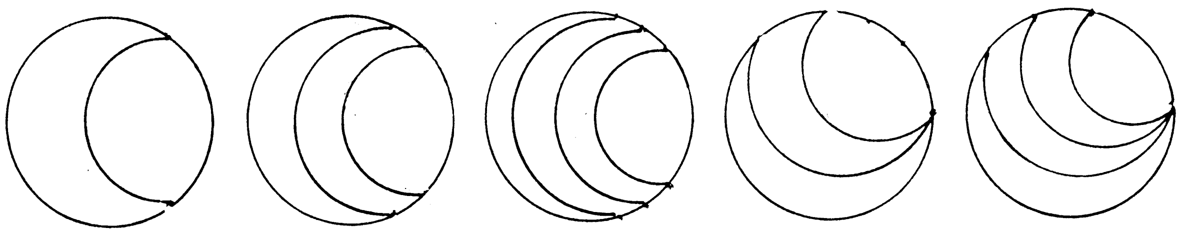
A single semi-circular cut can be used to divide a circle into two differently shaped parts (a crescent and a "digon") as illustrated. If the circle is of radius r then the area of the digon is: $r^2[\theta - \cos\theta \sin\theta + \frac{1}{2}\pi(\sin\theta)^2]$, where θ is in radians (degrees times $\pi/180$). In this formula θr^2 is the area of the sector OAPB, $(r\cos\theta)(r\sin\theta)$ is the area of the triangle OAB (so that their difference is the area of the segment APB) and $\frac{1}{2}\pi(r\sin\theta)^2$ is the area of the semicircle AQB.

By selecting appropriate values for θ the area of the digon can be made a simple fraction of that of the circle, as in the following Table and Figures.

Table of values of θ to give digon area as simple fraction of circle area. 31.218+ means the value lies between 31.218 and 31.219. Values found by step by step approximation, using a 'scientific' calculator.

area:	1/6	1/5	1/4	1/3	2/5	1/2
θ:	31.218+	34.200+	38.298+	44.472+	49.039+	55.541+

area:	3/5	2/3	3/4	4/5	5/6
θ:	61.655+	66.062+	71.427+	74.756+	77.043+



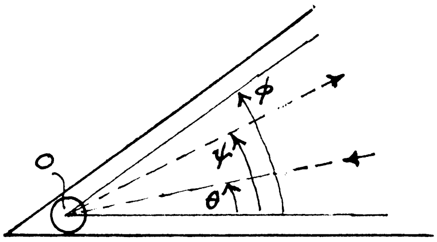
The diagrams are only intended as a guide and are not very precisely drawn.

Dissections continued on p43.

GEOMETRY

Triangular Billiards

Problem 3, the question of what happens when a ball is cued directly into a corner without a pocket so as to hit both sides of the angle simultaneously has now been answered satisfactorily by Professor A.G.MACKIE, who summarises his main results as follows:



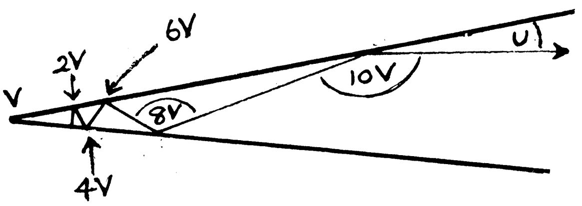
"The notation in the diagram is self-explanatory. We assume $0 < \phi \leq 90°$. When $\phi = 90°$ we have $\psi = \theta$ as is well known. But more generally we can show that $\psi = \theta$ when $\phi = 2\tan^{-1}(m^{-1})$ for $m = 1,3,5,\ldots$ i.e. for $\phi = 90°, 36.87°, 22.62°,\ldots$ In addition $\psi = \phi - \theta$ when $\phi = 2\tan^{-1}(m^{-1})$ for $m = 2,4,\ldots$ i.e. for $\phi = 53.13°, 28.07°, \ldots$.

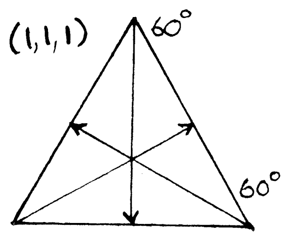
"The theory assumes flexible cushions and linear restoring forces proportional to the magnitude of the indentation on a given cushion and perpendicular to that cushion. The centre of the ball then acts like a particle subject either to two restoring forces at an angle ϕ to each other, or to just one such force, for the periods during which both cushions, or just one cushion, are respectively indented. The results I have quoted correspond to the rather special cases when two forces act until the centre of the ball is again at O after which no forces act. These apply for the given values of ϕ and for all θ in $0 \leq \theta \leq \phi$. There is another more obvious special case of this which holds for any ϕ but only when $\theta = \phi/2$. For all other cases there is a period when two forces act followed by a period when just one force acts. I have a computer program with graphics which can cope with any case.

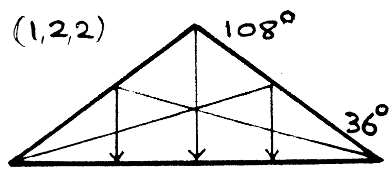
"Although the analysis is reasonably straightforward, it is really quite interesting and I am grateful to you for bringing it to my attention."

Fuller details will probably appear in a more technical journal.

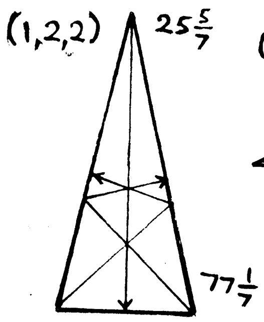
Problem 4. If a ball is projected into a corner of angle V at an angle U ($\leq$V) to one of the sides so as to hit the near-side first, under what conditions will the ball return along the same line? Obviously the ball must hit one of the sides perpendicularly. The general result is V=(90°-U)/n with 2n+1 bounces and n+1 points of impact. Thus U=90°-nV. The angles between the paths of the ball on its way out are even multiples of V.

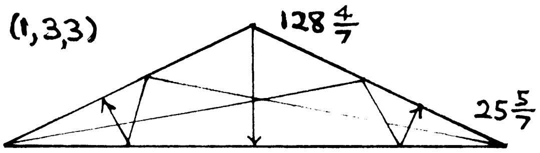


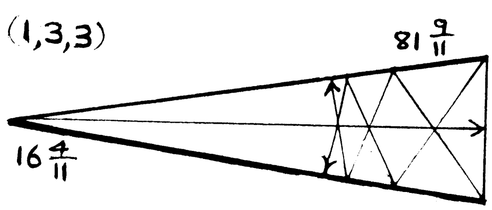




Problem 5. On a regular triangular table a ball cued from a corner to bisect the angle there will hit the opposite side at right angles and come straight back. This is the obvious case which can be denoted (1,1,1) meaning one bounce return from each corner. In the case of return after a single bounce from one corner the triangle must be isosceles. By symmetry the number of bounces taken by a returning ball at either of the other angles must be the same. All cases (1,n,n) are possible. The base and apex angles must be in the ratio 1 to 2n-1 (or vice versa). Hence if V is the apex angle we require V/2 +V/(2n-1) = 90° or else V/2 + (2n-1)V = 90°. Hence, either V = 180[(2n-1)/(2n+1)] or V = 180/(4n-1). The cases (1,2,2) and (1,3,3) are illustrated. The scalene cases are saved for next time.







Star Points: Solution postponed to next issue for lack of space.

NUMBERS

Cryptarithms

These are the solutions to the T.H.WILLCOCKS cryptarithms in the last issue.

(**4**) RACHEL x 2 = ALISON
378145 x 2 = 756290

(**5**) ALISON x 2 = RACHEL
365148 x 2 = 730296

(**6**) ALISON x 3 = RACHEL
243168 x 3 = 729504

I hope these haven't caused two much trouble between (the real-life) Alison and Rachel!

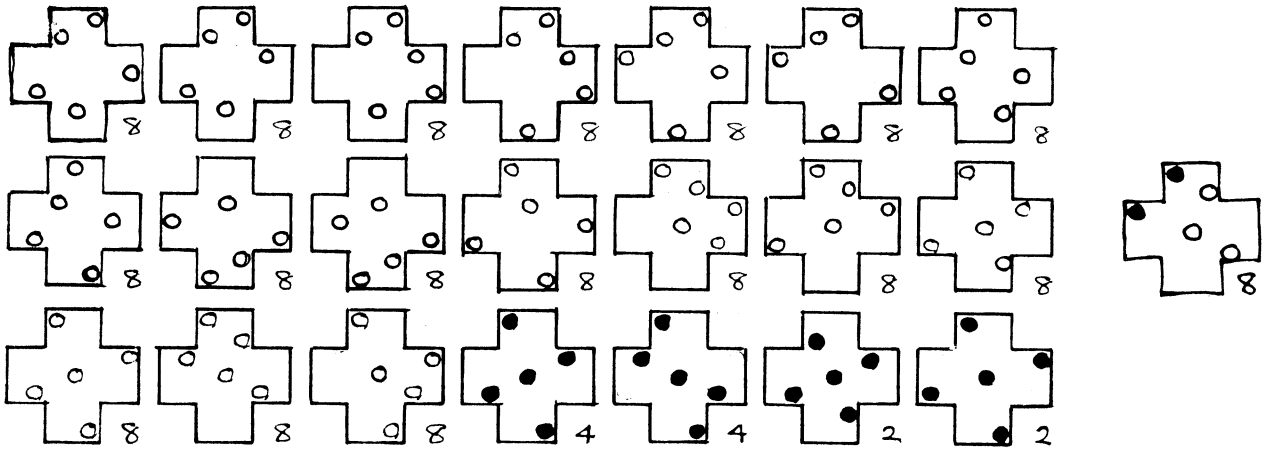
(**7**), Stephen TAYLOR offers the following cryptarithm, which he says is "just one of those things which get handed down through the family!" Can anyone identify a source for it? Some similar problems, but with only three terms in the addition, are given in Madachy's Mathematical Recreations (Dover 1979, originally published as Mathematics on Vacation, 1966) and also in Mathematical Diversions (Van Nostrand 1963, Dover 1975) by J.A.H.Hunter and J.S.Madachy. Most of the examples these authors give also require certain extraneous conditions to be satisfied (e.g. that THREE be divisible by 3, or that NINE be a perfect square). The only unconditional examples they give are (**8**) and (**9**):

(**7**). ELEVEN+THREE+THREE+ONE+ONE+ONE = TWENTY
(**8**). ONE+TWO+FIVE = EIGHT
(**9**). TWO+THREE+SEVEN = TWELVE

In his Christmas and New Year greeting this time T.H.WILLCOCKS poses the following cryptarithmic multiplication: Problem (**10**): AxGOLD = SMITH. A is one of the digits 2,3,...,9 and G,O,L,D,S,M,I,T,H are the digits 1,2,...,9 in some order. Show, without listing the solutions, that if one of G,O,L,D represents 9, the number of solutions is even. He also asks if anyone knows of any other general results relating to cryptarithms.

Enumerations

(**2**). My method of enumeration of the 5 Queens in unguard on the solitaire board depends on noticing that it is not possible to have two in the central 3x3. So we consider in turn positions with none and then with one in this area. There are 3 places the one can go: corner, edge or centre of the area. We thus find 21 patterns as shown below (or 22 with the special case with two Qs separated by an inward-pointing corner). Below each diagram is the number of different orientations of the pattern. These total 148 (or 156 including the anomalous case - Dawson claimed 152).



Enumerations and cryptarithms will in future appear in the general puzzle section.

Dissections (continued)

Polyhexes

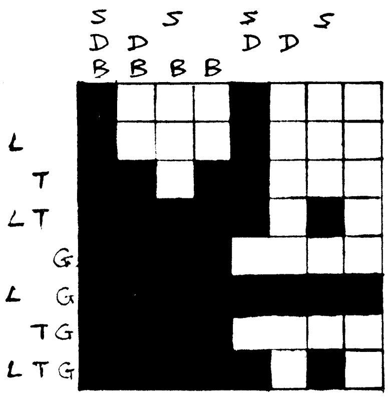
The clouds in Sopwith (see p 34) are polyhexes - that is "polyominoes" formed with hexagons instead of squares. All cases 1 to 4 are illustrated on p 34. How many are there of 5 and of 6 squares? With these we can pose similar problems to those considered for square polyominoes (as detailed in Chessics 28). For example: Arrange the 3 pieces of 3 hexagons and the 7 pieces of 4 hexagons to form a hexagonal array of side 4. The two asymmetric 4-pieces (shown shaded in the figure on p 34) may be placed either way up (i.e. we are using two-sided polyhexes). There are many other possibilities to try.

ANSWERS

Puzzle Questions and Answers

I have given up trying to classify these questions into categories such as Logic or Algebra, so will now simply call them puzzle questions, and these can be of any type that do not fit into the pages of special topics such as Transitions and Dissections.

3. Logical Grocers. The solution as given by the composer H.A.Adamson (1871-1941) who was "a powerful mathematician, third wrangler of his year" is couched in terms of Boolean style logical formulas. But we can translate it into easier visual form by using (unexpectedly but not inappropriately) a chessboard.

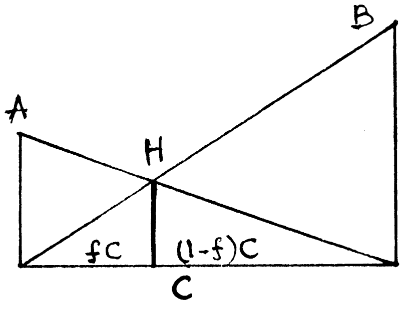


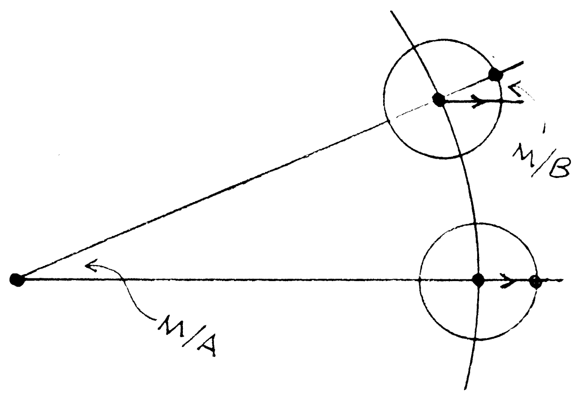
The 64 squares represent the whole of humanity. The first four ranks represent grocers, the odd ranks long-nosed persons, ranks 1,2,5,6 teetotalers. The first four files represent the brave, the odd files smokers, files a,b,e,f the dark-haired. The five conditions given now enable us to black in various squares to indicate that they are null (e.g. the fact that all long-nosed grocers are tee-total enables us to black in the whole of the third rank). The result is the pattern shown.

Clive Palmer comments that this type of diagram is known in the trade as a Karneugh Map - basically a Venn diagram for large numbers of sets.

The conclusion is that all grocers are non-brave (as indicated by the fact that the first quarter is entirely blacked in). Why grocers should have been chosen as the victims of this little joke rather than chess-players (say) might perhaps have something to do with food shortages at the time (during the first world war). The fourth condition can be seen to be totally irrelevant - it is there just to befog the issue.

6. Cross-Point. If the point directly below the crosspoint is a fraction f of the way along the base-line and H is the required height then H/fC=B/C and H/(1-f)C=A/C. (These equalities are consequences of similarity of triangles.) Thus f=H/B and 1-f=H/A. Thus 1-H/B =H/A from which we find the required formula: H = AB/(A+B). The height of the crosspoint is the same regardless of the distance C between the poles. I discovered this result for myself, but subsequently came upon the identical puzzle in L'Echiquier , 1926. Can anyone provide an earlier reference for it? The distance apart of the poles, C, is irrelevant.





7. Lunar Calendar. What we mean by a "month" normally is the time-span between two occurences of the same phase of the Moon, and the phases depend on both the Sun and Moon. At full-moon Sun, Earth and Moon are in line. If a month is M days then during this time the Earth has orbited through an angle of M/A of a cycle about the Sun, and the Moon has orbited through an angle of M/B about the Earth. As can be seen from the diagram, we require these two angles to be equal, or to differ by an exact number of cycles, that is M/B - M/A = n. If n=0 then A=B and we have perpetual full-moon, which we know is not the case. If $n \neq 0$ then n/M = 1/B - 1/A, that is M = n/(1/B - 1/A) which means M = AB/(A-B) when n=1. According to the only references to hand at present I find that A=365.25 days and B=27.32 days, whence M=29.53 which gives 12.37 months in a year, which seems about right.

8. Exchangeable Operations. The best that can be done with the inverse pairs of operations +,- and x,/ are the obvious cases: $s \boldsymbol{+} 0 = s \boldsymbol{-} 0$ and $s \boldsymbol{\times} (\pm 1) = s \boldsymbol{/} (\pm 1)$. The other pairs formed from these four operations give more interesting results:

$$s/(s-1) \boldsymbol{+} s = s/(s-1) \boldsymbol{\times} s \qquad s^2/(1-s) \boldsymbol{+} s = s^2/(1-s) \boldsymbol{/} s$$

$$s/(1-s) \boldsymbol{-} s = s/(1-s) \boldsymbol{\times} s \qquad s^2/(s-1) \boldsymbol{-} s = s^2/(s-1) \boldsymbol{/} s$$

In these identities of course any expression can be substituted for s (e.g. put s = t+1).

9. A Singular Riddle. The answer, in verse, is:

A man with one eye two plums must have seen,
One perfectly ripe, the other quite green.
The former he took, and ate it with pleasure,
The other he left to ripen at leisure.

I found this riddle and its solution in The Twentieth Century Standard Puzzle Book, 1907, a collection of puzzles from the London Evening Standard, edited by A.C.Pearson, but it is undoubtedly older. Clive Palmer offers an alternative, perhaps even better, solution:

Old Father Time was the Man in the Rhyme
The Plums turned to Dates with the passage of Time.

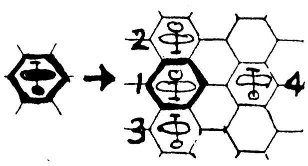
10. Domino Patterns. With a double-4 set of dominoes we cannot form quadrilles of either the French or English varieties (described in issue 1) since there are 15 tiles with 30 faces, and 30 is not divisible by 4. Can we perhaps put them into other groupings? The same can be asked of the double-5 set, which has 21 tiles. Here are two to construct:
(a) Arrange the double-4 set to form a rectangle showing ten rows of three faces alike.
(b) Arrange the double-5 set to form a rectangle 6x7 consisting of 6 squares of four faces alike (i.e. quadrilles) contained within a border of 6 rows of three faces alike, as in (a).
Readers are invited to try other arrangements - let me know your results for publication.

11. Multiplication Table. Consider the multiplication table for the natural numbers. The number 1 is the only one that occurs once in the (body of the) table. The prime numbers 2,3,5,7,11,13,17,... are those that each occur exactly twice in the table. What numbers occur three times in the table? What numbers occur four times? More generally (and much more difficult to answer): What is the first number to occur N times in the Table? This question leads to some interesting, if well known results in number theory.

x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

12. Integral Part Formula. There are some functions that are considered to be inexpressible by means of a formula. One such is the integral part of a real number, inp x (i.e. the largest whole number less than or equal to x). But with a little ingenuity this can in fact be formulated in special cases. The puzzle is to find a formula for inp (n/2) where n is a positive whole number. You may use addition, multiplication, subtraction, division and raising to powers.

13. The Four Rs. Another well known formula-forming puzzle is to express as many numbers as possible in terms of four fours, or some other set of digits. Rather than using fixed digits it is more instructive to try the problem with four Rs, each R representing the same digit, but its actual value not specified. All the usual notations applicable to digits may be used (e.g. positional notation, decimal points, recurrence dots). The puzzle was posed in this form by T.R.Dawson as long ago as 1916.



14. Sopwith Manoeuvres. The moves of the aircraft in Sopwith are explained on p 34. Some key manoeuvres are indicated in the diagram. You are required to get your plane, shown black, into each of the other positions shown in fewest moves, without of course using the Immelman turn (which would solve case 2 in 1). A secondary problem is to say how many ways there are of doing each manoeuvre. The reversal 1 is the key to the others.

Cryptic Clueing

By QUERCULUS

Several correspondents, expert in other puzzle domains, have expressed bafflement by our relatively unsophisticated cryptic crosswords. Here therefore is a quick guide to some of the more common types of clue encountered, illustrated by examples from the two previous issues of the Journal. Almost all cryptic clues consist of two parts - one a straight definition, the other a less direct indication.

Overclue. Sometimes it is possible to give several different definitions of one word, and these may be combined in one cryptic clue, so that the word is 'overclued'. e.g.: "For all practical purposes unreal" = VIRTUAL, incorporating two definitions, or: "Family initiate pop artists in medical class" = BLOOD GROUP, with three definitions.

Misdirection. This is the art of the conjuror. He seeks to make you look at or expect one thing so that you do not see something else. It can also be used in clueing, in various ways. e.g: "This sea becomes most wan" = ASHIEST (anagram of THIS SEA not of MOST WAN). The words used in a clue will often have two or more alternative interpretations, and the clue will be phrased so as to suggest one of the meanings when the other is intended. e.g. "medical class" in the 'overclue' example.

Overlap. It is often possible to economise on words by allowing the two parts of the clue to overlap, so that a word or phrase or part-word serves for both parts. Some times one part may be completely absorbed in the other, or both may be the same.

Puns. Occasionally the sound of a word, or its syllables, is considered instead of its letters. e.g: "You sound false and hesitant on this instrument" = EU/PHONI/UM.

Anagram. The disordering of the letters of one word or phrase to form another meaningful expression. In the early days of the crossword clues were sometimes given in the form of a direct anagram. e.g: "Cart horse (anag.)" = ORCHESTRA. In these more sophisticated times the existence of an anagram is usually hinted at less directly. e.g: "Be slow to make arm bends" = ELBOWS.

Reversal. It is debatable whether the reversal of a word is an anagram or not. For the classification of clues it is clearer to separate the two concepts. e.g. "Vehicle backing into police HQ" = DRAY (reversal of YARD).

Burial. A popular basic form of cryptic clue is to hide the word to be found among the letters of the clueing sentence. e.g. "To paint R in sickly hue shows inherent sense" = INTRINSIC, or "No M in alphabet begins to appear unrealistic" = NOMINAL (a burial with the head still sticking out of the ground!).

Partition. A word can sometimes be split up into two or more separate words, usually unconnected with its meaning as a whole, the cryptic part of the clue can then define these parts. e.g. "Dance from the laugh prohibition age" = HA/BAN/ERA. When this process is taken further the word may be split up into its very letters and the individual letters clued. This can however be taken too far and become a bad habit in my view - it is the lazy compiler's way of dealing with a word he cannot find a decent clue for.

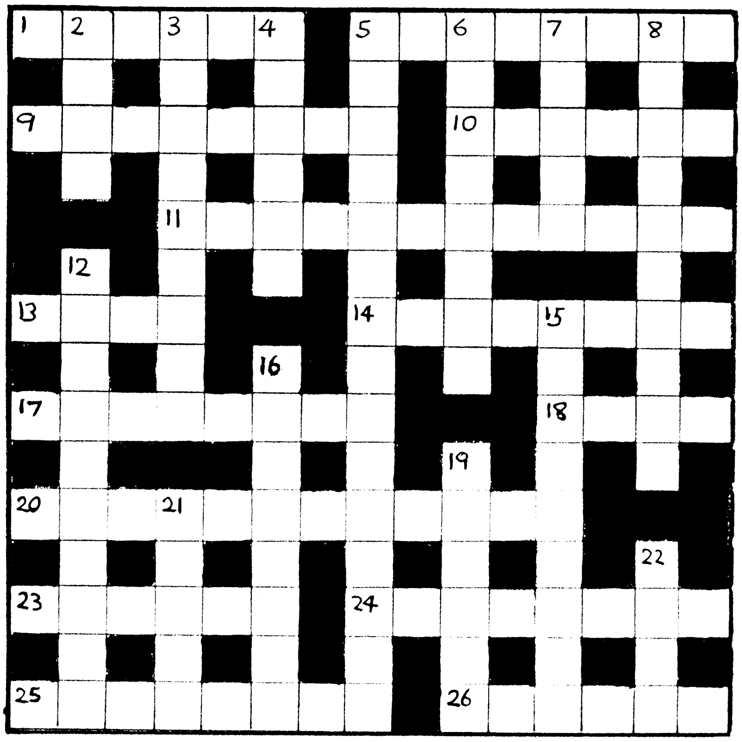
Dissection. Cutting the heart out of a word may leave another word, and these two 'components' can then be clued separately as for a partition. e.g. "Dance of an Egyptian in the desert" = S(ARAB)AND.

Unpunctuality. The punctuation in a cryptic clue is all part of the fun and is usually intended to mislead. e.g: Omission of inverted commas: "Land in America" = REALTY [Rufus, The Guardian, 7 July 1986]. Other devices are omission of capitals, or insertion of misleading ones; omission or insertion of spaces. Of course the mispunctuation should not be random or wholesale - it must enhance the clue.

These are only a few of the more common possibilities . Probably I have omitted some other important basic types. Many clues combine two or more of these basic methods. e.g. Reversal/Partition: "Builder gives back all different negatives" = STONEMASON (NO SAME NOTS reversed). Reversal/Dissection: "It's backward in mime to get out of step" = MI(STI)ME. Anagram/Dissection: "Sport brought to boot in autumn?" = F(OOTB)ALL, "Dance about Kew showing a lack of order" = CA(KEW)ALK. Dissection/Partition: "Dance to make learner deceiver gad about" = GA(L/LIAR)D.

If you didn't have a clue before, you should now be able to tackle the 27 opposite!

Cryptic Crossword 3. By Querculus



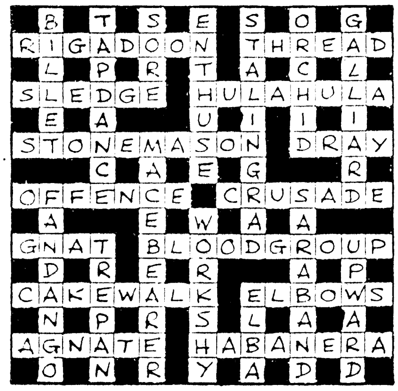
ACROSS
01. A sporran sometimes may contain payment. (6)
05. Wisdom of Troy? (8)
09. Farmer's preparation for a cautious swim? (5-3)
10. A long nib best writes in backward motion. (6)
11. Article on mineral twitch by short Scot professor. (12)
13. Passed the old way. (4)
14. One by-passes so rich a man in estimating tax. (8)
17. It's not our acouchy inside, it's the plantain eater. (8)
18. Divided by two. (4)
20. They have the appearance of the king of the jungle for miles around. (12)
23. Sing-song sung a la tub? (6)
24. Undeveloped isle none took in. (8)
25. Dwelling of orientals under canvas. (8)
26. The General, taken in by the Agent, names the Dwarf. (6)

DOWN
02. Each twinge is painful. (4)
03. Tailless cat spotter? (9)
04. More wild veg dye. (6)
05. Great birdman is one who believes in angels? (15)
06. Insurpassable State Reg. Orderly. (8)
07. Of the third power block. (5)
08. All but brown horse with little thanks. (10)
12. Make legation be open for further discussion. (10)
15. Rock, the beach sound. (9)
16. Kit hundred out in acre. (8)
19. Please observe shortened version by the sound of it. (6)
21. Rent out easel. (5)
22. Reaching the heights before the capital of Quebec. (4)

Crossword puzzle patterns are an interesting subject in themselves. The pattern this time is one that has been popular for a long time in The Guardian.

The spelling of one answer this time has one more letter than the Concise Oxford or Chambers' 20th Century Dictionaries allow. Querculus found it in the 1957 Guide to the London Zoo!

Crossword 2 SOLUTION



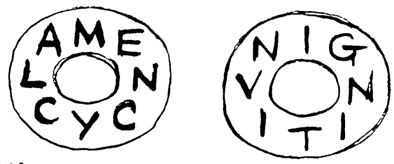
Themeword: DANCE

Change-Chains

ALICE QUEEN SNARK
ANILE QUERN SNAKE
ULNAE RUNES LAKES
QUEAN SNARE SCALE
QUEEN SNARK ALICE

Lifebelts

Identify the name of the ship!
Which letter should be changed to give a different name?
(When read from a different point).



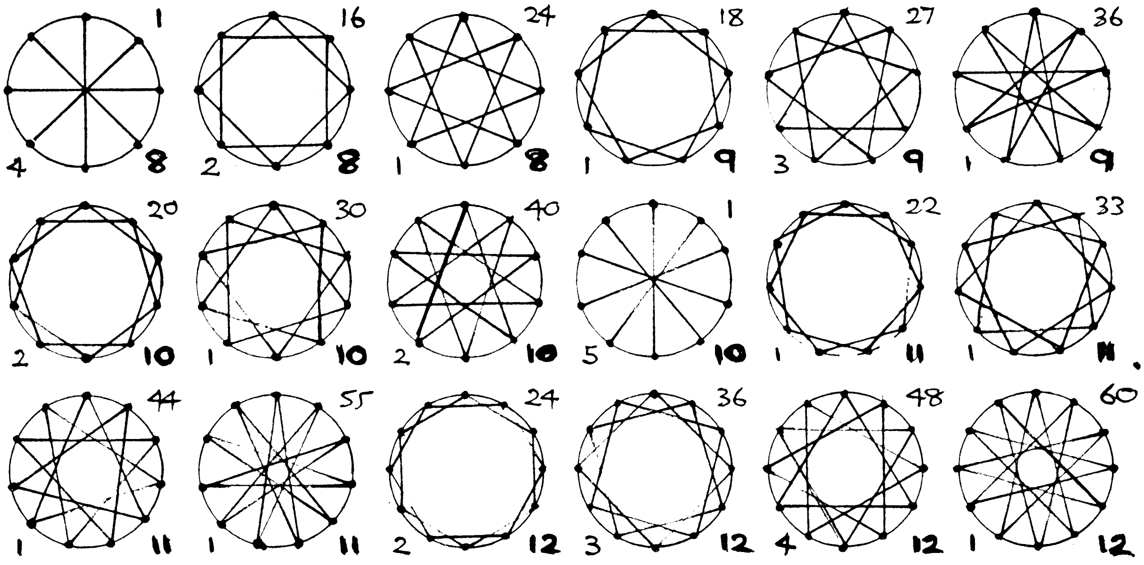
Other examples wanted.

Chess Problems
Solving Ladder

R.Brain	39
A.W.Ingleton	39
T.G.Pollard	35
D.Nixon	30
N.Nettheim	18
R.W.Smook	13
T.H.Willcocks	11

Stars & Asterisks

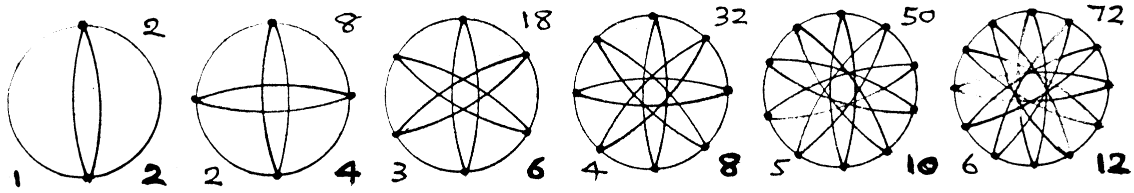
The condition for a star to consist of a single circuit is hcf(m,n) = 1 (hcf = highest common factor) [Coxeter, Introduction to Geometry, p36]. The number of points of intersection within and on the circle is mn (m circles of n) in all cases (single or multiple circuit) except for asterisks. When m=n/2 all the lines pass through the centre. If the digons in the asterisk are drawn as ellipses (or pairs of circular arcs) the number of intersections becomes mn again. Illustrated are all the stars for n from 8 to 12, except the 12-point asterisk.



Illustrated are the asterisks of 2 to 12 points with digons drawn as two arcs. It is interesting that the numbers 2,8,18,32 are the numbers of chemical elements in the successive periods of the atomic number sequence ending at the inert gases:

$(H\ He)_2\ (Li - Ne)_8\ (Na - Ar)_8\ (K - Kr)_{18}\ (Rb - Xe)_{18}\ (Cs - Rn)_{32}$

So perhaps calling these patterns "atoms" is not so fanciful after all!



Mathematical Art

Ian Shanahan writes in connection with page 16 (issue 1): "This page was particularly interesting, due to my work in computer music, a perfect wedding of science and art. 'Mathematical Art' is absolutely not a contradiction in terms. In fact most higher mathematics is artistic, as it regularly reveals intuition and 'elegant solutions'. Furthermore, all art, in some way, represents some mathematical truth. e.g. on my Yamaha CX5M music computer I've synthesized some stunningly beautiful musical sounds using the equations for frequency modulation (FM)"..."Also, visual art (and chess problems) express truth in Geometry (see T.R.Dawson's Ultimate Themes)."

The Pair Tree "is an example of a 'Fractal' . There is a recent branch of geometry relating to such strange and beautiful beasties called, predictably, 'Fractal Geometry' ...There is a wonderful book ...called The Fractal Geometry of Nature by B.N.Mandelbrot."

The front cover illustration is presumably another 'fractal'. Other patterns can be formed by similar rules, e.g. starting from two circles in the centre, allowing growth against two cells that are not adjacent, not allowing cells to grow if they will touch other growing cells, and so on. I am disappointed that readers have not so far sent examples of their own constructions - there are many possibilities to be explored.