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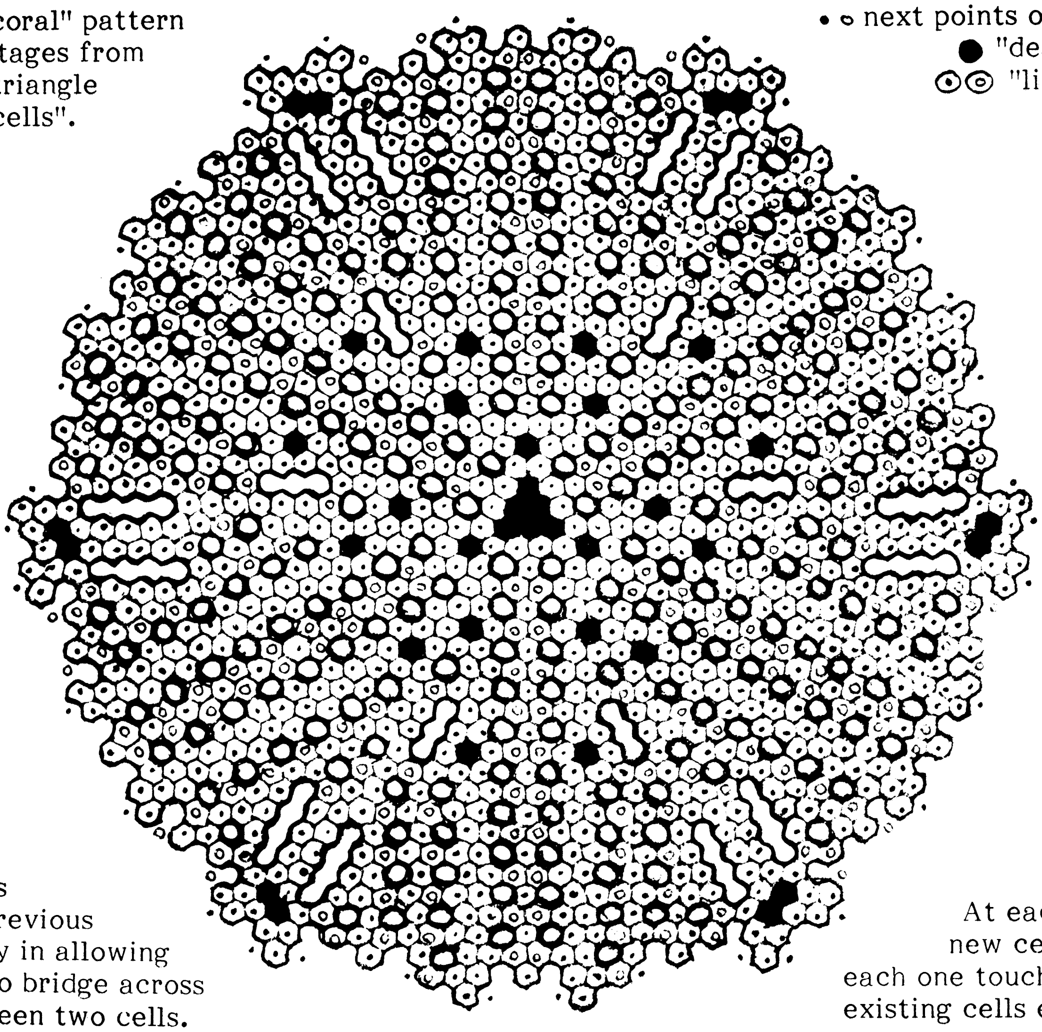
Growths & Processes

Another "coral" pattern grown in stages from a central triangle of three "cells".

• ○ next points of growth
● "dead" cells
⊙◎ "live" cells
O holes



This differs from the previous "coral" only in allowing new cells to bridge across a gap between two cells.

At each stage new cells grow each one touching two existing cells exactly.

Comparison with the pattern illustrated on the front of issue 3 will show how a very slight change in the rules of construction can produce great alterations in the result.

Contents

*** Prize Competitions.

THEORY OF GAMES

What's in a Game?

This article seeks to define the essentials common to all games, or to the principal types of game, and to clarify the terminologies used in describing and classifying them. Views of readers - on better choices of terms, points omitted, etc - would be appreciated.

1. Laws. A game is a formalised activity, governed by more or less detailed conventions, called the laws of the game. Laws are of two types - rules and regulations. The rules describe the essential mechanics of the play. The regulations outline customs and etiquette to be observed when the game is played in a professional manner (i.e. with money or reputation at stake). It may be necessary to have an independent regulator (umpire, judge, referee, arbiter, controller, etc.) to interpret the laws or even to keep order among the players. Breaches of the laws may be major, bringing the game to a premature end, or minor, when play may continue after payment of some penalty. In many respects the rules are akin to axiomatisations of mathematical systems, whereas the regulations are like legalistic codes of practice to govern human behaviour.

2. Sides. Play may be cooperative or competitive. Some of the players may cooperate to achieve certain objectives. Where this cooperation is formalised and permanent the cooperating players form a side. A side of two players is a partnership and of three or more players a team. In the case of teams with a large number of members it may be necessary to appoint a team-leader (captain), possibly non-playing, to direct strategy with the advice of the other team members. A game for one player is a solitaire, and a game for one team (all cooperating) a display (e.g. formation dancing, marching or gymnastics). Where there are two or more competing sides it is usual to distinguish the different sides by assigning them a colour (or, for example in football or horse-racing, patterns of colours). Items of equipment that come under the control of one of the sides may need to bear that side's colours. The spectators may also perhaps be considered essential 'participants' in some games. Competitive games are contests.

3. Positions. Any state that can be reached in the course of a game can be termed a position in the game. As well as actual positions that have arisen, coaches and puzzlers may well consider hypothetical positions, likely or unlikely to arise respectively. The laws will lay down how the game is to start - the possible opening positions - and under what circumstances the game is to be considered to end. In some games this may be due to the expiry of the time allowed for play, in others to the achievement of a terminal position (e.g. stalemate, checkmate, potting the black, laying the last card). A legal position is one that can be reached in a game played strictly in accordance with the laws.

4. Matches. The term game besides applying to any activity conducted under the specified laws is also applied to each particular example of it, from opening to conclusion. A series of games (not necessarily all of the same kind) between the same two sides, treated as if it were itself a 'game' is called a match. A match may sometimes be divided into a series of sets. A tournament is a schedule of games involving a number of sides (more than participate in one game) arranged to determine an overall winner. They range from all-play-all through swiss systems to knockout competitions.

5. Continuity. Games may be classified into sports and pastimes according to whether the positions through which the game passes form a continuum or a sequence. The same distinction can be made in terms of their requirements of physical skills of coordination of mind and muscle or the lack of such skills. At one extreme of sports, muscular strength is very evident, as in Weight-lifting or Shot-putting, while at the other great delicacy of control is exercised, as in Snooker or Shove-hapenny. Although Snooker combines both a discrete sequence of positions (those reached when the balls are stationary) with continuity (in movement of the player, his cue and the balls) the presence of the latter makes it a sport - in contrast for example the moves of the pieces in Chess do not demand unusual physical skill (and in computer Chess are not even physical displacements). The more active sports tend to be played out-doors.

6. Interaction. Another classificatory aspect of competitive games is the degree of interaction between the sides. Interactive sports are Football, Tennis, Baseball, etc in which one side actively prevents the other reaching an objective. Non-interactive sports are activities carried on either in parallel, i.e. as races or in series, i.e. as competitions, where the players take turns to show their abilities. Almost any activity can be turned into a competition. A rough classification of competitions is into: judged competitions (like Flower shows, Diving, Figure-skating, Knobbly-knees contests) and measured competitions (like High Jump, Javelin throw, Angling, Weight-lifting) and scoring competitions (Darts, Golf, Archery, Shooting, Five-stones).

7. Aims. When the game ends the laws provide a means of evaluating the success or failure of each side in the activity. In some games the distinction between winners and losers is absolute, in others, where a method of scoring is used the degree of success or failure can be assessed. If two sides do equally well there is a tie, and if all sides do equally well there is a draw. Some games (e.g. Nim) do not admit a draw. A position from which one side should win, draw or lose provided no mistake is made, may be termed a winning, drawing or losing position for that side. The opening position in Chess is probably even (drawing) though this has never been conclusively proved, and there have been famous claims to the contrary. It is difficult to determine in a complex game whether a position should be won or not. One can only calculate moves ahead with precision to a certain distance before the number of possibilities becomes overwhelming or chance factors accumulate to an unpredictable extent. When calculation can get one no further, the evaluation of the position must be based on looser judgments of a more general nature. In Chess for example this involves taking account of such features as space, material, time, vulnerability of King, potentialities for Pawn promotion, and so on. If the course of subsequent play becomes sufficiently clear then the losing player may resign (submit) rather than play on unnecessarily. Similarly a draw may be agreed without playing it out to a conclusion. The result has to stand, even if subsequently it is found that the players' judgment was in error.

8. Turns. In the case of a sequential game (pastime) the laws lay down what sort of actions may be taken in particular positions. A minimal change to a position is a move. A series of moves by one side, without intervention by another, is a play. Each change of position in the sequence of positions that constitutes the game is a turn of play. In some games all the sides move simultaneously (e.g. Diplomacy, Sopwith) in others the turn to play passes from one side to another. In this case laws are laid down to determine who has the first turn, and in the case of three or more players, who moves next. The most common rule is that the turn to play rotates in a fixed cyclic order. A go is a transformation of position that can be accomplished in a single turn of play by one side. An important feature of many games is that a player may not pass (i.e. forego his turn to play) - he must make a definite change to the position, even if the only changes he can make are to his disadvantage (the situation called zugzwang). The term 'move' is often misleadingly used when 'turn' is meant (this is because in many games there is one move per turn) but matters become much clearer if a distinction is observed.

9. Chance. Pastimes may be classified as games of chance and of choice. Games of chance tend to be those that are arithmetical and games of choice those that are geometrical but there are many exceptions. Games of chance employ number-bearing equipment like dice, cards, dominoes. In this class therefore we include such 'board' games as Ludo, Monopoly or Backgammon (really better termed race or track games).

10. Odds. A game in which the forces of the players, or their arrangement, are initially unequal is unsymmetrical. Games that try to simulate some real-world situation, e.g. war-gaming, tend to employ asymmetric starts, though usually the asymmetry does not represent a serious imbalance. Some abstract games are also asymmetrical, e.g. games of one against many (Fox and Geese) or of inner v outer (e.g. Hnefatafl). Most board games are symmetrical. When players are unequally matched in ability those with an advantage may have to bear a handicap. In Chess this used to take the form of odds of a piece or pawn omitted from the better player's forces, though this is seldom done these days when grading systems have become so well developed.

CARD GAMES

Ginger Rummy

This is a more 'strict' form of Space Rummy, which was described in issue 1, p5. A pack of 48 cards is used, omitting the 10s, with J, Q, K counting as 10, 11, 12. Each player receives 12 cards and aims to form it into three matched sets of four. A fully matched set consists of either four of a kind, four in sequence of suit, or four cards in sequence, one of each suit. These count zero. A set of four comprising three towards a matched set counts as one card - the odd one out. A set of four comprising two halves of matched sets count as two cards - the smallest in each part. A set with two from a matched set and two odd counts as three - the two odd and the smaller of the others. Any other counts all four. You may declare at any stage, in place of drawing and discarding, even at the first go. The three sets of four are scored quite separately, and it is up to each player to arrange them to minimise his score.

A Concertina Patience

By C.M.B.TYLOR

This game is based on the well known principle of a sequence of alternating colour, red 4 on black 5, black 3 on red 4 etc. This is modified in two ways: (i) the sequence may go up as well as down; (ii) it may 'loop round' from Ace to King or from King to Ace. Thus a possible sequence might be: red 3, black 2, red 3 black 2, red Ace, black King, red Ace, black King, red Queen, black Jack, etc.

There is no initial setting out; cards are simply dealt out, face upwards, one by one. If a card can be placed on an existing pile, it must be so placed; cards are then moved, one by one, from one pile to another until no more moves are possible. One dealt card can lead to a large number of moves, so that a deal can take some time. If a card cannot be placed on an existing pile, it is placed on its own to start a new pile, with no limit on the number of piles. The object is of course to get all the cards in the pack into a single pile. To do this, it is necessary to go right round at least once, to link red Ace with black Ace. It may be best not to try to keep the number of piles to a minimum in the early stages, so as to increase the probability of being able to place the last few cards.

I estimated (guessed) that I could get the game to come out once in between 20 to 50 deals. If it is considered that this makes it too easy, there are various ways in which the difficulty could be increased. (a) Changing course in mid-stream, i.e. moving a card from one pile to another, and then, on second thoughts, moving it back again, could be banned. (b) A limit could be placed on the maximum number of piles in existence at any one time, or else points could be deducted for each new pile used. (c) Cards in a pile could be placed directly on top of each other, so that the player would have to remember the contents of each pile.

Three Bridge Puzzles

By John BEASLEY

Three off-beat puzzles concerning Bridge are proposed. The first is a matter of mathematics, the others are exercises in construction.

1. A 'bidding system' is defined as a set of rules which always determines a unique call, given the contents of a player's hand and the preceding bidding. Prove that if each of the four players is bidding strictly in accordance with such a system then there is at least one bidding sequence that cannot occur. (The precise missing sequence naturally depends on the system in use; what is to be proved is that there is always at least one.)

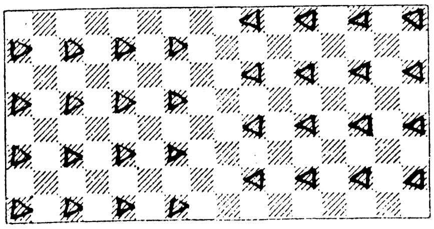
2. Construct a deal in which any contract, by any declarer, goes at least two down against best defence.

3. Construct a deal in which each of the four hands, if declarer, can make 3NT double-dummy even against best defence.

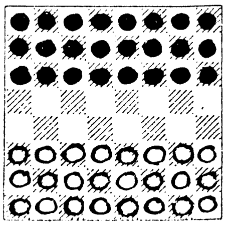
Solutions next issue.

BOARD GAMES

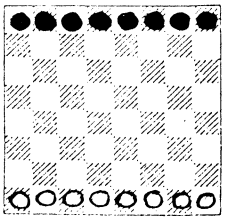
Here is a page of Draughts variants - and a competition.



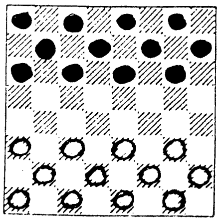
A



B



C



D

Deep Draughts

This game can be played on two ordinary Draughts boards placed end to end, but a special board is best, 8 x 16. [Such a board is commercially available for the game Rhythmo, a modernised version of Rythmomachy, produced by Quantum Games, and available from The Puzzle Box, price £8.45 plus 70p postage.] Sixteen pieces a side are used and start as shown in the diagram. Diagram **A**.

The pieces move (but do not capture) like chess bishops, diagonally, but if a piece moves it must go as far as it can in the chosen direction (forwards only). Directional pieces are used, and when one reaches the far end of the board it is turned round and comes back again in the opposite direction (its powers are not increased).

A piece captures by jumping over a piece on a square diagonally just in front of it. Having captured it must carry on as far as possible. If a further capture is then possible from the new position it must be taken. Capturing is compulsory. If there is a choice of captures you may choose which you prefer, regardless of the number that can be captured. A piece can capture, move to the end, turn round and carry on capturing in the same go, if there is a piece there to capture.

Double Draughts

The ordinary game of Draughts only makes use of half the squares of the board: the black squares. It could equally well be played on the white squares. Double Draughts is like two ordinary Draughts games played simultaneously on one board: one on the black squares and the other on the white. When it is your turn to move, you are free to choose which game to make your move in, unless a capture is possible in which case you must take. There is no huffing. Diagram **B**.

Quirk

The name of this game, apart from its English connotation, was chosen because of its similarity to the Arabic 'Quirkat' for a game on a small board that was an ancestor of Draughts (the Spanish called it 'Alquerque'). Each player begins with eight 'Quings' on his back rank. Quings move like chess Queens and capture like chess Kings. When they reach the opponent's back rank they promote to Keens, which move like chess Kings and capture like chess Queens. Diagram **C**.

Interlock

A Game Invention Competition!

You are provided with the name - Interlock - and the opening position - **D** - for a new game. All you have to do is to devise the rules! The game looks rather like Draughts at first glance, but you will notice on looking closer that the black pieces are placed on white squares and the white pieces on black squares. [The game could equally well be played with black on black and white on white - but then it is not so easy to see the pieces!]. The only rule insisted on is that the pieces remain on the same colour squares throughout the game - so, how do they interact! A prize of £10 is offered for the best entry to reach me before 1st November 1988. [The editor's decision is final of course.]

CHESS VARIANTS

Comments on Series-Play Chess

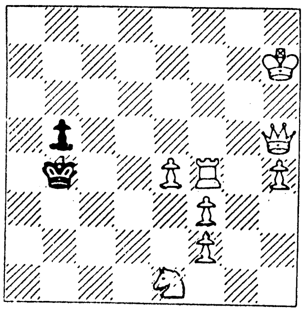
T.G.POLLARD: The section on Marseilles Chess and Scotch Chess led me to refer to Boyer's Les Jeux d'Echecs Non Orthodox. I have always adhered to the rule that check may be given only with the last move of the series, but Boyer allows check at any point, the player forfeiting the remaining moves of his series.

P.M.COHEN: Worth noting, I think, is that in AISE Progressive, giving check before the last alloted move of one's turn loses. Best version, I think, is one John McCallion taught me: No piece may move a second time if any mobile piece has not moved yet. Once all have moved, the count for all restarts at zero - thus a newly mobile piece does not get extra moves to catch up.

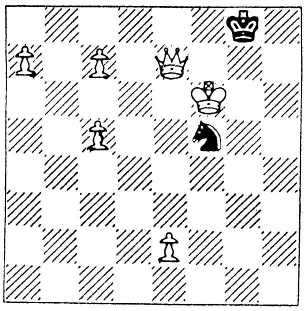
Theodor STEUDEL comments that there already exist a lot of Seriesselfmates with only two Black men. He provides the examples **A, B, C** below, showing four promotions.

Erich BARTEL sends example **D** below, for comparison with Ian Shanahan's **32.** He comments this may be the first problem showing this kind of incarceration.

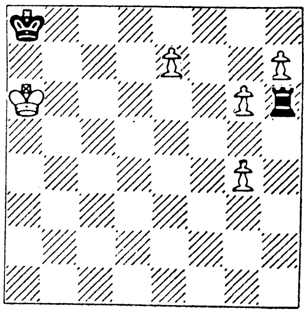
A. M.Tomasevic
Mat 1980, Ssm 34



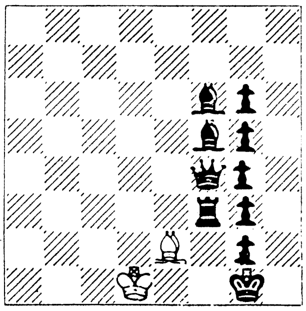
B. J.Kricheli
SE 1977, Ssm 19



C. B.Gelpernas
BCM.1977, Ssm 15



D. Erich BARTEL
Feenschach 1965, Shp 11



A. 1-3Sc4 4-7e8(B) 8-9Bc2 10-13Kd3 14Re4 15-19f8(R) 20-21Rfe3 22-26f8(R) 27-28Rfd2 29Qe2 30-33h8(B) 34Bd4 for Pxc4 mate. **B.** 1Qe3 2Qh6 3-7e8(B) 8c8(R) 9Rc6 10Bh5 11Kg6 12Rf6 13-15c8(S) 16a8(Q) 17-18Qag5 19Se7+ for SxS mate. **C.** 1e8(S) 2h8(Q) 3Qf6 4-5g8(R) 6-7Ra5 8-11g8(Q) 12Qg3 13-14Sb5 15Qc6+ for Rxc6 mate.

D. 1Kf2 2g1(B) 3g2 4-5Rh1 6Qh2 7g3 8Bh3 9g4 10Bh4 11g5 for Kd2 stalemate.

Circean Varieties

Circean varieties of chess are any in which 'captured' men reappear elsewhere on the board, unless the reappearance square is occupied. The original Circe Chess was invented exactly 20 years ago, and rapidly established itself as one of the 'standards'. The rule is that the captured piece returns 'home' - the home square for Knight or Rook being taken to be that of the same colour as the square on which it is captured (since it is impossible to establish which of the two original squares it occupied) - and the home square for a Pawn being taken to be that in the same file as the capture square. For example, in **61** the pieces at d2,d3,e2,e3,e6,f2 if captured reappear at f8,g8,a8,b8,d8,f7 respectively. One of the inventors of Circe, Jean-Pierre BOYER, died in 1986, but was able to complete a selection of his compositions which has now been published as a special issue of Rex Multiplex (Price 80 Francs from Denis Blondel, 22 Allée des Bouleaux, 94510 La Queue en Brie, France).

In King-Circe (better termed Total Circe) the Circe rules are extended to apply to the King as well, so that the K may be captured, and is only considered to be checkmated if it cannot escape destruction. Neutrals are treated as white or black at the choice of the player whose turn it is to move. Thus if White captures the NPg3 in **62** it reappears at g7 and if Black takes it it reappears at g2. Neutral Ps promote to Neutral pieces. Antipodean Chess which I invented in 1976 (Chessics 1) is a Circean variety in which a captured man reappears at the 'antipode' - the square a (4,4) leap away, e.g. Qb8 in **63** reappears at f4. The antipodean rule does not apply to the Ks in **63-4.**

Informal Tourney: Hans GRUBER has kindly agreed to judge original chess problems appearing in the 1987-88 issues. I hope the prospect of an award will spur composers to action!

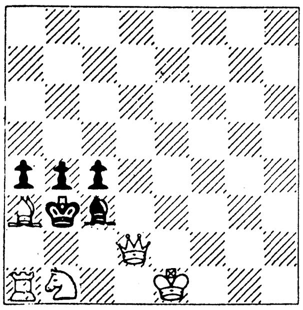
CHESS PROBLEMS

Caissa's Kaleidoscope

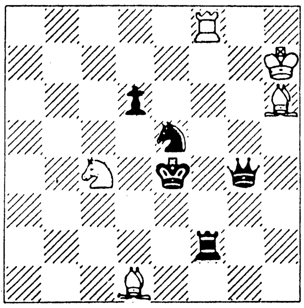
Seriesselfmate (**49**) see examples opposite. Helpmates: the two sides cooperate to find a mate of the Black King in the number of pairs of moves stated (so in hm 2½ White moves first). Problem **51** is a multiplet, the other parts being: (b)c5-d6, (c)f4-c2, (d)e3-c3, (e)e3-c8, all hm2½. The 'Opting Pawn' (**52**) has the option of moving one or two steps forward, no matter on what rank it stands. In seriesmate, White plays all the moves.

The Circean Varieties are explained opposite. Note that **56** - **60** end in stalemate. Problem **61** is dedicated to Alexander George. Grasshopper (**64**) see last issue. Problems **55** and **64** are related; the Nightrider g5 can make any number of Knight moves in one direction in one move (capturing only on the last) e.g. to e6,c7,a8 or e4,c3,a2.

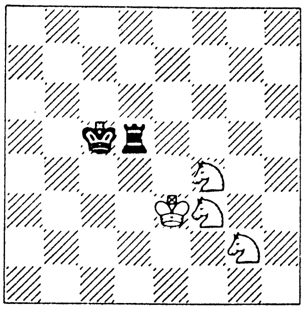
49. N.NETTHEIM
Seriesselfmate in 12



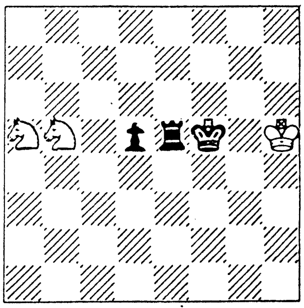
50. S.J.G.TAYLOR
Helpmate in 2 (2 ways)



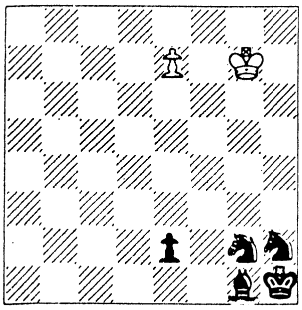
51.E.HOLLADAY
Helpmate 2½ see text



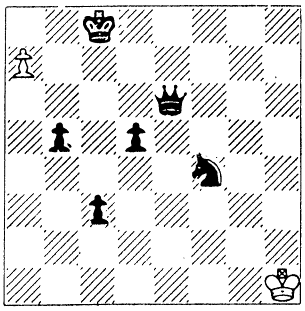
52. F.M.MIHALEK
hm3 Opting P (3 ways)



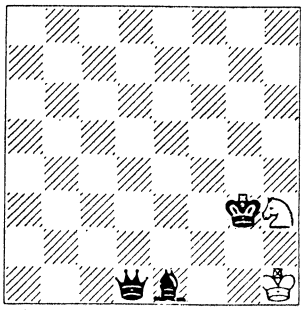
53. Th.STEUDEL
Sm 3 Circe (b)g7-f6



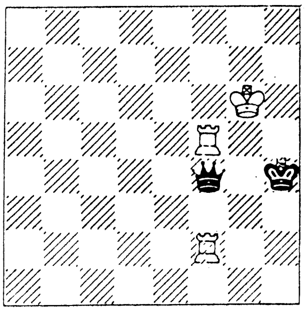
54. Th.STEUDEL
Seriesmate 8 Circe



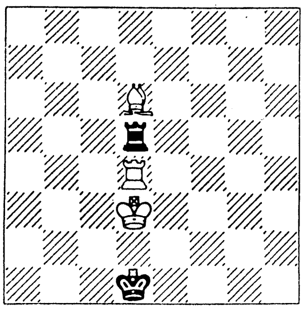
55. G.P.JELLISS
hm 3 Circe



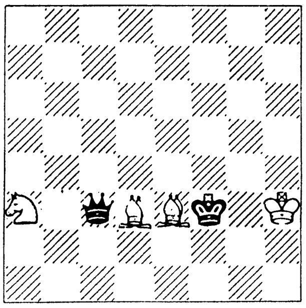
56. E.HOLLADAY
hp 2 Circe (2 ways)



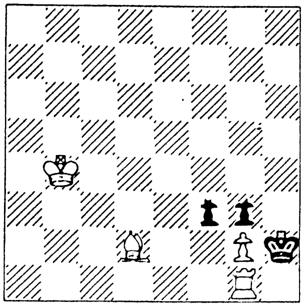
57. F.M.MIHALEK
hp 2½ Circe



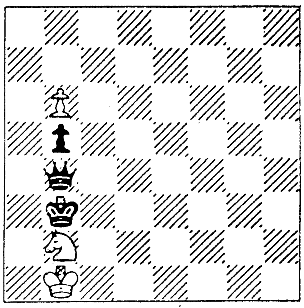
58. F.M.MIHALEK
hp 2½ Circe



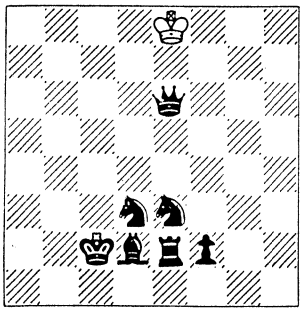
59. Erich BARTEL
hp 2½ Circe (4 ways)



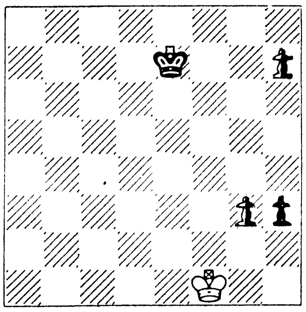
60. E.HOLLADAY
hp 3½ Circe



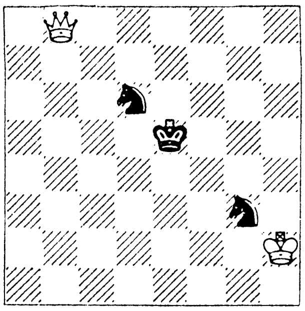
61. I.SHANAHAN
hm5 King Circe



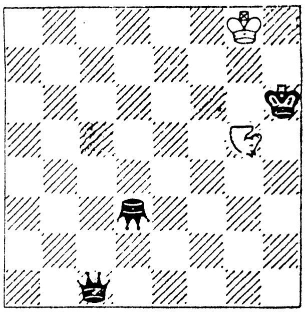
62. I.SHANAHAN
hm5 Circe Neutral Ps



63. G.P.JELLISS
hm2 Antipodean



64. G.P.JELLISS
hm3 Antipodean



G-hopper & N-rider

Solutions to reach me by 1st July

TRANSITIONS

Sliding the 15-1 Puzzle to Magic Squares

By Len GORDON

A problem often stated for the well known 15-1 puzzle, is to go from an ordered arrangement to a magic square in the least number of moves. There are two problems here. One is to select the magic square which allows the least moves, and the other is to find those moves. Since there are 8 times 880 possible 4x4 magic squares, finding the one(s) that allow the least moves is a big problem. Here are the results of an exhaustive computer search. Magic squares that can be reached in less moves than any published previously were found. Barring computer bugs, these are the best possible.

My brother, Jerry, and I have both written computer programs for sliding block puzzles. Jerry selected various magic squares and solved them for minimum moves with his computer. By repeated trial, he discovered one that could be reached from Loyd's inverted start in 36 moves (as against Loyd's 50). This was exciting, so we decided to try an exhaustive search. I coupled a magic square generator with a sliding program and let it run. In 12 hours it found 3 squares (2 in addition to Jerry's) that could be reached in 36 moves; none that could be reached in less. **B'** shown previously (on page 40) requires a unique sliding sequence, **B''** and **B'''** shown below can each be reached by two paths.

For the no inversion start, the computer ran for 6 hours. It found one square other than Dudeney's that could be reached in 37 moves, and one and only one that could be reached in 35 moves (**C'** on page 40). All three require unique sliding sequences.

The above are notes I sent to Ed. Hordern just before he finished his book on Sliding Piece Puzzles in 1986. When I sent the note I had also investigated the case presented as Problem **5** on page 24 of the G&P Journal. There are four magic squares that can be reached in 42 moves. I give one solution to each. Two of them have a second solution, the other two have 14 solutions each. Note certain similarities in the patterns.

The transformation you attribute to Robert Ely (**A** to **A'** on page 40) will take at least 50 moves, instead of 42. The 'lift and replace' form of the puzzle is equivalent to the above form, as can be seen by replacing each number N by 16-N, which does not affect the magic property of the final square reached.

For 3x3 sliding puzzles, there is no point in using a computer. Just had a nice California afternoon. Here is what I found in the sun. There is only one possible magic square. It transforms to the natural order you specify in Problem **4** in 21 moves as shown in the first solution below. Two other cases, in 19 and 23 moves, are also shown.

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

→ → → →

7	8	13	2
4	11	14	1
10	5	0	15
9	6	3	12

6	8	3	13
1	15	4	10
12	2	9	7
11	5	14	0

13	1	10	6
2	14	5	9
7	11	0	12
8	4	15	3

9	4	3	14
2	15	8	5
12	1	6	11
7	10	13	0

```
DDLUL DLDRR ULDRU RULLL DRUUR DDRDL ULULU RD
                LLUUL URDRU LDRDD LUULD RDLUR RRUUL DLDRR DLUUL UL
                        LLULD RRULU RURDD LURDL DLUUU RDLLD RRDRU LULUL DR
                                UULLU LDDRU LDRRD LLURR RULLU RDLDR DRULU RULDL LU
```

6	5	8	11
10	9	4	7
1	2	15	12
13	14	3	0

B''

6	1	8	15
10	13	4	3
5	2	11	12
9	14	7	0

B'''

7	0	5
2	4	6
3	8	1

→ → →

6	7	8
3	4	5
0	1	2

7	8	0
4	5	6
1	2	3

8	5	2
7	4	1
6	3	0

```
URDLL URULD RDLUR RDLUR U
               UULDR URDLD LUURR DLDL
                              ULDRR ULURD DLURD LLURD LUU
```

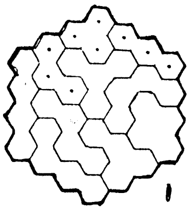
[This beats my effort at problem **4** by 14 moves! GPJ]

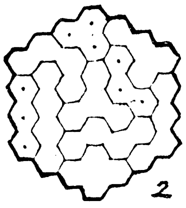
Solution to Problem **6** to appear next issue.

DISSECTIONS

Polyhexes

One further 4-hex should be added to the Sopwith diagram on page 34 of the last issue, since the J-shaped piece is also asymmetric, so gives another form when turned over. The number of (turn-over-able) 5-hexes is 22 (as illustrated by Solomon Golomb on p136 of Polyominoes) and the number of 6-hexes is 84 according to my own count, including the piece with a hole. (I would be glad of an independent check of this enumeration.) The six diagrams below solve the problem of arranging the 3 pieces of 3 cells and the 7 pieces of 4 cells to form a 37-cell hexagonal array. They show the six possible positions for the straight 4 cell piece. The inset 10-cell triangle in diagram 5 can be rotated or reflected to give alternative solutions of this case and of case 1.


1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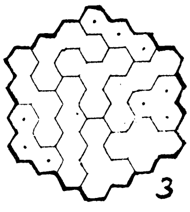
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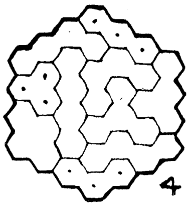
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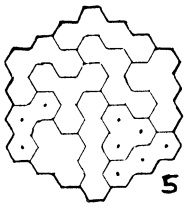
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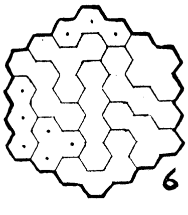
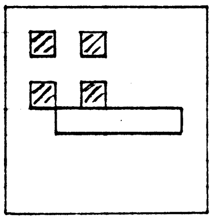
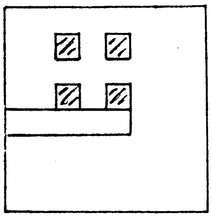

6

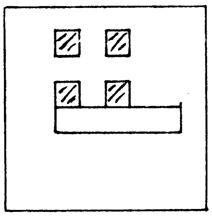
The number of cells in a hexagonal board with n cells to a side is the n-th "hexagonal number". What is the sum of the first n hexagonal numbers? Demonstrate graphically. Are there hexagonal numbers that are also (a) square or (b) triangular? Can the 5,3,1-cell pieces be arranged in a size 15 triangle? Can the 12 symmetric 5-pieces be arranged to form a size 5 hexagon, minus the centre cell? These are just a few suggestions.

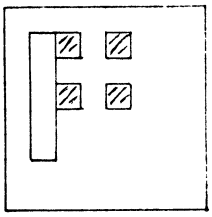
Pentominoes

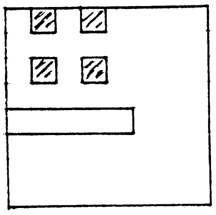
Sivy FARHI asks you to complete the following dissections, by placing the 11 missing pentominoes (all different of course). The solution in each case is unique!

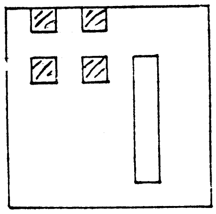








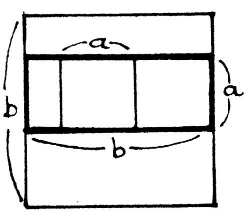




Squaring the Rectangle

The ratio of the shorter side to the longer side of a rectangle may be called its "squareness". This fraction equals 1 in the case of a square and approaches 0 for a long thin rectangle. The number also expresses the ratio of the area of the rectangle to the area of a smallest square containing it, or the area of a largest square contained in it to the area of the rectangle, i.e.: $s = a/b = a^2/ab = ab/b^2$ (diagram **A**).

A



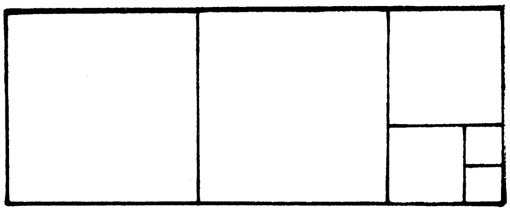
s

$= a/b$

$= a^2/ab$

$= ab/b^2$

B



$m_1 = 2$

$m_2 = 1$

$m_3 = 1$

$m_4 = 2$

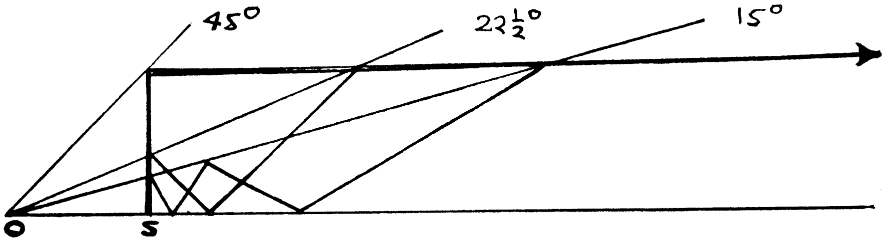
$s = 5/13$

Any rectangle can be divided up into squares by the process of removing from one end of it a square of the largest possible size (side a), then applying the same procedure to the smaller rectangle that results. If the numbers of squares removed in this way, of the first, second, third, etc. sizes are m_1, m_2, m_3, ... what is the squareness of the rectangle? The type of formula that results is well known, but I have not seen it given this geometrical visualisation before. (Diagram **B** illustrates the case of a rectangle 5x13).

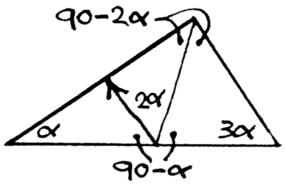
What is the squareness of a rectangle with $m_1, m_2, m_3, m_4 = 1,2,3,4$?

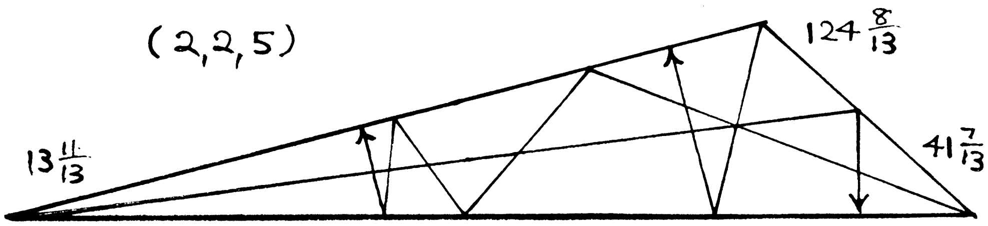
Triangular Billiards

Problem 4 (further note). If two mirrors are hinged at O and a ray of light is shone vertically from S on the base mirror then the outcoming ray will be parallel to the base mirror when the angle between the mirrors is 45/m degrees, where m is the number of reflections on the top mirror. Furthermore, the outcoming ray is always at the same height above the base mirror, namely a height equal to the length OS. Could there be some practical application of this device?



Problem 5 (continued). If a ball cued to bisect an angle of a triangular table returns to the corner after one bounce, then (as noted last time) the triangle is isosceles, i.e. its base angles are equal. In the case of a two-bounce return the base angles must be in the ratio 3 to 1, as illustrated below.

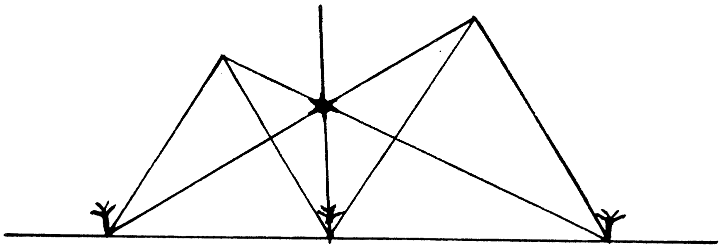




If a two-bounce return is to be possible from two corners, in a non-isosceles triangle, then the angles must be in the ratios 1 to 3 to 9, i.e. they must be 180/13, 3(180)/13 and 9(180)/13. The number of bounces from the other corner must be five. This (2,2,5) case is illustrated above. The case (2,3,n) is saved for next time.

Star Points

Problem 1 (proposed on p 26). To find the location of the star seen above the palm trees with three rays at 60 degree intervals leading down to the bases of the trees was the question. A pretty answer is to construct equilateral triangles (i.e. pyramids!) between the trees, and join the bases of the outer trees to the tops of the pyramids. These lines will cross at the required star point. [I sent this some years ago to the Journal of Recreational Mathematics but the editor cut out all the fanciful elements of the story, much to my annoyance, and I'm not sure if it actually appeared.]



The star point is also the point where the circumcircles of the two triangles intersect. This observation provides a simple proof of the equality of the angles, by Euclid III.27 (angles on equal arcs are equal - to half the angle subtended at the centre of the circle).

Problem 2. If we now replace the three collinear points in the preceding question by three points forming a triangle, we can ask the same question. Where in the plane of the triangle can we locate a point (or points?) such that the lines AP, BP, CP, (extended past P) form a star at P? That is the lines form six angles of 60 degrees. A construction for the position of the point(s) is also required. [Solution to appear in issue 6.]

QUESTIONS

Puzzle Questions and Answers

There is a general theme of order and permutation, alphabetical or numerical, running through the selection of puzzle questions this time. Answers to the puzzles in issue 3 follow on the next two pages. Have you noticed the way the shortest questions tend to have the longest answers? The shortest question with the longest answer is - WHY?

A Self-Documenting Sentence. Tom MARLOW offered the following remarkable sentence as a Christmas and New Year greeting, with the comment that he has been trying to produce a personalised sentence of the sort. Can you produce something?

"This sentence contains three A's, one B, three C's, two D's, thirty three E's, seven F's, one G, six H's, ten I's, one J, one K, one L, one M, twenty four N's, sixteen O's, one P, one Q, nine R's, twenty six S's, seventeen T's, five U's, five V's, four W's, four X's, four Y's and one Z."

Alpha Pairs. Loretta BRUCE poses this intriguing alphabetical puzzle. Using every letter of the alphabet once only put two letters in front of each of the 13 three-letter words below, to make them into five-letter words. For example, Q must obviously pair with U, and can go before ITS to form QUITS or IRK to form QUIRK.

ANS, AIR, ASH, ASS, ECH, ELL, INK, IRK, ITS, OWL, IDE, RED, WAY

Code Words. What is the systematic connection between GNU and FOX and APE and EMU - apart from all being three-letter life-forms? What is the longest word that can be formed on the same principle?

Prime Words. Michael KINDRED offers a prize of a 'My Word' card game, manufactured for his group by Waddingtons, for the best answer received to the following poser: "A prime number is one which is only divisible by itself and one, so how about a 'prime' word, which has no anagrams in it apart from itself. The prize will go to the person submitting the longest prime word from which it is impossible to make any 2, 3 or 4-letter words. Chambers 20th Century dictionary latest issue is referee. Scrabble letter values will be taken into account in the case of a tie." And if more than one solver comes up with the best answer, the first to reach the editor wins. The same time limit will apply as for the chess problems (i.e. to reach me before 1st July).

Permutable Words. The related question of finding a set of 2, 3, 4, 5 or 6 letters whose permutations include as many English words as possible was posed by Solomon GOLOMB in Polyominoes (1966, p78). He comments that it is difficult to find sets of 4 letters where more than 5 of the 24 possible permutations are English words, and that for 5 letters a score of 8 out of the 120 permutations is an excellent score.

Permutable Primes. While we are on the subject of permutations and primes, an interesting question is: Is there a largest permutable prime - and if so what is it? By a 'permutable prime' I mean a prime number, expressed in the usual denary notation, that remains a prime if its digits are permuted in any other order (e.g. 37 and 73).

The Alphabetical Cube. There are 26 letters and 27 cells in a 3 x 3 x 3 cube. What is the maximum amount of disorder we can introduce if we letter the cells of the cube? We can measure this disorder by the number of moves necessary to restore the letters to some regular alphabetical order by sliding them one at a time from cell to cell - the central cell being initially, and finally, empty. This is a 3D sliding block puzzle.

Probably Deranged. A 'derangement' is a permutation that alters the position of every element in an arrangement. It is often illustrated by the feat of putting all one's letters into the wrong envelopes so that they go to the wrong people. What is the probability of making a derangement in the case of a large number of letters and envelopes? The assumption being that any permutation is as likely to be chosen as any other.

ANSWERS

Cryptarithms

(**7**). ELEVEN + THREE + THREE + ONE + ONE + ONE = TWENTY
505153 + 69755 + 69755 + 235 + 235 + 235 = 645368
(**8**). ONE + TWO + FIVE = EIGHT
621 + 646 + 9071 = 10538
(**9**). TWO + THREE + SEVEN = TWELVE
106 + 19722 + 82524 = 102352

(**10**). A Cryptarithmic Multiplication by T.H.WILLCOCKS.

Let N be a number of n digits, R a number of r digits ($n>0$, $r>0$), and $1<A<10$ (A a digit). Consider the product $A(N.10^{r+1} + 9.10^r + R) = P.10^{r+1} + Q$ (say) and the product $A[(10^r-1-R).10^{n+1} + 9.10^n + (10^n-1-N)] = K.10^{n+1} + L$ (say), i.e. Q and L are numbers of r+1 and n+1 digits respectively (possibly starting with zeros). After some juggling we have: $P=AN+(R-1)$, $Q=10^r(10-R)+AR$, $K=A(10^n-R)-1$, $L=10^{n+1}-A-AN$, from which we have: $K+Q=10^{r+1}-1$(=999...) and $P+L=10^{n+1}-1$(=999......).

For example 7x3294=23058 (i) and 7x5967=41769 (ii) and here 32+67=99, 4+5=9, 230+769=999 and 58+41=99. Hence if the result of multiplication (i) is known, we can immediately write down the result, without calculation, of multiplication (ii).

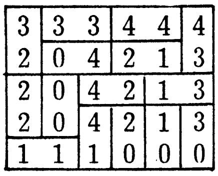
The given cryptarithm is a special case in which N,R,P,Q between them contain the digits 1,2,3,4,5,6,7,8 once only. As these are all incongruent (mod 9), so will be the numbers resulting from their subtraction from 9. Hence to any given solution there is a 'complementary' one, and hence the number of solutions is EVEN.

For example a solution is 2x9267=18534. The complementary is 2x7329=14658, where 267+732=999.

This result can be applied to other scales of notation. For example, in the scale of seven: 5x2463=16341 and 5x3642=25503 where 24+42=66, etc.

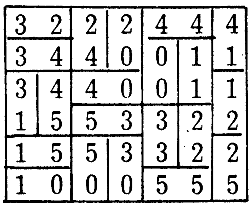
Domino Patterns

Solutions to the two problems proposed are as follows, where the domino-ends form rows of three alike, or squares of four alike. Diagrams **A** and **B**.



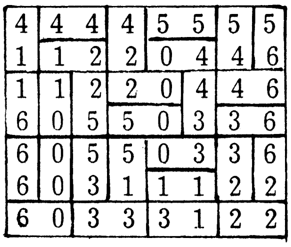
3	3	3	4	4	4
2	0	4	2	1	3
2	0	4	2	1	3
2	0	4	2	1	3
1	1	1	0	0	0

A



3	2	2	2	4	4	4
3	4	4	0	0	1	1
3	4	4	0	0	1	1
1	5	5	3	3	2	2
1	5	5	3	3	2	2
1	0	0	0	5	5	5

B



4	4	4	4	5	5	5	5
1	1	2	2	0	4	4	6
1	1	2	2	0	4	4	6
6	0	5	5	0	3	3	6
6	0	5	5	0	3	3	6
6	0	3	1	1	1	2	2
6	0	3	3	3	1	2	2

C

In a double-N set of dominoes, numbered [0,0] to [N,N], there are a triangular number of tiles: (N+1)(N+2)/2 and thus a rectangular number of ends: (N+1)(N+2), as the above arrays illustrate. Diagram **C** is the best I have been able to do with the [6,6] set. The 3s and 1s at the bottom form an L-shape instead of a square or straight tetromino.

Multiplication Table

The number of times a number occurs in the table is equal to the number of distinct divisors that it has. Thus 1 has one divisor (itself) and occurs once. A prime, p, has two divisors, 1 and p, and occurs twice (1xp=p and px1=p). A power of a prime, p^k, has k+1 divisors, namely 1, p, p^2, ..., p^{k-2}, p^{k-1}, p^k and occurs k+1 times. Any number is of the form: n= $1(p_1|x_1)(p_2|x_2)(p_3|x_3)$... where p_1,p_2,p_3,... are different primes and '|' means 'raised to the power' [a useful notation, since typewritten subscripts to superscripts tend to be mistaken for ordinary numbers!]. This number must occur in the body of the multiplication table $(x_1+1)(x_1+2)(x_1+3)$... times, since the number of occurrences is equal to the number of ways of expressing the number in the form ab, and p_r may occur in the factor a either 0,1,2,... or x_r times, which gives a total of x_r+1 possibilities.

The numbers that occur three times are thus the squared primes (4,9,25,49,121,169,...) those that occur four times are cubed primes (8, 27, 125, 343,...) and the products of two distinct primes (6, 10, 14, 15, 21, 22, 26, 33, 34, 35, 38, 39, 46, 51, 55, 57, 58, 62,...) those that occur five times are the fourth powers of primes (16, 81, 625, 2401,...) those that occur six times are the fifth powers of primes (32, 243, 3125,...) products of a prime with the square of another prime (12, 18, 20, 28, 44, 45, 50, 52, 63,...) those that occur seven times are the sixth powers of a prime (64, 729, 15625,...) those that occur eight times are the seventh power of a prime (128, 2187,...) a prime times a cube of another prime (24, 40, 54, 56, 88, 104,135, 136, ...) or products of three distinct primes (30, 42, 66, 78, 102, 105,...) and so on.

To determine the first number that occurs N times in the multiplication table it is necessary to factorise N in all possible ways and then compare the magnitudes of the numbers 2^x (where x+1 = N), 2^x3^y [where (x+1)(y+1)=N], $2^x3^y5^z$ [where (x+1)(y+1)(z+1)=N] and so on. When N is a prime the answer is thus 2^{N-1}, but the general case is not so straightforward. The sequence of numbers that are the first to appear 1, 2, 3, ...,8 times runs: 1, 2, 4, 6, 16, 12, 64, 24, certainly not a simple increasing sequence!

Integral Part Formula

Required is a formula for the integral part of n/2 (n a positive whole number). This can be formed as follows: According as n is odd or even we have: $(-1)^n = -1$ or 0, so then $1-(-1)^n = 2$ or 0, and $\frac{1}{2}[1-(-1)^n] = 1$ or 0, and $n-\frac{1}{2}[1-(-1)^n] = n-1$ or n, and so the formula is:

$$\mathrm{inp}(n/2) = \tfrac{1}{2}(n-\tfrac{1}{2}[1-(-1)^n])$$

which equals (n-1)/2, that is n/2 - 1/2 when n is odd and n/2 when n is even. Similarly:

$$\mathrm{inc}(n/2) = \tfrac{1}{2}(n+\tfrac{1}{2}[1-(-1)^n])$$

The Four Rs

The following results were listed by Dawson in 1916: 1 = RR/RR, 2 = R/R + R/R, 3 = (R+R+R)/R, 4 =R/R + √(R/·R˙), 5 = √(RxR)/(·R+·R), 6 = R/·R˙ -√(R/·R˙), 7 = (R-·R˙-·R˙)/·R˙, 8 = (R-·R-·R)/·R, 9 = R/·R - R/R, 10 = RR/R·R, 11 = R/·R + R/R, 12 = (R+·R+·R)/·R, 13 = R/·R + √(R.·R˙), then there is a gap to: 17 = (R+R+·R˙)/·R˙, 18 = R/·R˙ + R/·R˙, 19 =(R+R+·R˙)/·R˙, 20 = R/·R + R/·R, 21 = (R+R+·R)/·R, and then the further cases: 27 =(R+R+R)/·R˙, 30 = (R+R+R)/·R, 55 = RR/(·R+·R), 81 = (RxR)/(·R˙x·R˙), 90 = R/·R˙.x R/·R, 99 = RR/·RR˙, 100 = RR/·RR, 111 = RRR/R, 999 = RRR/·R˙, 1110=RRR/·R

T.W.MARLOW writes: There is a powerful trick which you may not accept as legitimate: $\log_{\sqrt{\sqrt{R}}}(R) = 4$, and generally for n root signs in the base the value is 2^n. Two Rs easily make by the methods shown above 1, 3, 9 or 10. Combinations of these values easily give all values up to 36 except 21 and 27, and these are solved as above.

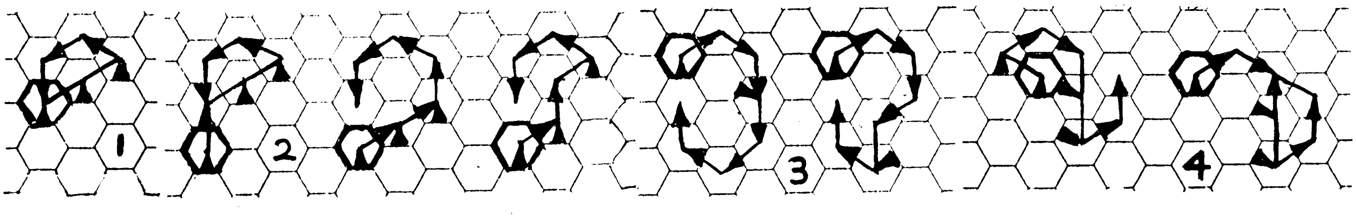
Sopwith Manoeuvres

(1). The reversal on the same cell is achieved in 5 moves minimum in two ways - one to the right and the other to the left: RS^2LT^3 or LS^2RT^3 (i.e. two slips to the right and three left turns, or two left slips and three right turns).

(2). The Immelman manoeuvre, reversal in the cell ahead,takes one more move, either an Ahead move interposed in the (1) manoeuvre (six ways) or following the same routes but making an extra pair of turns that cancel out. Thus there are 12 ways altogether.

(3). To chase your own tail takes 7 moves, and can be done in 6 ways, consisting of six turns, all in the same direction, with a 'backward' move interposed where appropriate.

(4). The side-ways slip to the right also takes 7 moves. There are only 2 ways, depending on whether the left slip is made as the first move or the last. The other six moves are in each case $RT^2RS^2LT^2$.



WORD PATTERNS

Crossword Puzzle Patterns

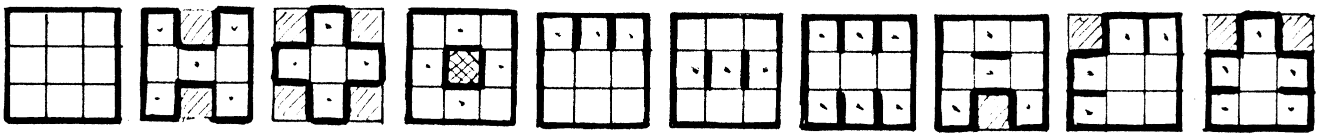
A crossword puzzle pattern consists of an array of squares such that words of two or more letters can be written across and down, one letter to a square. We also assume that the pattern is 'connected', i.e. it cannot be split into two separate crosswords without breaking at least one of the words. In other words the clue-squares form a polyomino.

Any square-edge across which a word is to be entered is shown by a light line, all other edges by a heavy line. Round the outside of the pattern these heavier lines will form an enclosing frame. Internally these lines will either form 'barriers' between adjacent clued squares or 'blocks' separating clue squares from unused squares. Any unused squares within the frame will usually be blacked in, unless they form an exceptionally large 'hole'.

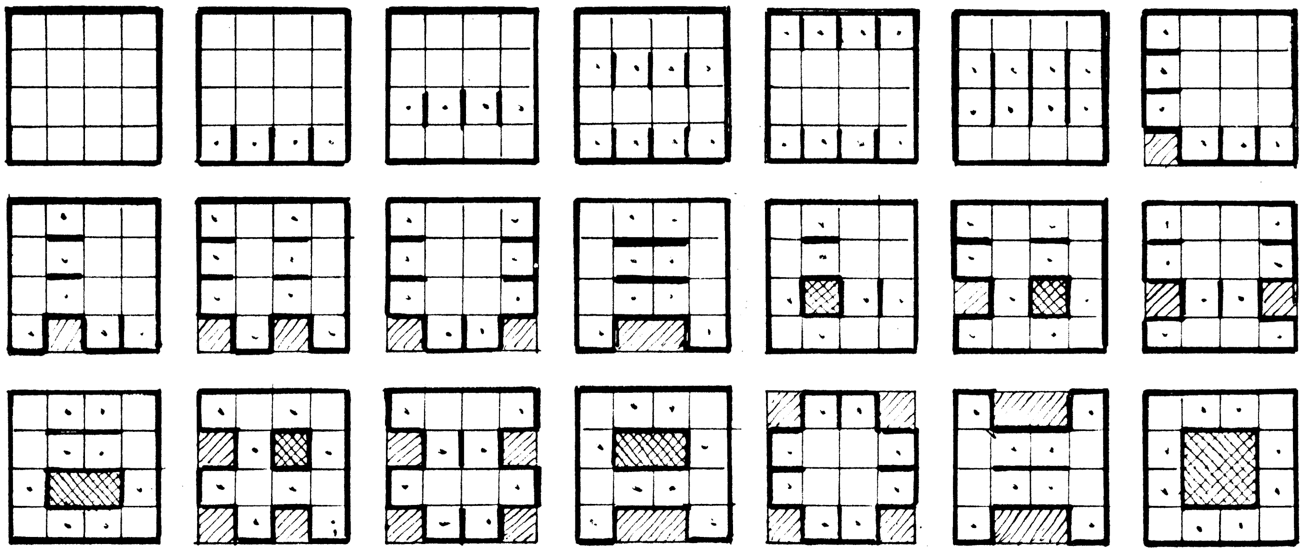
Thus four types of pattern can be distinguished: (1) Open patterns having no barriers or blocks (2) Block patterns, having blocks but no barriers (3) Barrier patterns, having barriers but no blocks (4) Mixed patterns having both barriers and blocks.

Often extra squares round the outside are blacked in to alter the shape of the outline (usually to make it rectangular) but these are not essential.

Another requirement for a crossword puzzle pattern is that it should not contain too high a proportion of unchecked letters. These are shown in the diagrams by dotted squares. A rule that is characteristic of many good crosswords is that every unchecked letter must be next to a checked letter. This eliminates 'cul-de-sacs' of two or more squares and 'tunnels' of three or more. It also implies that the number of unchecked letters in a word of N letters cannot exceed (N+1)/2. Most crosswords also avoid the use of two-letter words. With these restrictions there are just ten possible patterns within a 3x3 square. Interestingly, they are all symmetric in some way:



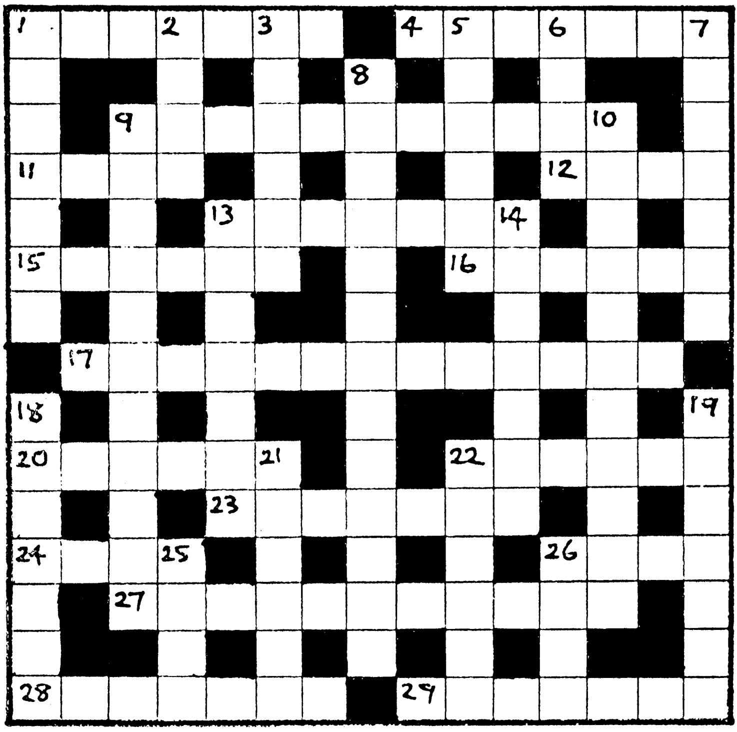
If we further disallow the use of three-letter words then the number of possible patterns within a 4x4 square is reduced to the following 21. Only three are asymmetric.



These patterns include forms that are more in the nature of 'Word Squares' than Crossword Puzzles. Word Squares are characterised by a high concentration of doubly clued letters. To reduce the number of possibilities in the case of larger squares we will need to impose even more stringent conditions. One would be to prohibit more than two successive checked letters in any word. We also need to keep down the number of unused squares. Most crossword patterns avoid any 2x2 areas of blocks. We will investigate the consequences of these restrictions next time.

Cryptic Crossword 4

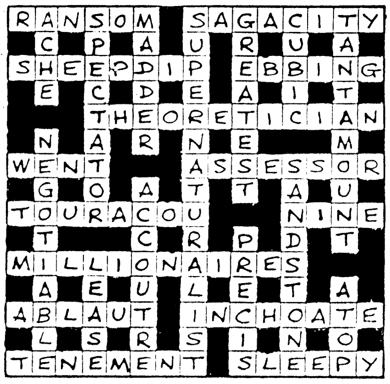
By QUERCULUS



ACROSS
01. Moustache mistakenly hides drug man. (7)
04. Elemental Indian Queen amid upper classes at Minehead. (7)
09. Told of sixth degree of kinship. (11)
11. Sister, there's nothing in a name. (4)
12. Sweep along like backward French aristocrats. (4)
13. Defence contains simian in part. (7)
15. Gold glow hair? (6)
16. He, ex hypothesi, is Conservative. (6)
17. Mine within fastidiously and steeply. (13)
20. Rub it tenderly where hurt. (6)
22. Penance for quiet separation. (6)
23. Fail to appear due to dateful schedule. (7)
24. Soothing unguent confuses soporific ungulate. (4)
26. Sad, obscene, sportsman. (4)
27. I scrap Ultra code details. (11)
28. Overhaul sleek TT machines in kitchen. (7)
29. Try goblin apiece. (7)

DOWN
01. No clear view of eye piece. (7)
02. Manner of men I'm among. (4)
03. Pretend to be a priest. (6)
05. Earning £10 note back. (6)
06. Untouchable head. (4)
07. Dons may turn up for work then. (7)
08. Difficulty concerning short calculation I state under hypnosis. (13)
09. Frank impression. (11)
10. Shogi scroll reveals class mates. (11)
13. Dee Corp. go ahead in recession. (7)
14. Considered, although too central. (7)
18. Monument Nobel is known for. (7)
19. Lengthen slang sentence. (7)
21. Stinger in tail of hornet and beetle. (6)
22. Nothing small about obstacle race. (6)
25. Could be a bit tempting. (4)
26. Hillside up in the Arbroath area. (4)

Crossword 3. SOLUTION



R	A	N	S	O	M	■	S	A	G	A	C	I	T	Y
■	C	■	P	■	A	■	U	■	R	■	U	■	A	■
S	H	E	E	P	D	I	P	■	E	B	B	I	N	G
■	E	■	C	■	D	■	E	■	A	■	I	■	T	■
■	■	■	T	H	E	O	R	E	T	I	C	I	A	N
■	N	■	A	■	R	■	N	■	E	■	■	■	M	■
W	E	N	T	■	■	■	A	S	S	E	S	S	O	R
■	G	■	O	■	A	■	T	■	T	■	A	■	U	■
T	O	U	R	A	C	O	U	■	■	■	N	I	N	E
■	T	■	■	■	C	■	R	■	P	■	D	■	T	■
M	I	L	L	I	O	N	A	I	R	E	S	■	■	■
■	A	■	E	■	U	■	L	■	E	■	T	■	A	■
A	B	L	A	U	T	■	I	N	C	H	O	A	T	E
■	L	■	S	■	R	■	S	■	I	■	N	■	O	■
T	E	N	E	M	E	N	T	■	S	L	E	E	P	Y

TOURACOU is the one from the Zoo Guide (Chambers and Oxford both give TOURACO).

Uncommon English

The following are a few extracts from a proposed dictionary of nonexistent but plausible words:

Antipaganda. Counter-propaganda.

Grissole. An unkind name for something intended to be a rissole.

Hooligang. An organised collection of troublemakers.

Immotion. Inward emotion, Deep feelings not outwardly expressed or inexpressible.

Litterature. Rubbishy writing.

Propogate. Move in hops - as on a pogo-stick.

Spurium. An element of doubt.

Further examples are invited.

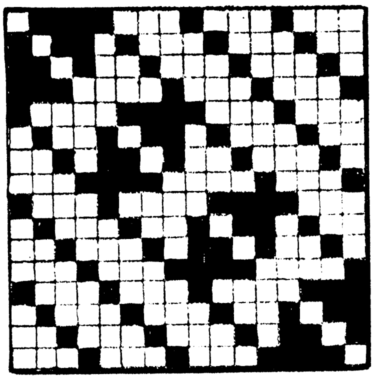
Lifebelts

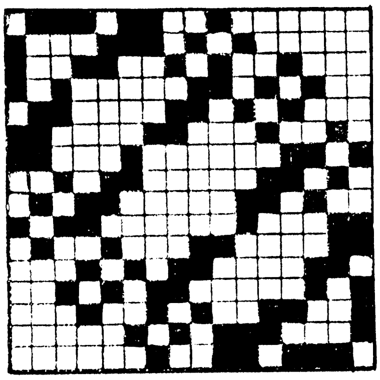
(1) CYCLAMEN and CLEMENCY
(2) INVITING and, opposite direction, IGNITION

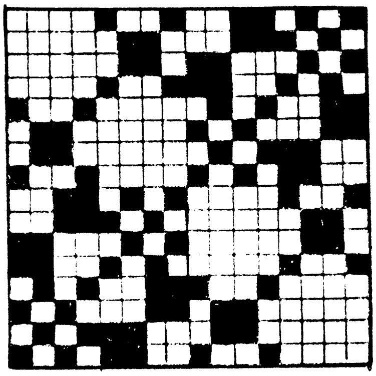
BINARY ARRAYS

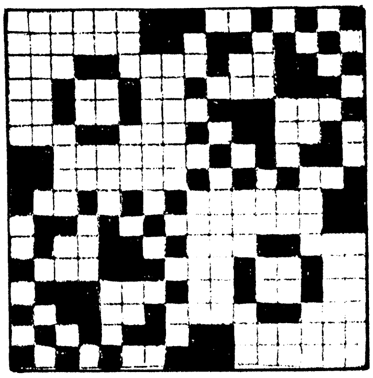
Chessboard Mosaics

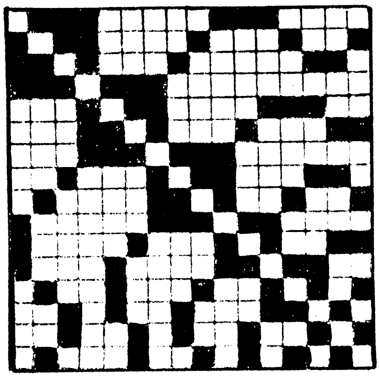
The palaces of the chessboard Kings have their rooms all tiled with mosaics, and below are several examples. Each room is of course square and is tiled with 256 tiles (i.e. 16 x 16) in two colours (shown here as black and white). The patterns of the mosaics are designed by first numbering the squares of a 4x4 board from 1 to 16 in some fashion and then laying a black tile as the mth in row n of the mosaic to indicate that a Rook move from square m to square n on the 4x4 board is possible. The white tiles indicate the cases in which such a move is not possible. The method of numbering used to produce the first pattern is the straightforward left-to-right, top-to-bottom procedure. Can you deduce from the patterns the other numbering schemes from which they are derived?

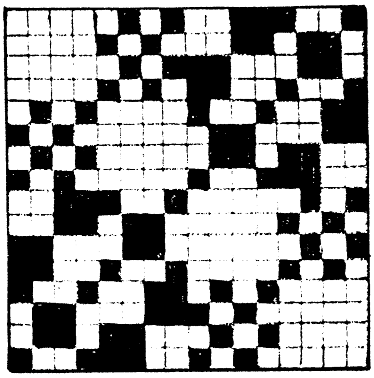












The numbering schemes cannot be fully deduced from the patterns, because the same pattern will result if the ranks or files of the 4x4 square are permuted (since this does not affect which squares the Rook can reach). In all cases the main diagonal (1,1) to (16,16) will be white, since a Rook on square x cannot move to cell x. Also the pattern will be symmetric about this main diagonal, since if the Rook can move from x to y it can move also from y to x. In the case of a symmetric numbering of the 4x4 board the mosaic derived will be symmetric also about the other diagonal, since the numbers x and 17-x are opposite one another across the axis or centre of symmetry. In the case of axial symmetry the secondary diagonal (1,16) to (16,1) will be black, since x and its complement 17-x are then in the same rank or file and the Rook can move between them, but in the case of central symmetry this diagonal will be white. It is particularly interesting to note that in this case the pattern can be rotated 90 degrees and will yield an alternative numbering. Two questions: What is the alternative numbering that results from rotating the first diagram? (the result is surprising) and: Can the numbering that results from the rotated pattern be the same as that from the original?

This is the beginning of a new regular section on "binary arrays".