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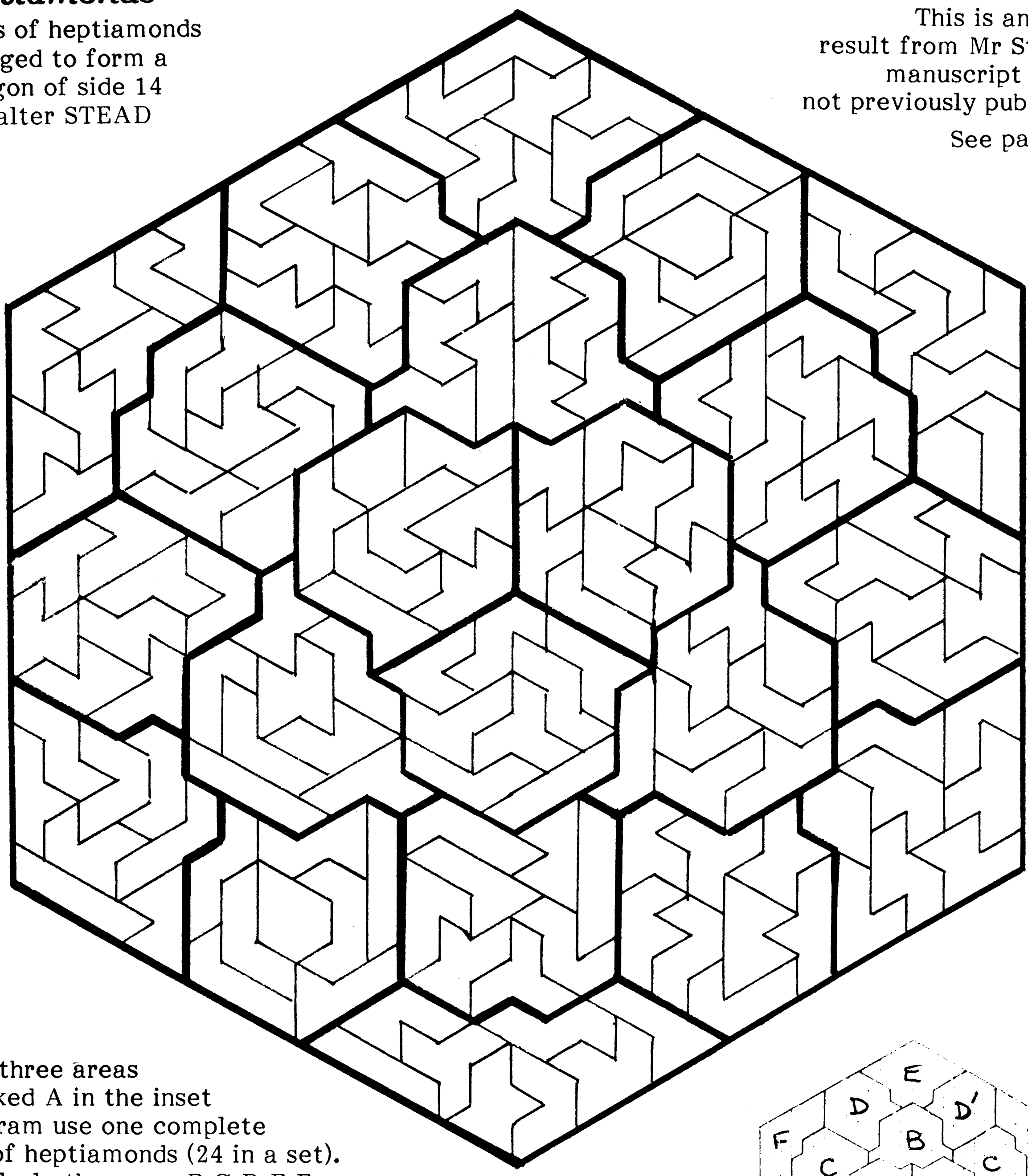
Contents: See page 67.

Heptiamonds

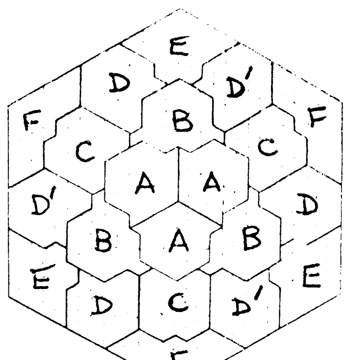
7 sets of heptiamonds arranged to form a hexagon of side 14 by Walter STEAD

This is another result from Mr Stead's manuscript notes not previously published

See page 80.



The three areas marked A in the inset diagram use one complete set of heptiamonds (24 in a set). Similarly the areas B,C,D,E,F. Areas D' reflect areas D.
Mr Stead describes these constructions separately as "3 simultaneous, similar decks, each of 8 heptiamonds" and he gives the shapes pictorial names as follows: A=sunfish, B=bison, C=shield, D=jug, E=UFO, F=bat. They are dated Aug-Sept 1966.



THE NIM FAMILY OF GAMES

The name Nim comes from an Old English word meaning 'take', from which we also get the word 'nimble' meaning 'quick in movement or in mental apprehension'. It is applied to a whole family of games, mostly played by the taking of counters with the object of being the player who takes the last one. There are always two players and every position is either winning or losing for the player whose turn it is to go. There are no ties. Each such game has a 'misere' variant in which the aim is to avoid taking the last counter, but the strategy is usually just a slight modification of that for the 'direct' game.

Battle of Numbers

The earliest account of a Nim-type game that I know of is the Battle of Numbers described in the Problèmes plaisants et délectables of C.G.Bachet, published in 1612. The players take turns in calling out a number, each time adding on a number from one up to a specified limit, t (say 6), and the winner being the first to reach a finishing total, T (say 100). The numbers to try to leave your opponent are T - m(t+1), that is, 2, 9, 16, 23, 30, 37, 44, 51, 58, 65, 72, 79, 86, 93 in the example. When played with counters, 100 are placed in a heap and up to t are taken each time and you must aim to leave a multiple of t+1. Then if your opponent takes s you take (t+1) - s of course.

The play in Cribbage, which was invented around 1640, uses the same principle, with 31 as the target total, but the choice of numbers to be added is restricted to those on the cards in your hand, and the other Cribbage characteristics dominate the Nim aspect.

Nim

The quintessential Nim however dispenses with the numerical limit, t, by arranging the counters in several heaps. The only rule then is that a player must take one or more counters, all from the same heap. Thus if there is just one heap, the player to go can win by scooping the lot, and if there are two heaps he can win if he can equalise the two. With more heaps the strategy is not so obvious, it was first explained in 1902 by C.L.Bouton in the Annals of Mathematics. He showed that the player to go can win if he can 'even-up' the heaps, in the sense that the numbers of 1s, 2s, 4s, 8s, 16s, etc (i.e. powers of two) in the heaps total to even numbers. For example, if there are six heaps containing 1, 2, 3, 4, 5, 6 pieces then there are 1+2+3+4+5+6=21 ones in it, and 0+1+1+2+2+3=9 twos, and 0+0+0+1+1+1=3 fours. Thus the player must 'remove' a one a two and a four to even up. This he can do by removing one counter from heap 4.

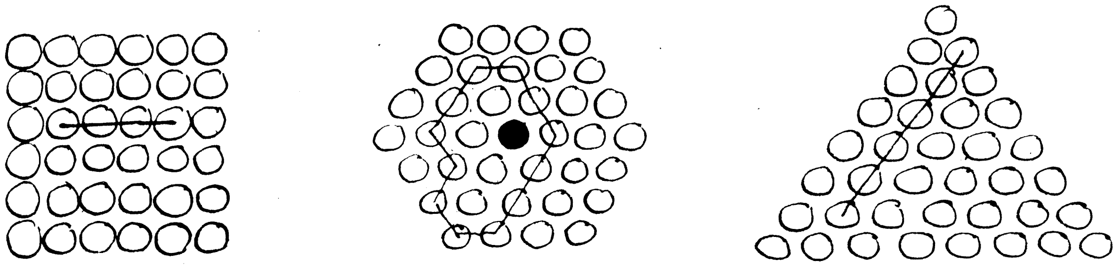
Nimmity

I have coined the name Nimmity (Greek 'mitos' = thread) to apply to a range of Nim type games in which the rule of play is to take one counter or else a group of counters in unbroken sequence along certain lines. [Regulation: The whole take must be made in one motion, the removal of your hand from the layout signifying the end of your go, after which you may not return to take some more, nor to replace any.] This is a very fruitful concept, leading to an enormous variety of possible new forms of Nim play. Here beginneth a nimiety of nimmities! For example:

Rank and File. The counters are arranged in a square array (e.g. on a chessboard). The first player may only take groups that lie along the ranks, and the second player only along the files. By taking a group that crosses both main diagonals the first player can thus prevent the second making a take that will leave a symmetric position.

Black Spot. The array is a hexagonal honeycomb formation with a black counter on the centre point. The lines are closed paths without crosslinks (e.g. three counters each touching the other, or six counters surrounding another). Only complete circlets can be taken, not single counters or open paths. The black counter must always form part of at least one circlet. The object is to force your opponent to take the black spot (because it is part of the last circlet to be taken).

Triangular Nimmity. The array is a triangular honeycomb formation. The lines are the straight lines of cells in the three directions parallel to the edges of the triangle. A single counter or a group of counters may be removed from anywhere in a line, provided the removal does not disconnect the pattern of the remaining counters, i.e. does not result in two areas of counters with no counter in one adjacent to any in the other. The triangular shape, in contrast to a hexagon or square, means that the second player cannot create a symmetric position by an equal, parallel take on the other side of the board. The connectedness condition means this is sort of a 'one heap problem'!



Possible opening positions and first moves in the games. Can anyone say which player should win from these opening arrays? The analysis is not at all easy even in smaller cases.

About the Journal

Apologies for the late appearance of issue 5, which has been combined into a double-size issue with number 6. This has worked out so well - providing more room for some longer articles - that I propose to double-up the summer issue next year also (that will be 11+12).

There has been much talk in the news pages of postal zines about plans afoot for the publication of a 'professional' zine on the lines of 'Games and Puzzles' that ran from 1972 to 1981. Several different organisations or individuals have publicised their intentions, but none has yet managed to produce the goods. Meanwhile The Games and Puzzles Journal does actually exist! It may not yet meet all your ideals, but given a little further support, in the form of subscribers, contributors and perhaps advertisers it could flourish.

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CARD PLAY

Three Bridge Puzzles

By John BEASLEY

Here are the solutions to the questions posed in the last issue.

1. That there is always at least one bidding sequence that cannot occur. This is simply a matter of counting. There are 35 possible bids, ranging from 1C to 7NT. Each independently may assume one of 22 states: (i) not bid; (ii) bid immediately after preceding bid, not doubled; (iii) bid similarly, doubled immediately, not redoubled; (iv) bid similarly, doubled after two passes, not redoubled; (v) bid similarly, doubled immediately, redoubled immediately; (vi) bid similarly, doubled immediately, redoubled after two passes; (vii)-(viii) bid similarly, doubled after two passes, redoubled immediately or after two passes; (ix) bid after one pass, not doubled; (x)-(xv) bid after one pass, doubled or redoubled as in (iii)-(viii); (xvi)-(xxii) bid after two passes, otherwise as in (ix)-(xv). Additionally, the first bid of all may be preceded by three passes. There are thus 23×22^{34} possible bidding sequences, but only $52!/(13!)^4$ possible deals. [The comparison of factorials with powers is just an assertion about numbers; my actual technique is to write down the fractions and cancel out until the result becomes obvious. Approximate cancellations come in useful; for example, we can safely cancel 7x9 against 8x8 when we are trying to prove that the term containing the former is the smaller.] Note that this proof applies even if the players are using different systems; all that is necessary is that each player sticks strictly to his own system.

2. A deal in which any contract by any declarer goes at least two down against best defence. Perhaps the simplest such deal is **A** below. Whatever the contract, the defence can now take the first eight tricks. A slight modification gives **B**. Now the defence can always take at least eight tricks (though not always immediately), and nine if the trump suit is that in which the opening leader is void. Furthermore, the second deal might well attract some bidding.

A

```
            S x
            H x x x x
            D A K Q J
            C x x x x
S x x x x           S x x x x
H A K Q J           H x
D x x x x           D x x x x
C x                 C A K Q J
            S A K Q J
            H x x x x
            D x
            C x x x x
```

B

```
            S -
            H x x x x
            D A K Q J 10
            C x x x x
S x x x x           S x x x x
H A K Q J 10        H -
D x x x x           D x x x x
C -                 C A K Q J 10
            S A K Q J 10
            H x x x x
            D -
            C x x x x
```

C

```
            S Q 10 9
            H -
            D A 8 7 6 5 4 3 2
            C K J
S -                     S K J
H A 8 7 6 5 4 3 2       H Q 10 9
D K J                   D -
C Q 10 9                C A 8 7 6 5 4 3 2
            S A 8 7 6 5 4 3 2
            H K J
            D Q 10 9
            C -
```

3. A deal in which each of the four hands can make 3NT double dummy against best defence. Consider a suit distributed as follows: Q 10 9 / K J / A 8 7 6 5 4 3 2 / –. This yields seven tricks to the side holding the long suit only if there is a side entry to the long hand, for if the ace is played on the first or second round then the third round must be won in the short hand, and if the ace is held up until the third round then the opponents can take the first two tricks. If a preliminary discard is made from the short hand, however, then a first-round duck sets up seven immediate tricks, and there is no need for a side entry to the long hand.

Now consider a deal comprising four such suits, C. Suppose that the defence starts by ducking a heart. Declarer discards a spade from dummy and ducks a spade, setting up seven spade tricks to add to the ace of diamonds and the heart already in the bag, and the defence can take at most three more tricks before allowing him in to claim them. Similar play occurs if the defence takes one or more aces before ducking a heart or club, though declarer must be careful always to answer a heart duck with a spade and a club duck with a diamond; and if the defence leads a spade or a diamond at any time then declarer makes an overtrick.

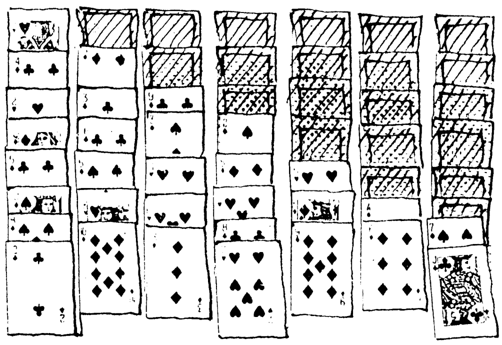
Complete Information Card Game

G.J.SUGGETT writes: "Stephen Taylor's description of 'Psychological Jujitsu' (issue 3, page 35) as the only card game he knows where all players have at all times complete information regarding the state of the game inspires me to mention the game that I know as Besicovitch's game (although I have seen an entirely different game described under this name). The game was supposedly invented by Professor A.S.Besicovitch who, reputedly, offered a fiver to anyone who could beat him (this was twenty years ago when £5 meant something!). The game is one not only of complete information (it is usually played with the hands face up on the table) but also of perfect symmetry in the initial distribution of cards. It is played by two people.

"A 36 card pack is used (2, 3, 4, 5 of each suit removed. Ace counts high). The pack is separated into red and black and only one half (say the blacks) dealt. Whatever one player has been dealt in spades, his opponent is given in hearts, similarly for clubs and diamonds. This procedure ensures the symmetry of the initial hands with (H, S) (D, C) as complementary suits. One player now chooses a trump suit. To preserve the symmetry his opponent will use the complementary suit as trumps. Thus if one player is using spades as trumps, his opponent will be using hearts. There is also an (initially empty) pile of cards in the centre of the table. Play alternates, with the player who chose trumps playing second. On his turn a player has the following options: (i) If the pile is empty he must lead a card from his hand to start a new pile. (ii) If the pile is not empty he must either (a) lead a card to the pile which 'beats' the top card on the pile (i.e. is a larger value of the same suit, or one of the player's trumps) or (b) pick up the complete pile. The first player to empty his hand of cards is the winner."

A Navy Patience

This patience has become popular in my family circles since it was introduced by my youngest brother who learnt it in the Navy. All 52 cards are laid out row by row as illustrated, in 7 rows of 7 with three extra cards tagged on. The 21 face-down cards are usually closed up beneath the face-up cards.



On any of the 7 uncovered cards you may place the next lower card of the same suit, if it can be seen in one of the other columns. Any cards on top of the moved card are moved with it to the new column. When a face-down card is exposed it is turned face-up. When a column is emptied the space may be filled by any King (with any cards on top of it). When an Ace is uncovered it is placed above the layout, then the Two on top of that, and so on, until the whole pack is sorted. Some plays of the patience can be quite long, others quite short, before reaching a blocked position, and quite a number of re-deals may be needed before it comes out. It requires a lot more patience than I have nowadays - I like my patiences to resolve every time - but I'm told that I'm missing the whole point of the thing!

A Note on Royal Carpet Patience

Tom MARLOW: I have long known your Royal Carpet Patience in a slightly different form as follows. Lay out a 3x3 array. Then either play 2 cards onto 2 in the array that total 11, or play 3 cards onto J, Q & K in the array. Success is to dispose of all cards onto the array and comes quite frequently, indeed it is not too difficult to succeed with an array of only 8 positions.

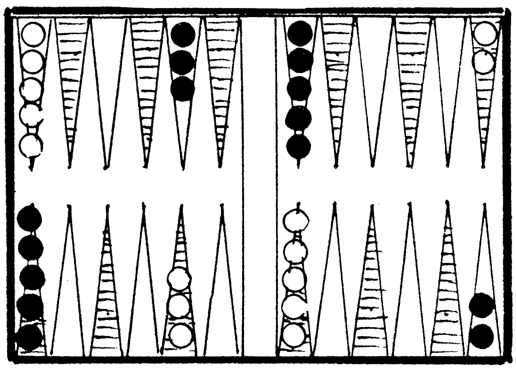
BACKGAMMON - A Challenge from the East

By Dennison NIXON

My first Backgammon set was won in the Sunday School sports when I was about eight years old, but I did not realise what it was because it was in the usual form of a shallow box with a draught-board on the outside. The coloured points on the inside I imagined to be merely decorative and the 12 + 12 draughtsmen included seemed intended for that game, as indeed they were. It was not until several years later that I discovered from Foster's Complete Hoyle that the coloured points were actually those of a backgammon board and that 15 + 15 men were required, though only 12 + 12 men were supplied, no doubt some oversight by the marketing firm.

From that time I have played the game regularly, especially during my 16 years with British Steel where we found it ideally suitable for lunch-time play, a game averaging only about 10 minutes. As we had a steady influx of new-comers, I must have taught dozens of people to play the game, and always drew attention to the fact that Backgammon is one of the few games where there is almost always a chance that the under-dog may win in the end. Also, to indicate that the game had at least one distinguished exponent, I would inform them that Charles Darwin played two games almost every evening with his wife Emma, and kept records of the results running into thousands of games.

Opening Position for 'Hit'



After almost a life-time of regular if not too intensive Backgammon I was therefore much surprised to find in a local public library the following statements in a book entitled Backgammon, Games and Strategies by Nicalaus and Basil Tzannes: "Plakoto and Moultezim are much more interesting than Hit" the name the authors use to distinguish our Western form from the two Greek and Middle-Eastern games. Again: "Plakoto is the king of Backgammon games. Moultezim is a serious game for the fundamentalist, the pure strategist, the complete Backgammon player. A player who does not know these two games is not really fulfilled." The authors are two Greek brothers, now living in the U.S.A. and since the avowed purpose of the book is to amplify these statements and to introduce the two games to the western world, I felt impelled to read their book, which I have done twice, including the several illustrative games they give on all three types of Backgammon. This I found most stimulating and must agree that they present a very persuasive case for the two Greek games.

My regular Backgammon opponent is my Bridge partner, with whom I am engaged (once a fortnight) in a marathon match at Hit, points running already into the 400s, but we do not use the doubling cube which seems to me not really in the spirit of the game. Results, on standard scoring, show that a double game occurs about once in every seven, a treble only once in about 90.

In view of the Tzannes brothers' claims we have now played a series of games at both Plakoto and Moultezim and I find them both really challenging, as the authors claim. My opponent still prefers Hit but I myself find the Greek games at least as enjoyable to play, but must have more experience of the new games before deciding on their respective merits as against Hit with its vast traditional attraction. The following descriptions of the two games assumes a knowledge of our Backgammon, which is easily obtainable if not already held.

Plakoto

All 15 men of each player commence on his one-point and move all round the board to the opponent's home table (immediately opposite) where, when all 15 have arrived and none are under capture, they may be borne off in the usual way, with double and treble games scorable, also in the usual way.

A point is held by two or more men and no enemy man may move there. A single man on a point is a blot and can be made prisoner by an enemy man moving to that point. The enemy man now holds both point and prisoner, which is released only when his captor moves away. The capturing side may move as many more men as desired to the point.

If the 15 men on either player's one-point are reduced to a single man this is known as his 'mother' and is subject to a special hazard. If a mother is imprisoned by an enemy man, the enemy immediately scores a treble game unless his mother is still also on his one-point and thus liable to capture. If that capture happens, the game is declared a draw (the only draw in Backgammon). Play may be necessary after the capture of the first mother until the opposing mother is captured or moves from the one-point.

The throw of an early high double is of special importance in Plakoto as it enables a man to move right round the board, even sometimes from the one-point, and it may capture a man in his home table. Such a capture is a great and lasting advantage and may well lead to a win. One or two blots in one's home table are almost inevitable in the early stages but it is always advisable to weigh up the chances of their being hit before making them.

Primes (6 held adjacent points) may include one or more captured men and are just as important as in Hit.

Moultezim

All 15 men of each player commence on his one-point but the two home tables are not opposite one another, each player's home table, with one-point, being on his left. Also, all men move in the same direction, anticlockwise. A single man holds a point and may be joined by more men of the same side, but bars any opposing man. As in other Backgammon games, each player's aim is to move his men right round the board (in this case to the opponent's outer table) and when all have arrived, to bear them off, with usual scoring of double and treble games.

At the beginning of the game each player is allowed to move only one man until he gets this man into the opponent's home table, after which he may move any man. This freedom from restriction may come into effect after the first move of a throw.

Neither player is allowed to form a prime in his own home table but must always leave at least one point unoccupied.

More Quotes from the Tzannes Book

"In an informal game, cups are a nuisance. Just throw the dice with your own hands and enjoy the 'feel' of the game."

"Plakoto is here explained in detail for the first time for readers of the English language. It is undoubtedly the most exciting form of Backgammon."

"...the game of Hit is not Challenging enough for a mature player. In Greece and the Middle East, where Backgammon is part of daily life, a player who knows only Hit is scorned as an unfulfilled player. Maturity, they say, starts with Plakoto and reaches its peak in Moultezim."

"A typical game of Hit lasts 6 - 8 minutes, but Plakoto and Moultezim each take twice as long." Our experience shows 10 minutes for Hit and about 40 minutes for either of the other two.

Unfortunately the book, originally published in 1977, is at present out of print.

Deep Draughts: In the diagram on page 53 of the last issue one set of the pieces should be Black, since once they have reached the end of the board and turned round they would otherwise be indistinguishable from the opposition.

Postal Soccer Games

By Alan Parr
Hopscotch, 6 Longfield Gardens, Tring, Herts, HP23 4DN

A recent survey showed that the number of people playing sports games (particularly soccer management games) by post is at least comparable with that playing Diplomacy. Indeed, if everyone playing in any such games, some of which may well have over fifty players, is included, the total may well substantially exceed that playing any other family of postal games. Of course, no one actually knows just how many are involved, particularly since many of the games and their players have little contact with the world of the traditional Diplomacy zine, but there is no doubt at all that by any standard the number is large.

All such games work on a broadly similar basis. The player takes the part of the chairman/manager/coach of a soccer club whose resources include some cash and a number of players. These players will naturally be of varying skill levels, able to operate in definite positions, of varying age and perhaps even character. For each match - and normally one session of play will contain two or three games for each club - the manager must select not only the best players available to him but must also use them in the most appropriate way, using the best formation and tactics - which will of course vary depending upon the opposition, venue, and so on. He sends these details to the GM (gamesmaster, or organiser) of the game, as do all the other managers. It is the GM's monumental task to record and evaluate all these details and to play each and every match by comparing the strengths of the two sides by applying a special formula. Whether he uses simple dice or a computer this formula will produce the match result and very probably the goalscorers and times of the goals as well. He then types up the results, league tables, administrative details and so on, and arranges for each manager to receive a copy of the updated information preparatory to playing the following session - with a little bit of luck he may be able to put his feet up and relax for a full couple of days before he has to start worrying about the next session.

What most definitely makes life interesting for the manager is that he is in control of all aspects of his club, so he decides not only whether to pull a forward back into midfield for an away match, whether using a sweeper is worthwhile, etc, but also conducts transfers, directs coaching policy, and so on. He may also have to make decisions on whether his side will play hard, use the offside trap, settle for a draw when playing away, etc. He may need to bear in mind the absence of a star player through injury, suspension, or call up for international duty. Perhaps he will have to consider the lurking presence of a known hardman in the opposition defence or be aware of the foibles of individual referees. (No one league will present all these problems - indeed, every game has slightly different rules - but in each he will face problems which must be handled in a sensible and realistic way, and though the best managers are likely to achieve the most success, plain old-fashioned luck will play a small part.)

I've tried once or twice to trace the history of such games. It seems clear enough that the very first game can be traced back to Clive Booth and Chimaera. My guess is that Clive's game began in 1976 (David Phillips suggests around August 1975). In this game teams used a goalkeeper, defenders, midfield players, and forwards, with every side having to employ exactly three MFs, their strengths being added either to the defence or the attack or split 50/50 between the two. A side's FWs were then compared in total strength to the opposition total DF strength to give a percentage chance of scoring from a chance (e.g. if A's attack totals 18 and B's defence 12 there is a 18/(12+18) = 60% chance of scoring from an attack). The actual number of chances basically depended on the total strength of the side.

More or less contemporaneous with Clive's game (though in fact perhaps a shade later, since the rules acknowledge Clive's pre-eminence) were those used in Leviathan. The most obvious thing about them is just how different they are - there's certainly no impression of their borrowing extensively from Clive's set. Indeed, they come across as a much more primitive game, with only four strengths of players (effectively levels 2-5) coded by colour - thus a yellow player at level 5 was the strongest available. All teams had to utilise a 2-3-5 formation (ten years after Ramsey's 4-3-3, and nearly twenty years after the first whispers of 4-2-4 began to emerge from the continent!) and no player could be used away from his qualified position. The overall team strengths determined the number of scoring opportunities, so that a side with total strength 52-55 would get between three and six (the exact number determined by dice) chances each half, while a much weaker side with 32-36 levels would only get between zero and three chances. Further dice throws decided the actual outcome of the shots, with the home side having a higher chance of success.

I suppose both of these games were pretty primitive by today's standards, and I can see why they attracted a fair amount of flak from some quarters. Nevertheless, in some ways they were relatively advanced, and in one important aspect - that of finance - they were already more complex than United games are today. In any case, of course, their importance lies in their pioneering the path which so many other games were to follow in the next few years.

Editor's Note. The above article is based on two that appeared in Alan Parr's Zine Hopscotch (Nos 37 and 41 in 1983-4). A recent issue of Hopscotch (No 77, July 1988) reports the result of 'The Sixth International United Champions' Tournament' in which 45 teams took part (20 from Germany, 19 from Britain, 4 from USA, 2 from Netherlands). After preliminary elimination rounds 16 teams played a two-leg home and away knockout competition. The line-up for the semifinals consisted of: 'One of Germany's greatest sides, Essens Legendare Fussballtruppe, under Klaus-Jurgen Fleischer; Blackwater Edge, managed by the top American Dan Stafford; Twickenham Academicals, the first ever winners of the competition back in 1982, managed by Andrew Whiteley; and the reigning champions, Last Laugh, under Andrew Spencer.' The aggregate scores were: TA 3 ELF 2, LL 4, BE 2, and in the final: LL 2, TA 3, so Andrew Whiteley with Twickenham Academicals became the first manager ever to win the International competition for the second time.

Other Zines

Back in issue 1 I promised a list of overseas zines - there must be many, but those that I have actually seen are mentioned below, together with other periodicals of interest. I hope to give fuller reviews of some of these next time (especially WGR).

Veni, Vidi, Vici. Thomas Mendel, Vaillantlaan 403, NL-2526 AA Den Haag, Netherlands. Although published in Holland this is in German. Mainly Sports Simulations and Dip.

Hispiduri. Jutta and Paul Merken, Lubbeweg 21, 2940 Wilhelmshaven, West Germany. A horse-racing simulation (Win Place and Show), other sports and business games.

Spielwiese. Johannes Schwagereit, Hilgardring 32, 6750 Kaiserslautern, West Germany. Besides the usual includes a gun-fight game, High Noon, Cluedo and Finchley Central!

Domino Principle. Erik Joustra, Topaasplantsoen 404, 2403 DK Alphen aan den Rijn, Netherlands. Mainly sports simulation games, football, cycling, baseball etc.

Diplomacy World. Larry Peery, Box 8416, San Diego, California, 92102, U.S.A. Issue 47, Summer 87, was an 80-pager, devoted entirely to Diplomacy and its variants.

World Game Review. Michael Keller, 3367-I North Chatham Road, Ellicott City, MD 21043, U.S.A. ($8 for 4 issues). Covers much the same field as G&PJ. Particularly good on dissections, chess variants, cryptography, review and classification of games.

Wargame News and **PBM** (Play By Mail). M.W.Costello, 'Emjay', 17 Langbank Avenue, Rise Park, Nottingham NG5 5BU. These are two A4 computer-produced magazines devoted mainly to games reviews and trade-oriented news (subs £10.50 and £5.50 UK).

3D Sopwith

Slightly disappointing is the fact that the flying in Sopwith is purely two-dimensional. It could equally well be taken to simulate some form of desert tank battle (with the clouds as oases or sandstorms perhaps). I therefore propose a slightly more elaborate version in which the planes can fly at three different levels - cloud level or above or below the clouds. The level occupied by a plane can be indicated by a level-marker occupying the same cell. Two extra moves: Climb and Dive, both leading to the cell directly ahead, but one level higher or lower, are the minimum necessary. Firing can be allowed in these two directions also, but the range is reduced to one or two cells (e.g. firing upwards from level 1 of cell 1 enters cell 2 at level 2 and cell 3 at level 3). Planes entering the same cell must do so at different levels, otherwise they collide and each suffers two units of damage (and forfeits any burst of fire scheduled for that move).

WHAT'S IN A GAME?

Continuing the article in the last issue, concerned with various general concepts that are applicable to a wide range of games, and with the classification of games.

11. Good and Bad Moves. As noted in section **7** a position may be termed winning, losing or drawing for the side whose turn it is to play. What is meant by saying that a move in a position is good or bad? If my analysis is correct these terms can have very different meanings according to the type of position. Good and bad moves are often annotated ! or ?.

In a winning position there must be at least one move that retains the win, that is converts the position into a losing position for the other player. Any such move is a good move, and any other is bad. A bad move that retains the draw may perhaps be described as poor, while one that gives the winning chances to the opponent is presumably wicked! If you have a winning position, then by playing good moves you retain the advantage and eventually win, but it is better to win in style, which means in the fewest moves possible. Thus, among the good moves you need to choose the best, which are those that win quickly. Such play shows what is called brilliance. A position in which there is just one best move may be termed a problem position - and the unique move that wins brilliantly is the key.

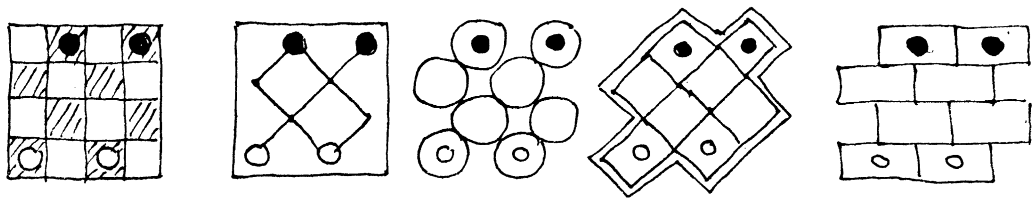
In a drawing position similarly there must be at least one move that retains the draw, and this is a good move too. Any other move is bad. In the 'Kilkenny Cats' situation in Chess (K v K) the draw is absolute: no move by either player can win or lose. The same goes for other degenerate positions (K,B v K; K,S v K; K,XB v K,XB). The position K,S,S v K is not quite degenerate however, since while White cannot force a win, Black can let him win by playing stupidly. The same goes for K,S v K,S; K,S v K,B; K,B v K,S; and K,XB v K,YB. Thus if the opening position is even (as is generally supposed in Chess) then for a win to occur at least one bad move must be played. Brilliance (as defined above) is also, surprisingly, to be found in drawing - this is what is shown in 'Draw' endgame studies.

The characteristic of a losing position is that no matter what move a player makes he is bound to convert it into a winning position for the opponent. Thus in a losing position there are no good moves in the above sense. All the player can do is to try to induce the opponent to make a mistake; by complicating the situation, i.e. multiplying the number of choices open to him so that he has more chance of making a mistake; by delaying tactics, so that the opponent's choice of moves is spread over a longer series in which there is thus again a greater chance of error; or by seeking a more problematic situation, i.e. reducing the number of good moves available to the opponent.

This probabilistic style of thinking on the part of the players is also applicable in drawing and winning positions, the purpose being to reduce the ratio of good moves to other moves so that the chance of an error is increased.

12. Board Games. A board game is characterised by positions (see note **3**) in which pieces (or men) are assigned to places (or cells). The rules of the game specify in what ways a player may choose to transform one position into another. The cells collectively constitute the board on which the game is played, and the pieces the forces employed.

The cells take the form of squares in the case of Chess, but in other games may be hexagons, dots, points of intersection of lines, peg holes, cups, and other forms. The same game can often be played on a wide variety of different-looking boards; only custom specifies one design in preference to others. For example, all the following designs would be suitable for a game of Miniature Draughts:



The physical appearance of the board only serves as an aid in moving the pieces according to the rules. In some cases the spatial arrangement may be inadequate or misleading; as for example in the use of a flat board for playing Cylinder Chess. The structure of a game lies in its rules, not in the board.

The powers of a piece consist of those rules of the game that govern its play. Pieces of different powers need to be of different physical design (like chessmen) or to bear the title or symbol of its status, as in Shogi or some pocket magnetic sets. Some games employ only pieces of one type, and in such games the distinction between the structure of the board and the powers of the pieces becomes blurred. In games which involve pieces of several different types the cells of the board may be linked together by the moves of the various pieces in quite different ways.

Games like Chess in which the rules restrict us to one piece per cell and one cell per piece may be termed monic games. Games not following this rule are thus presumably de-monic! They employ pieces that occupy more than one cell (for example, many Sliding Block Puzzles) or cells that can accommodate more than one piece (for example the 'points' of Backgammon or the 'cups' in the seed-sowing games of the Mancala family).

Comments of Readers

Peter KINGS writes concerning section 10 of this article, on Odds: "The second half of the sentence devoted to Chess strikes me as avoiding the issue, that is, does a satisfactory method of handicapping exist in Chess? The all-pervasive grading system may help to prevent players of widely differing abilities having to play each other (although it does happen). However it is no answer to the problem of how significantly ill-balanced players can yet be provided with the opportunity for a good contest. Pawn and move was an earlier attempt but it was found to change the structure of the game. Also, in the long ago, some club players were seemingly content to be described as 'knight' players (i.e. receiving those odds against the club's best) but today we live in an egalitarian age and a low number is judged to be less painful! As Hooper & Whyld mention in their Companion, the only satisfactory method of handicapping is by time, although this has yet to be seriously employed. Bobby Fischer 20 secs, P.R.K. 20 mins might, for example, have produced a contest. It is interesting to note that the great oriental game Go (Japanese) or Wei-Ch'i (Chinese) has a very satisfactory handicapping system."

Michael KELLER makes the same point, about handicapping in Go. He also comments on other sections of the article: "Theory ofGames has a specific meaning in mathematics (referring to a methodology for analyzing strategy games, usually involving matrices) and is perhaps best avoided as a page heading." [It also has a specific meaning in English - which takes priority over specialist jargon in my book! G.P.J.] (4) Matches: "The corresponding American terms are 'round robin' ('all-play-all') and 'elimination' ('knockout')." (6) Interaction: "There is a distinction in my mind between two types of interactive sports. Directly interactive sports (American football, soccer, hockey, basketball) are those in which players can physically interfere with the opponents - these are also often called contact sports. Indirectly interactive sports (baseball, cricket, tennis) generally have little, if any, interference between opposing players. The dividing line, of course, is not absolutely sharp; even in track, basically a non-interactive sport there can be some interaction (e.g. a 1500m run, where runners may try to box others in)." (8).Turns. "I like your distinction between 'move' and 'turn' (this comes up in complex board wargames too, where there is often a long 'turn sequence' describing the types of moves possible in a turn and the order in which they may be made). Another piece of terminology I would like to see followed more closely is the card game distinction between 'deal' (one round of play in which all or some of the cards are dealt out to the players) and 'hand' (the set of cards held by a player - this is often carelessly used as a synonym for 'deal')." (9) Chance. "Monopoly is not a race game; the term 'track game' tends to obscure the nature of the game."

Philip COHEN comments similarly: "Races are generally not non-interactive at lengths above dashes; there are strategic considerations, not to mention jostling and such if racers aren't confined to their lanes for the duration. Have you considered how Nomic (in Douglas Hofstadter's magnificent Metamagical Themas, ch 4) fits into your scheme?

Further details on Go and Nomic would be appreciated.

CHESSICS

Anthony Dickins Memorial Tourney

The next issue of The Problemist will include an announcement of a composing tourney in memory of Anthony Dickins, who died on 26 November 1987, age 73. The tourney will be for Series Problems of any type on Cylindrical Boards (including the anchor-ring or torus) and will be in two parts: (a) orthodox pieces (b) fairy pieces - not more than two types. The Judge will be Cedric Lytton. Entries, not more than 3 per proposer altogether, should be sent to me: G.P.Jelliss, 99 Bohemia Road, St Leonards on Sea, East Sussex, TN37 6RJ. The closing date for entries is 30th September 1989.

Chessays

I am well on the way to re-issuing the Chessays series and expanding it with some further titles. Future titles in the series will be strictly of 16 pages, A5 size, with paper cover and priced at £2 per issue. Production difficulties, and cost, have forced me to give up the card covers and larger format that was attempted with No.3. Fuller details will be announced in the next issue of the Journal.

Comments and Corrections

Ian SHANAHAN prefers a version of my Problem **11** with Ps f4,f6,e7 moved to h2,g3,h4. This increases the length to Shm 35 and makes all the WPs "structurally necessary". I prefer the published version however since it shows the task (of BK circuit eating all but two of the complete W forces) in pretty (arrow pattern) and economical form (including the number of moves). Not all tasks are Maxima.

Harald GRUBERT corrects **17** by making the piece on c6 a Black Rook, and corrects **18** by moving the Black King to d6 and the Black Rook to e5.

Stephen TAYLOR corrects **34** by moving Black Bishop f4 to g5.

J.C.DUMONT (writing some years ago) informed me that Echecs Marseillais was first published by Albert Fortis in 1925 in the Marseilles paper Le Soleil.

Grasshopper Chesses

Philip COHEN: Since Grasshoppers are weak they tend to come in swarms; the G-Chess I've played (described in Games magazine in the last year or two) uses 8 each on the second rank (Pawns go on the third).

Grasshopper combined with King: Some alternative examples of shortest games to mate of one K+G by the other are the following, dating back to FCCC days: (a) GPJ, 1K+Gc3 Sf6 2Pf4 Sc6 3Pf5 Se4+ 4K+Gxh8 mate (b) C.C.L.SELLS (now C.C.LYTTON), 1Pe4 K+Ge6 2Qh5 Qe8 3Sc3 Pa6 4Pe5 mate (c) J.D.BEASLEY 1Pa4 K+Gc6 2Ra3 K+Gb6 3Pd3 K+Gxb1 4Qd2 mate. A surprising assortment of different mate positions.

As several correspondents have noted the solution quoted in issue 2 (page 22) was one move-pair too long (presumably this was my first try at it and I had forgotten to update my records). Here is another solution by C.C.FRANKISS (Brasil): 1Sh3 Sh6 2Sf4 Sf5 3K+Gc3 c6/Sc6 4K+Gxh8 mate, using the S instead of the P to guard the Black Scorpion's escapes.

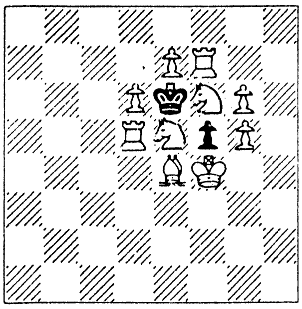
Caissa's Kaleidoscope

There are some longish stipulations this time - solvers take care! Thanks to the composers of **66** and **71** for the dedications. In **72**: Fers = (1,1) leaper, i.e. single-step Bishop; (a) is a usual Serieshelpmate in 8, (b) is a Serieshelp-discovered-mate in 21 with 'Royal Precedence' (i.e. where there is a choice of move order the King goes first - your opinion as to whether this dual-avoidance condition is justified would be valued). **74** is a zeroposition - i.e. the diagram is not itself a problem. In **75** 'Circe Malefique' means captured men reappear on the other side's home squares. In **76** c2 is a White (1,3)R=camelrider, and f1 is a Black (1,2)R= Nightrider, One player makes a series of 3 moves, then the other makes 2 moves, the second giving mate. The captured men in Antipodean Chess reappear a (4,4) leap away. In **77-78** the Zebra is a (2,3) leaper, the Bison is a (1,3)+(2,3) leaper. **80** is the composer's first problem I believe.

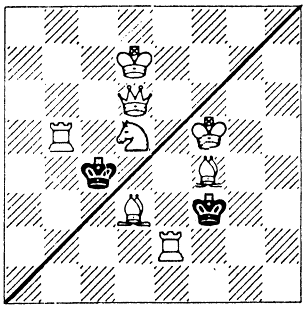
MORE ORIGINAL COMPOSITIONS URGENTLY REQUIRED!

All problem compete in an Informal Tourney: Judge 1987-8 Hans GRUBER.

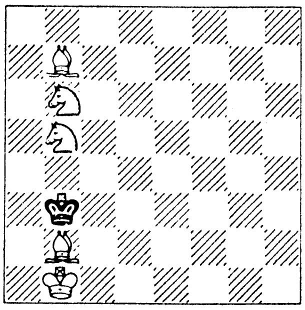
65. L.N.BORODATOV
Mate in 1. (b) Rotate the board 90° left. 'Road Signs'.



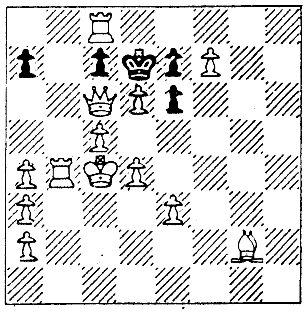
66. F.M.MIHALEK
Mate in 3. Whole board for each. Dedicated to G.P.Jelliss/Journal.



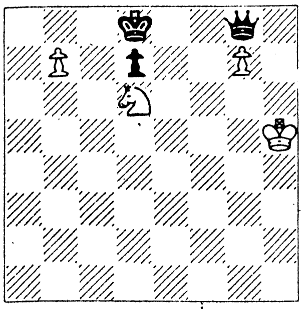
67. E.HOLLADAY
Mate in 5, two ways. (i.e. two separate key moves).



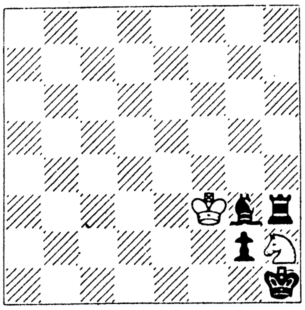
68. L.N.BORODATOV
What were the last ten single moves?



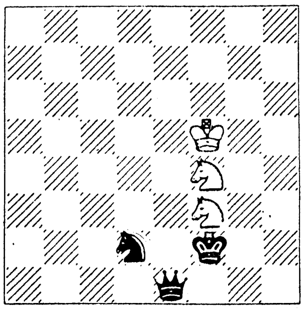
69. Erich BARTEL
Helpstalemate in 2 with set play.



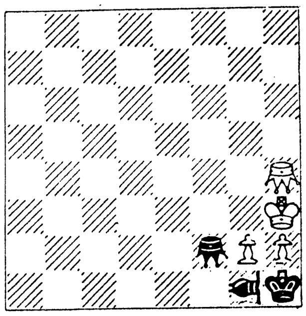
70. F.M.MIHALEK
Helpmate in 2
(b) interchange h2-h3
(c) remove Rh3



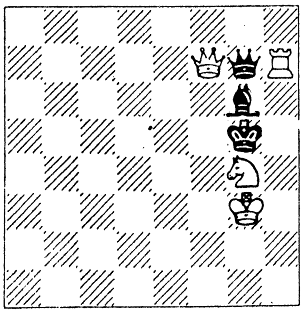
71. E.HOLLADAY
Helpmate in 3.
(b) Black Rook d2.
J for Jelliss.



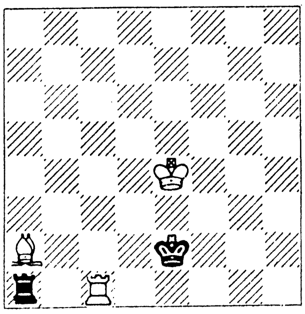
72. G.P.JELLISS
Grasshoppers.
Non-capturing Fers.
(a) Shm8 (b) See text.



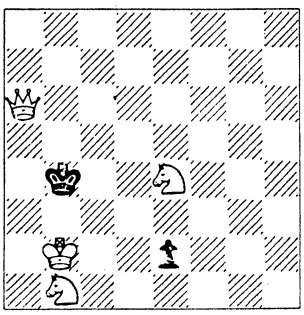
73. F.M.MIHALEK
Helpstalemate in 2 Circe Chess.



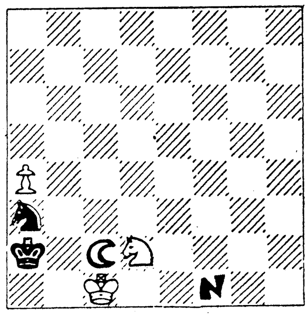
74. E.HOLLADAY
Helpstalemate in 2½
Circe - zeroposition.
(a) e2-d2, (b) a2-h2
Two ways in each.



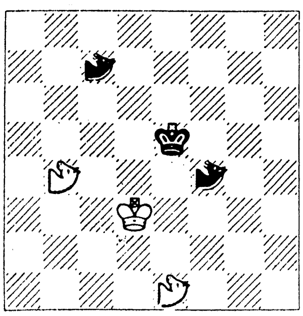
75. Erich BARTEL
Helpstalemate in 2 in 4 ways. Circe Malefique.



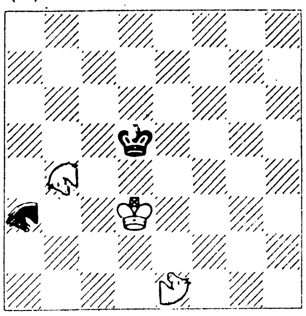
76. G.P.JELLISS
(1,2) and (1,3) Riders.
Antipodean Circe.
Shm 3+2. Duplex.



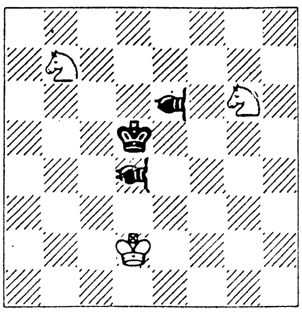
77. H.GRUBERT
Helpmate in 2, duplex. Zebras.



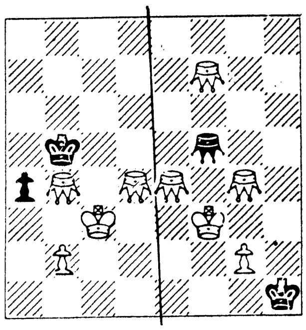
78. H.GRUBERT
Zebra e1. Bison a3,b4.
Helpmate in 2.
(b) b4-f4.



79. G.P.JELLISS
Ferses. Helpmate in 2 with set play.



80. V.PRIBYLINEC
Grasshoppers.
Helpmates in 2.



Solutions to reach me by 1st November.

CHESSICS

CHESS PROBLEMS - Solutions and Comments

The extra time allowed for solving in issue 3 has not resulted in any more solvers and has just got me confused, so I am reverting to the original plan of publishing the solutions in the immediately following issue. Two lots of solutions therefore follow.

Solutions to Issue 3.

29. MIHALEK. (a) 1Qd3+ Kg4 2Qd5 Kh4 3Qg5 mate. (b) 1Qe1 Kg2 2Kg4 Kh2 3Qg1 mate (c) 1Qd4 Ke2 2Kg2 Ke1 3Qd2 mate. The checking key floored several solvers.

30. WHITEFIELD. 1Qd5 PxQ 2Bf4 (not 2Bg5 or Bh6 else reflex mate) Pd4 3Sd2 Pd3 4OOO Pxe2 5Kb1 Pxe1=Q mate.

31. WHITEFIELD. 1Se3 2Rxc2 3Rxc1 4Rxb1 5Rb2 6Rf2 7Sxd1 8Se3 9Sf1 for OOO stalemate. A nice sequence - first time I have seen Bk clearing the decks for Wt OOO.[D.N.]

32. SHANAHAN. 1Rh8 2-8Kg1 9Rh1 10Rgh2 11g2 12Qh3 13g3 14g4 15-16Bh4 17g5 18Bh5 19g6 for Kxe2 stalemate. Familiar manoeuvres skilfully welded. [D.N.] The initial position resembles a sword and the final position a shield. [I.S.]

33. HOLLADAY. (a) 1Bb4 Kc6 2Bf3. (b) 1Rd7+ Ke6 Bg4. (c) 1Rb5+ Kc6 Be8. (d) 1Rb1 Kc2 Bg6. (e) 1Bf2 Kc4 2Bf7. (f) 1Bf7+ Kc5 2Bf2. (g) 1Bf3+ Kd6 2Bb4. (h) 1Bf7+ Kd6 2Bg3. (i) 1Bf3+ Ke5 2Bc3. Is this the record for number of 'twins' with near-perfect echo mates? [A.W.I.] Are these all the possible mates? I don't think so! [E.B.]

34. TAYLOR. 1Bd6 Bf4 2Bc5 Sb5. and 1Re6 Sa4 2Re3 Bc3. Unpins. Cooks: 1R on rank Sd1 2Be5 Be3. [T.G.P., A.W.I., E.B.]

35. TAYLOR. 1Rf2 Sb4+ 2Ke3 Sf3. and 1Qxg3 Sd3+ Kf3 Sf6. Well matched pair. [R.B.] Very difficult especially after his **34** which is quite easy. After finding the single-check Rook mate a single-check mate from the Bishop seemed hopeless but is achieved after a brilliant key captures what looks like an essential WP. [D.N.]

36. HOLLADAY. 1Se1 Sa5 2Pb3 Kb4 3Kb2 Pb5 4Sd3. and 1Ka3 Sc5 2Pb4 Sa6 3Kb3 Pb6 4Sd4. The set play 1...Sd4 2Sxd4 Ka4 3Sc6 Pb5 4Pb3 in worth noting. [R.B.] Amazing that chameleon [echo] is forced on b-file, but unobtainable on a-file. [D.N.]

37. MIHALEK. (a) 1Pd5 Qf4+ 2Be4 Kb4. (b) 1Be5 Qf5 2Kd5 Pe3. Echo ideal pin stalemate.

38. MIHALEK. 1Rf7 Pxf7 2Be8 Pxe8=Q 3Sd7 Qxd7. Letter T becomes I. Ideal pin stalemate.

39. HOLLADAY. (a) 1Se3+ Kb4 2Kb3 Ka4 3Sd5. (b) 1Sd3 Kd7 2Sc5+ Kd6/Ke8 3Sb7+/Sb7 Kd7/Kf8 4Kf7/Sd6. Asymmetry. Ideal stalemate. Stalemate set in (a).

40. HOLLADAY. (a) 1Sa5+ Kb6 2Sb7 K- 3Pd5(+) Kb6 4Kb8. (b) 1Kd8 Kb7 2Sa5+ Kb6/Ka7/Kb8 3Kc8/Kc8/Sb5 Ka7/Kb6 4Sc4/Kb8. Reflection asymmetry. Stalemate set.

41. WONG. On 8x8 board: 1Bg5 Rb6 2Bf4 Rb5 3Be3 Rb4 4Bd2 Rb3 5Bxb3 Ba2 6Bc3 mate. Cook: 1Bd2 Rc7 2Be3 Rd7 3Bf4 Re7 4Bg5 Rf7 5BxR B- 6Bf6 mate. (E.B.] Roman march.

42. VAUGHAN. 1Kb6 Gd5/Gd7/Gc7 2Ka7 Gc7/Gc7/Gd7 3Ka8 Gb7 mate. 3 'ways'. This was sent for my Grasshopper collection and is dated 4/10/77.

43. INGLETON. 1Kf2 Gg7 2Gg1 Rh6 3Kg2 Rb2+ 4Kh1 Ga1. and 1Kg2 Rf6 2Kh1 Rf4 3Gg4 Gg3 4Gg2 Rh4. Both possible mates cleverly engineered. Impresssive with only 4 pieces. [R.B.]

44. JELLISS. Left: 1Qa5 Eb6+ 2Ka4 Rb4 double check mate. Right: 1OO Rg2+ 2Kh1 Rg4 3Rf3. Antipin of Rg4 by Eg8. The left part was solved correctly only by T.G.P.

45. JELLISS. 1Mc5 Ke6 2Ke4 Be7 3Rc6. and 1Mc3 Be5 2Ke2 Ke4 3Rc4. (0,2) translation echo.

46. JELLISS. 1Mc3 Kd5 2Ke2 Ke4 3Rc4. and 1Kg4 Ke4 2Mf6+ Bd4 3Re6. Reflection echo. The difficulty with these two lies in visualising the mating configuration. [R.B.] These bifurcating leapers necessitate double-jointed thinking! [T.G.P.] Black B antipinned by WM.

47. JELLISS. (a) 1Ka3 Mb6 2Ka2 Kc2 3Ka1 Sa3. (b) 1Kb5 Md6+ 2Ka5 Kc4 3Ka4 Bb4. I've an idea I have published this somewhere before, but cannot recall where. (Schach-Echo?)

48. JELLISS. 1Sg6+ Kg8 2Me7 Kh7 3Sf8 mate. (Compare A.H.Kniest, Caissas Froeliche Tiefgarage, 1970 (13L) BKg8, WKg6, WSh7, WSf7, WGg5, Mate in 3, 1Se5 Kh8 2Gd5 Kg8 3Sf7.)

This does not qualify in the informal composing tourney, since I find it was on of those that was used in the 1978 solving competition at Canterbury (see The Probelmist, Nov 78, p266).

Parallel Time-Stream Chess. Dr Tylor's answer to the problem on page 38 (issue 3) is that: White changes 3Qf3 into Be6, leading to 3...Pfxe6 4Sc3 Pg5 5Qh5 mate.

Solutions to Issue 4.

49. NETTHEIM. 1Bb2 2Sa3 3OOO 4-6Qa1 7Kb1 8-9Bd2 10Kc1 11Sb1 12Qb2+ for Bxb2 mate. Finale hard to find and even harder to reach. [D.N.] Nicely controlled shuffling. [A.W.I] The author challenges readers to revise the problem so that it includes promotion.

50. TAYLOR. 1Rf6 Bb3 2Kf5 Sxd6 and 1Qe2 Rg8 2Kf3 Sd2. A fine setting of R&Q 'Kritikus' [E.B.] Pleasingly unified and quite difficult. [R.B.] Difficult with echoed self-pin. [D.N.]

51. HOLLADAY. (a) 1Se5 Rd6 2Sfd3+ Kd5 3Sgf4. (b) 1Sh2 Ke5 2Sh4 Rd6 3Sg4. (c) 1Sfd4 Kc4 2Kd2 Rc5 3Sge3. (d) 1Kc2 Kc4 2Sd3 Rb5 3Se3. (e) 1Se3 Kc6 2Sc4 Rc5 3Sd4. Another quart from the composer's pint pont. [R.B.] Find 3 horses much more difficult to control than 2. [D.N.] Two pairs of echoes, but (e) is an odd man out. [A.W.I.]

52. MIHALEK. 1Re4/Re6 Sc7/Sc3 2Ke5 Kg5 3Pd4 Sc4. and 1Re3 Sc6 2Ke4 Kg4 3Pd3 Sc3. The author had the third solution as a twin using a 'superpawn' but the editor preferred to use the 'opting pawn' since not sure of the rules for the former.

53. STEUDEL. (a) 1Pe8=B (not Q) 2Bc6 3Bg2(Sg8) mate. (b) 1Pe8=Q (not B) 2Qe3 3Qg1(Bf8). A pity about Pe2. [E.B.]

54. STEUDEL. 1Pa8=S 3Se6(Qd8) 4Sf4(Sb8) 5Sd5(Pd7) 6Sc3(Pc7) 7Sb5(Pb7) 8Sa7 mate. Merry-go-round! [E.B.] Layout strongly hints at S-tour! [A.W.I.]

55. JELLISS. 1Kh3(Sb1) Sc3 2Bg3 Sd1(Qd8) 3Qh4 Sf2 mate. Pleasing miniature ideal Circe mate. [A.W.I.] A 'find'. [D.N.] A wonderful Circe play and mate. [E.B.] D.N.]

56. HOLLADAY. 1Kh3 Kh5 2Qg3 R5f3. and 1Qg5+ Kh7 2Kh5 R2f4. Fine/Nice echoes. [E.B.,

57. MIHALEK. 1Bg3 (not b4) Rc5 2Rb4 Rc1 3Rb1. Pin ideal stalemate.

58. MIHALEK. 1Bc5 Qd4 2Sb1 Ke3 3Kg3. Resurrection of both Bs prohibited - excellent. [TGP] Good try with the other B pinning the Q. [D.N.] Harmonious pair of one-liners. [A.W.I]

59. BARTEL. 1Bc3/Bh6/Bg5/Bf4 Pf2 2Bh8/Bf8/Bd8/Bb8 Pxg1=R/B/Q/S(Ra1) Rxg1 stalemate. Smart idea, economically realised. [R.B.] A novel way to show AUW - pleasing. [T.G.P.]

60. HOLLADAY. 1Pb7 Ka3 2Pb8=R Qb3 3Rc8 Pb4 4Rc3. Not sound with BRb4, since PxRa8.

61. SHANAHAN. Qe8(Ke1)++++ Ke2(Ra8) 2Rd8 Kf2(Pf7) 3Kd1 Ke3(Sb8) 4Sd7 Kd3(Sg8) 5Se7 Kd2(Bf8) mate. Walling up the Queen. [G.P.J.] For those unfamiliar with Circe Kings they are only in check when attacked, if (a) they are at home or (b) their home square is blocked and cannot be vacated.

62. SHANAHAN. 1Ph2 NPh8=NS 2NPg2+ Kg2(NPg7) 3Ph1=R NPh8=NQ(NSb8) 4Rh8(NQd1) NSd7 5Rf8 NQd7(NSg8) mate. Cooks: 1Ph2 Kg2 2Ph1=S Kh1(Sg8) 3NPg2+ Kh2 4NPg1 =NQ+ NQg7+ 5Kd8 NPxg8=NR mate. [A.W.I.] 1NPg2+ Ke2 2NPg1=NQ NPh8=NR 3NQd4 NRh7+ 4Ke8 NRd7 5Ph2 NQd7(NRa8)++ mate. 1NPg2+ Ke2 2NPg1=NR NPh8 =NR 3NRg5 NRh3(Ph7) 4Ke8 NRh-g3 5NRg7 NRg7(NRh8) mate. [R.B. both] This 'promoted NPs + Circe field is not only very tricky - it is also extremely difficult to find anything which has escaped K.Gandew, who has worked intensively on the possibilities (as I once discovered to my cost!) [R.B.]

63. JELLISS. 1Kd6 Kg3(Sc7) 2Kd7 Qe8. Neat demonstration of Antipodean possibilities. [R.B.]

64. JELLISS. 1Qg5(Nc1)+ Kh8 2Qg6 Nd3(Gh7) 3Gh5 Nf7 mate (G/Qxf7? Nb3 still checking).

The following solving ladder includes some amendments to previous lists.
My comment that **28**(a) was not solved by anyone was erroneous, since both R.B. and A.W.I. solved it. A point is given for finding the intended solution or a cook to a problem, but there is no extra point for finding both - just the kudos of having your name cited as cook-hunter.

Solving Ladder

	1	2	3	4	Total
E.Bartel.	-	-	22	11	33
R.Brain	13	25	40	28	106
P.M.Cohen	-	-	3	-	3
A.W.Ingleton	13	26	29	22	90
N.Nettheim	-	18	-	-	18
D.Nixon	13	17	31	15	76
T.G.Pollard	13	22	41	26	102
R.W.Smook	13	-	-	-	13
T.H.Willcocks	11	-	-	-	11

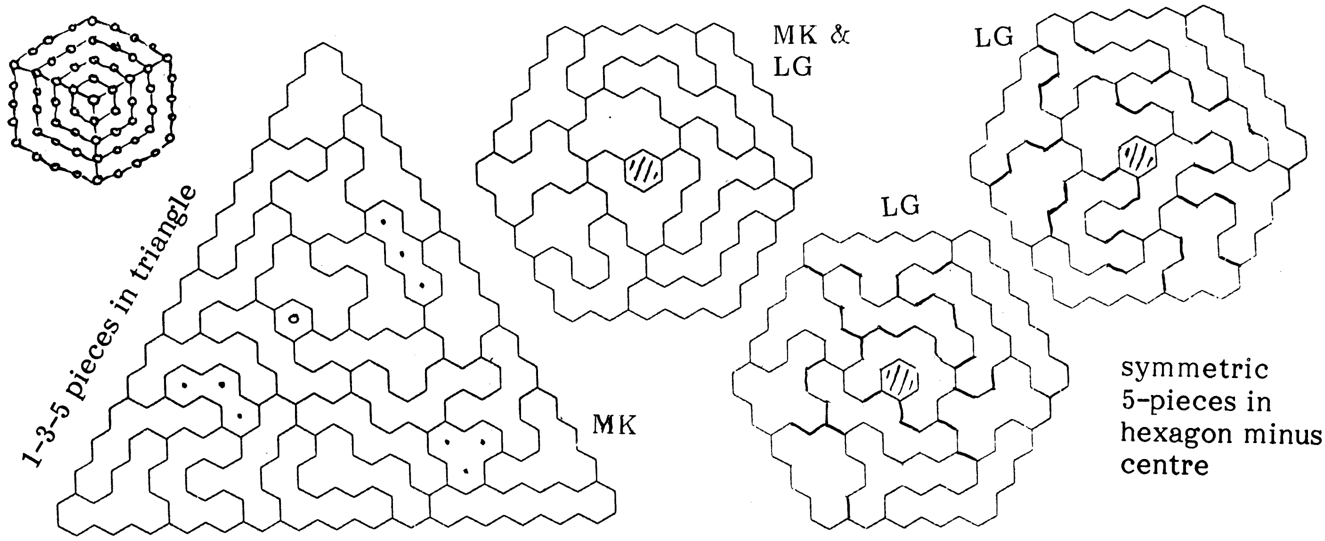
The Moose,

Insidious beast is the Moose,
For when it's turned out on the loose
The King it entangles
And snipes from all angles
With subtlety cooking his goose!

T.Gordon Pollard
(Comment on **48**)

Polyhexes

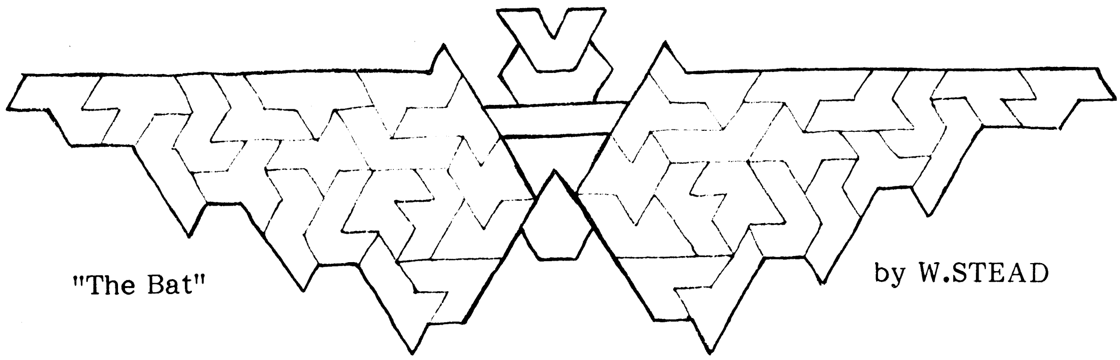
Looking at a cube corner-on, its outline is that of a hexagon. By this visualisation we can see that the sum of the first n hexagonal numbers is n cubed. Hence the nth hexagonal number is $n^3 - (n-1)^3$, which is equal to $3n(n-1)+1$, the formula previously given. The first number both square and hexagonal is $169 = 13^2 = 8^3 - 7^3$. The first number both triangular and hexagonal is $91 = (13 \times 14)/2 = 6^3 - 5^3$. The answers to the other two questions posed are provided by the diagrams:



These solutions are by Michael KELLER and Leonard GORDON. Working independently both came up with an identical solution to the hexagon problem - the two pieces at the bottom left corner of this case can be rotated to give a fourth solution. M.K. provided the artwork. He also corrects the total of 6-piece polyhexes to 82. Some remarkable dissections using the whole set appear in No 7 of his World Game Review.

Heptiamonds

Walter STEAD's notebooks contain some fascinating dissections with 'polyiamonds', i.e. shapes formed from regular triangles. There is only one of each of the sizes 1, 2 and 3, three of 4 pieces, four of 5 pieces, twelve of 6 pieces and 24 of 7 pieces. 'The Bat' below, composed 30th March 1967, shows all the 43 one-sided heptiamonds (5 symmetric in the centre and 19 pairs of asymmetrics - thus 19 + 5 = 24 the number noted above). The front cover result dates from Aug-Sept 1966.



Geometric Jigsaws

"Fully interlocking" jigsaw puzzles depend on pieces that have "bobbles" and "nibbles" that fit together. If the body of each piece, apart from the bobbles and nibbles, is taken to be square, how many different shapes of jigsaw piece are possible? Can you pick out sets of these pieces that will fit together to form a square or rectangle? No shape is to be used more than once.

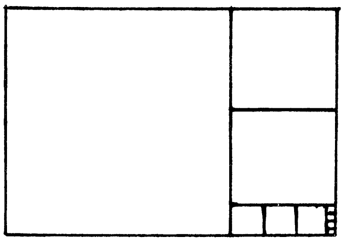
DISSECTIONS

Squaring the Rectangle

If we remove $m_1$squares of size axa from one end of a rectangle axb to leave a rectangle smaller than axa, then the "squareness" of the smaller rectangle, the ratio of its shorter to its longer side, is $s_1 = (b-m_1a)/a = b/a - m_1 = 1/s - m_1$, where s is the squareness of the larger rectangle. Thus $s = 1/(m_1+s_1)$.

If we continue the process and similarly remove m_2 squares from the second rectangle to leave a third rectangle of squareness s_2 then $s = 1/(m_1+1/(m_2+s_2))$. And for the complete dissection process we can write: $s = 1/(m_1+1/(m_2+1/(m_3+1/(m_4$ and so on. This type of expression is called a "continued fraction".

To calculate a continued fraction one has to work from right to left. By using a sign | to mean "divided into" it can be written from left to right: $s = ...+m_2)|1+m_1)|1$. For example, when $m_4,m_3,m_2,m_1 = 4,3,2,1$ we get $s = ((((4)|1+3)|1+2)|1+1)|1 = 30/43$.

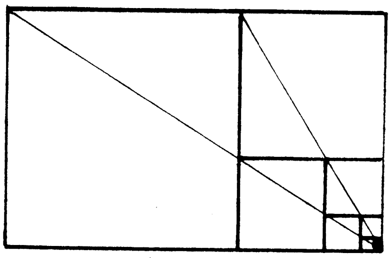


$m_1=1$
$m_2=2$
$m_3=3$
$m_4=4$
$s = 30/43$

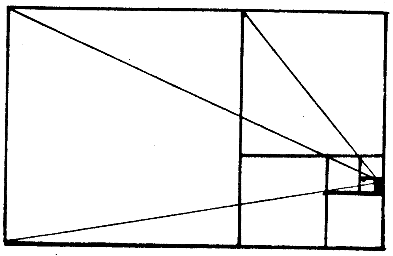
In practice the dissection process terminates after a number of steps because the squares get smaller and smaller until they go beyond the accuracy of our measuring or drawing devices. In the fantasy world of mathematicians however they may be supposed to go on indefinitely. If the process terminates then the ratio is 'rational', if it goes on for ever it is 'irrational'. This is one of the neatest ways of defining the distinction between rational and irrational numbers in mathematics (together they make up the 'real' numbers).

Is it possible to have $m_1,m_2,m_3,...$ all equal to 1?

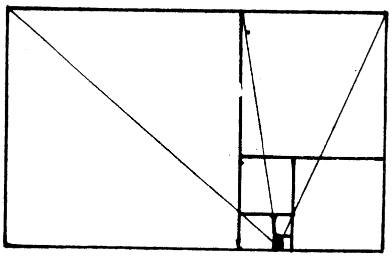
In a finite dissection the last m, say m_n, cannot be 1, since this would mean the remaining rectangle is a square, of the same size as the m_{n-1} we removed at the preceding stage. Thus a dissection in which one square is removed at every stage cannot be finite. In this case we have $s = 1/(1+1/(1+1/(1+1/(1+...$ which evidently implies $s = 1/(1+s)$, that is $s^2 + s - 1 = 0$. Solving this quadratic equation gives $s = (\sqrt{5}-1)/2 = 0.61803398...$ This number is known as the "golden" number or the "divine proportion", among many other soubriquets.



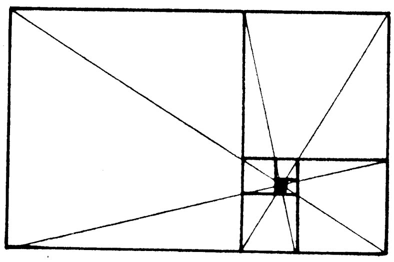
1111...



1110...



1101...



1100...

These four diagrams show the dissection of the golden rectangle into squares in various sequences. If 1 denotes left or top and 0 denotes right or bottom, then we can always orient the rectangle so that the sequence begins 11. The next choices can then be 11, 10, 01 or 00. By repeating the same four choices we get the four cases illustrated. The guide lines join corners of similarly placed squares.

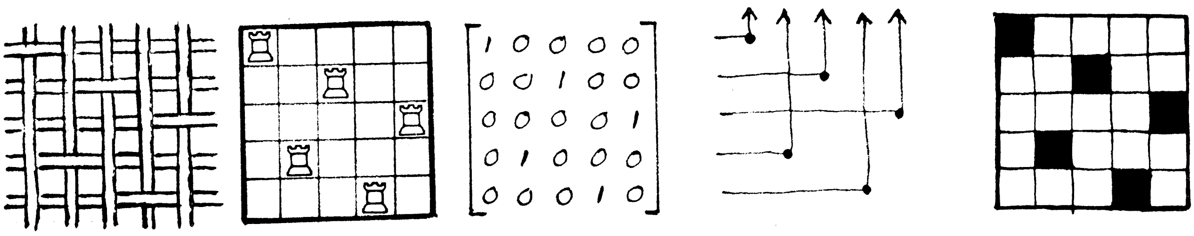
More on this in the puzzle section under "Golden Sequences".

What other rectangles produce the same number of squares at each step?

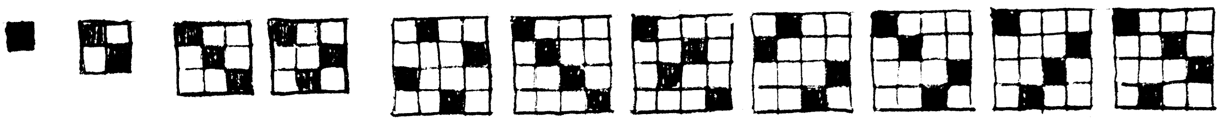
BINARY ARRAYS

SATIN SQUARES

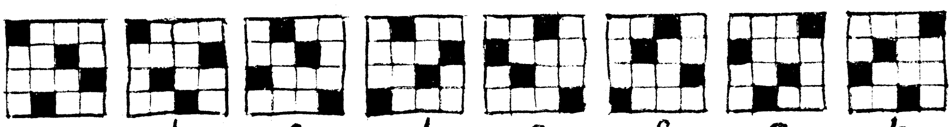
A satin square is an array, n by n, with n similar marks entered in it, one in each rank and file. In chessic terms it can be taken to represent an arrangement of n Rooks, none guarding any other. The name 'satin' of course derives from weaving, where it refers to a weave in which the 'repeat' pattern, showing where the weft threads (ranks) pass over the warp threads (files), has only one attachment point per thread. Mathematically it can be taken as a permutation matrix, which when multiplying any other matrix permutes its ranks or files (under suitable definition of matrix multiplication).



How many geometrically different satins are there of size n ? A partial answer to this question is given in Maurice Kraitchik's Mathematical Recreations (Dover reprint 1953 page 245) much of which appeared earlier in his L'Echiquier column around 1925. The work is completed in an article by D.F.Holt, 'Rooks inviolate' in The Mathematical Gazette 1974 pages 131-134. The following are all the cases of sizes 1 to 4.



A theorem by William Burnside (1852 - 1927) states that the number of geometrically different patterns of a given type in a shape unaltered by n symmetry operations a, b, c, ... is (A+B+C+...)/n, where A is the number of cases unaltered by operation a, B the number unaltered by b, and so on. One of these operations, we may take it to be a, must always be the identity. Thus A is the total number of cases without regard to symmetry.



a b c d e f g h

In the case of a square pattern there are 8 symmetry operations. The illustration shows their effects on the asymmetric 4 by 4 satin. The trivial 1 by 1 satin is the only one unaltered by reflection in the horizontal (d) or vertical (h). For all other cases (n > 1) we have D = H = 0. Evidently also we always have B = C (cases unaltered by diagonal reflection) and F = G (patterns unaltered by 90° rotation) and we know that A = n! . Thus, with E the cases invariant to 180° rotation, the formula simplifies, for n > 1 to:

$$T = (n! + 2B + E + 2F)/8$$

Denoting by E_n, F_n the values of E and F for the n by n case, we find that when n is odd $E_n = E_{n-1}$ (since the odd mark must be in the centre cell and by deleting it - and the rank and file in which it stands - leaves a square of side n-1, still with E-symmetry). Similarly $F_n = F_{n-1}$. Also $F_n = 0$ when n is of the form 4k+2 or 4k+3 (since the marks other than in the centre cell must occur in sets of 4, one in each quarter).

When n is even we have $E_n = nE_{n-2}$ (since the mark in the first rank can be in any of n cells, and deleting it and its 180° cousin leaves a satin of side n-2, with E-symmetry). Similarly, when n is 4k we have $F_n = (n-2)F_{n-4}$ (since the mark in the first rank can be in n-2 positions - not the two where the rank crosses the main diagonals - and deleting it and its three cousins reduces to a satin of side n-4).

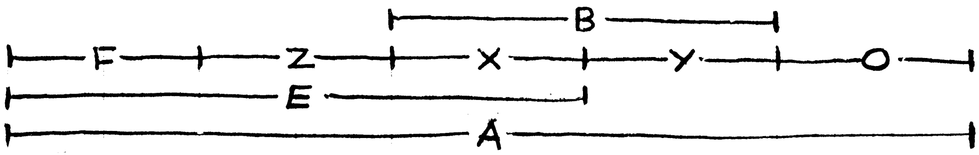
BINARY ARRAYS

From these recurrence relations we can derive the explicit formulae $E_{2k} = k!2^k$ and $F_{4k} = (2k)!/2(k!)$, given by Kraitchik. For diagonal reflection symmetry Holt reasons that if there is a mark in the first square of the diagonal, deleting it gives a satin of side n-1 with diagonal symmetry; and if there is not a mark in the first cell then it is in one of n-1 other positions, and deleting it and its reflection gives a satin of side n-2. Thus he arrives at the more complex recurrence relation: $B_n = B_{n-1} + (n-1)B_{n-2}$.

These three recurrence relations are sufficient to enable us to calculate T_n. The results, up to n = 10, are shown in the upper part of the following table.

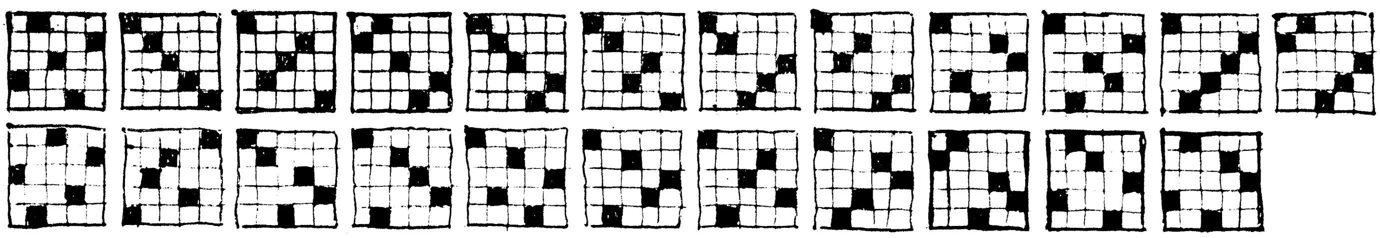
n	2	3	4	5	6	7	8	9	10
A	2	6	24	120	720	5040	40320	362880	3628800
B	2	4	10	26	76	232	764	2620	9496
E	2	2	8	8	48	48	384	384	3840
F	0	0	2	2	0	0	12	12	0
T	1	2	7	23	115	694	5282	46066	456454
F/2	0	0	1	1	0	0	6	6	0
Z/4	0	0	0	0	7	7	74	74	882
X/2	1	1	3	3	10	10	38	38	156
Y/4	0	1	2	10	28	106	344	1272	4592
O/8	0	0	1	9	70	571	4820	44676	450824

To break down the T cases into mutually exclusive classes however, we will require also to know the number X_n of satins with double diagonal symmetry. A recurrence for this can be derived in a manner exactly analogous to that given by Holt. For n odd we must have a central mark, and deleting this we find $X_n=X_{n-1}$. For n even we can have a mark at the end of one of the diagonals in the first row (2 choices) and deleting this and its mate leaves a size n-2 satin, or we can have a mark in one of the n-2 other cells and deleting it and its now three images leaves a satin of size n-4. Thus $X_n=2X_{n-2} + (n-2)X_{n-4}$.

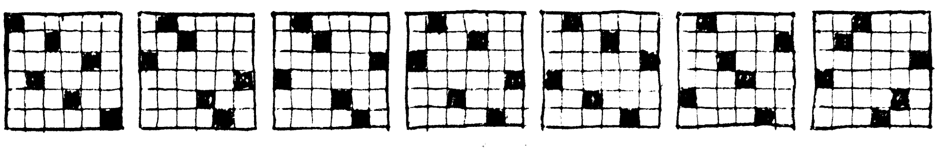


The diagram shows a partition of all the satins (A) into five classes according to symmetry. Class X is the intersection of classes E and B. Z = E-(F+X) is the class of 'purely' 180° rotary patterns, and Y = B-X is the class of patterns with a single axis of reflection. Class O = A-(E+Y) consists of all asymmetric cases. Each satin of type O has 8 different appearances according to orientation, but those of types Y and Z have 4 and those of types F and X only 2, so to get the numbers of geometrically distinct cases we divide by these factors. The lower part of the table completes that given by Kraitchik.

The 23 cases of size 5.



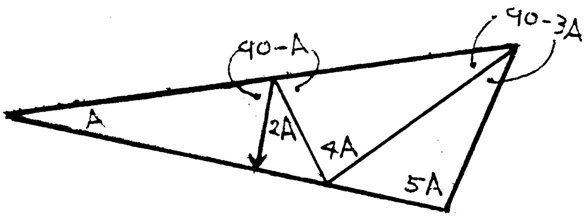
The 7 singly-centrosymmetric (Z) cases of size 6.

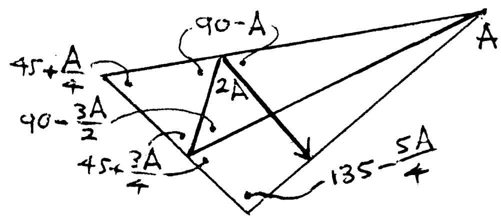


GEOMETRY

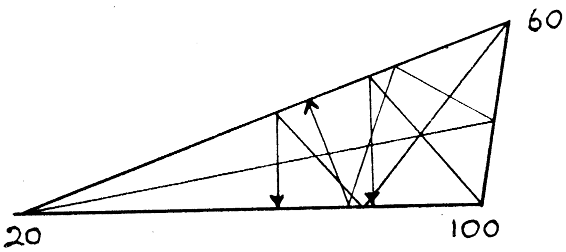
Triangular Billiards

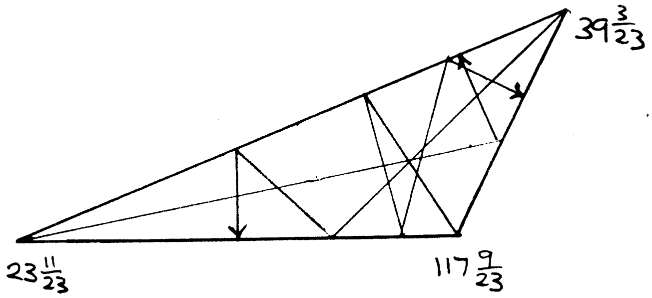
Problem 5 (continued). In the case of a return after three bounces, there are two types of path possible, as illustrated below. In the first the angles opposite the vertex from which the ball is cued must be in the ratio 1 to 5. In the second the angle of this vertex must satisfy the condition $45 + 3A/4 < 90$, that is $A < 60$, so that the other angles, $B = 45 + A/4$ and $C = 135 - 5A/4$, lie in the ranges $45 < B < 60$ and $60 < C < 135$.

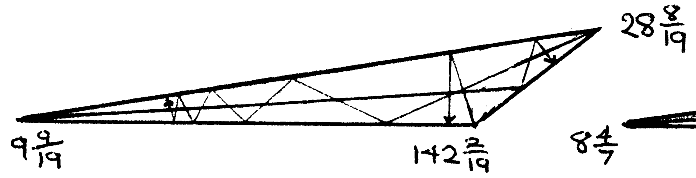


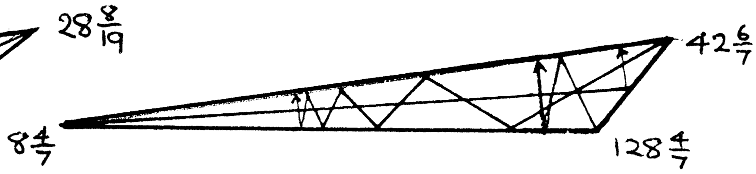


To form a triangle of the (2,3,n) type we require a pair of angles in the triangle to be in the ratio 1 to 3 (as noted last time). The first type shown above gives four solutions:

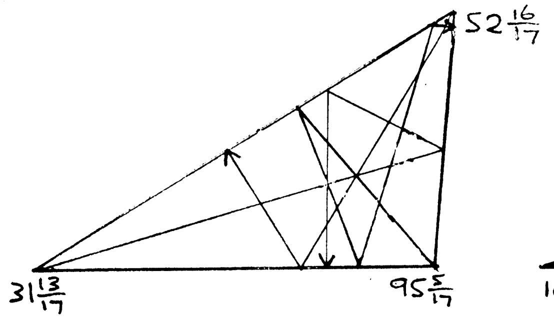


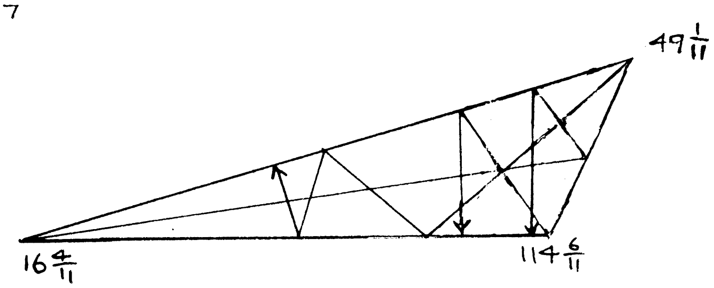






The second type gives two solutions. (Note that we can have 3A=B or 3A=C but not 3B=A or 3C=A, because B>45, C>60 and A<60.) Upon drawing the (not perfectly accurate) figures however, I note that two of the shots go perilously close to the pockets! - do they go in?





Here is some more Triangular Billiards on less oddly shaped tables:

Problem 6. On a regular triangular billiard table with pockets at the corners a ball is cued from one pocket to another (but not directly along the cushion). What is the fewest number of 'moves' (i.e. straight paths) that the ball must make? And at what angle to the cushion must the ball be cued to achieve this feat?

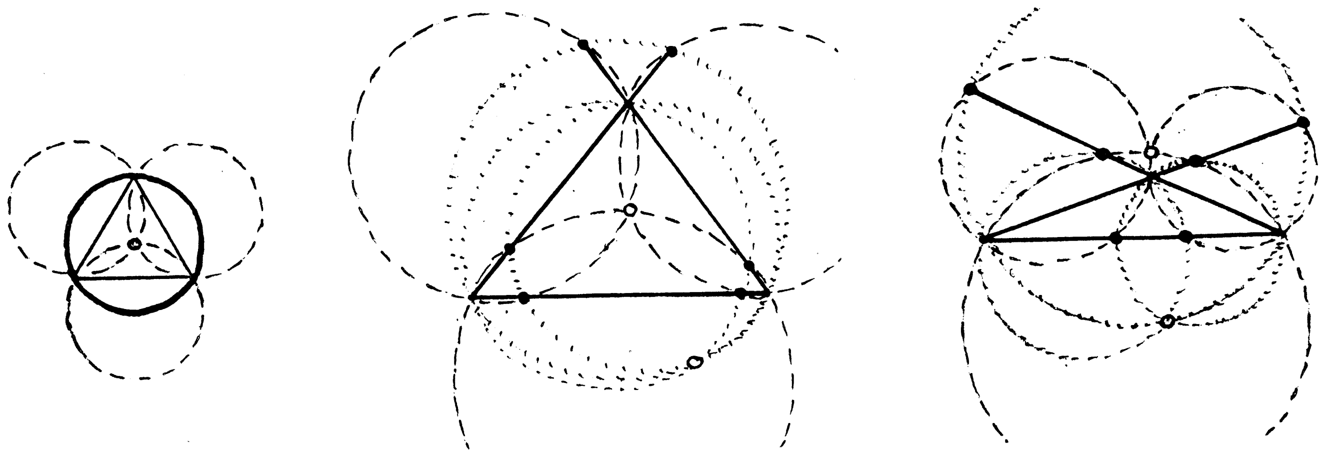
Problem 7. On a half-square triangular billiard table a ball is cued from one of the 45 degree corners, bisecting the angle. After how many bounces will it next be travelling in the same direction as it was at the start? (By "in the same direction" we mean in the same or a parallel line, and in the same sense along the line or its parallel).

Problem 8. A ball is cued from a 45° corner of a half-square triangular table to meet the opposite side at a point m nths of the way along the side from the right angle. Will it eventually reach a pocket? (The pockets being at the corners and 'eventually' meaning in a finite number of moves.) If so, which pocket, and how many moves?

GEOMETRY

Star Points

The question (issue 4, p58) was, where in a triangle ABC can we find a point P such that the lines AP,BP,CP (produced) form a 'star', i.e. six angles of 60°. In the case of a regular triangle the centre point and all points on the circumcircle solve the problem, but in all other cases there are just two star points. One of these can be constructed by drawing regular triangles outwards on the sides of the given triangle - the circumcircles of these triangles all meet at the required star point. The other star point can be found similarly by drawing regular triangles inwards on the sides of the given triangle. The other six points of intersection of these six circles all lie on the sides of the given triangle (in the exceptional cases when one of the angles of the triangle is 60° or 120° these six points reduce to four since one of the star points is at the 60° or 120° vertex itself).



In the case of a triangle with all angles less than 120° one of the star points is within the triangle and solves Fermat's problem of minimizing PA+PB+PC (see H.S.M.Coxeter, Introduction to Geometry, 2nd edn, 1969, p22).

4-gon Conclusion?

A polygon is called equilateral if all its sides are equal, equiangular if all its angles are equal, and regular if it has both these properties. Triangles have the helpful habit of being equilateral if and only if they are equiangular but this is not true for polygons with more sides. Equilateral 4-gons are diamonds. Equiangular 4-gons are rectangles. Regular 4-gons are squares. An incyclic polygon is one that has an 'incircle' - touching all its sides. A circumcyclic polygon has a 'circumcircle' through all its vertices. A cyclic polygon is both incyclic and circumcyclic. All triangles are cyclic. What 4-gons are cyclic?

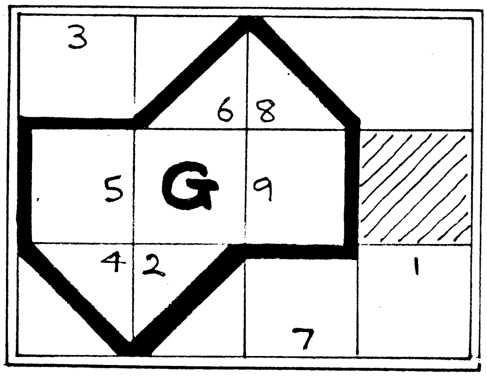
The Centres of a Triangle

One of the most interesting topics in Geometry from a recreational point of view is that of the 'Centres' of a Triangle. A regular triangle has one centre - its centre of symmetry - which serves for all purposes, but in a scalene triangle the Incentre (centre of the inscribed circle), Circumcentre (centre of the circumscribed circle), Centroid (centre of gravity if the triangle is cut from a uniform plate) and Orthocentre (point where the three 'altitudes' cross) are all different points. In obtuse triangles the Circum- and Ortho- centres are not even inside the triangle itself. The fact that the Centroid lies on the same line as the Ortho- and Circum- centres (the Euler line of the triangle) is a high-point of most Geometry courses. When is the Incentre on the same line?

Here is another Centre of a triangle: the 'centre of similitude' of the incircle and circumcircle of the triangle. Construct this point and show its relation to the Incentre and Circumcentre. By the centre of similitude of two similar figures I mean a point from which a straight line meets the first figure at distance x and the second at distance y and the ratio y to x is constant. The construction is quite easy and results in a pretty figure. Our interest in Geometry will be more aesthetic than academic in these pages.

TRANSITIONS

Get My Goat



The diagram shows Tom MARLOW's clever solution to the 'Get My Goat' puzzle (p 40 of issue 3). The fence is a different shape, but still encloses the goat, thus fulfilling the conditions of the problem. The sequence is: 9G12G, 93453, 76354, G2197, G279.

Mr Marlow also reported that the solution to problem **C75(b)** in <u>Sliding Piece Puzzles</u> can be reduced by one move: In the solution for **C75(a)** change move 192 to 9DR and 193 to 10LD, complete the 211 moves and add 9LU.

Sliding Magic Squares

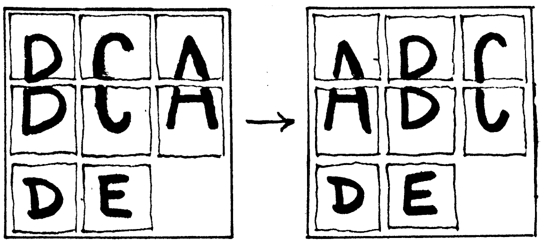
The following technical notes left over from Len Gordon's article in the last issue may be of interest to readers who have computers with which to tackle these problems.

It is not often that a computer search can improve on long-standing solutions to a difficult and popular puzzle. Dudeney ran a newspaper puzzle column, so the results he presents in his books often reflect the best of his readers. The very nature of a truly good puzzle is that it is not amenable to 'brute force' search. Simple tests luckily ruled out most of the magic squares quickly, but a fair percentage had to be tested by sliding. The search time increases exponentially with the number of solution steps.

Reported times are for a compiled BASIC program on a 2.8 Mhz TRS-80. For this type of problem on an 8-bit machine, compiled programs usually run about 50 times as fast as ordinary BASIC, and about 1/8 as fast as assembly language programs. Since completing this study I have switched to an 8 Mhz IBM-pc type of computer and am using a QuickBASIC compiler. This has resulted in a speed-up factor of about 4.5. In addition, John Harris has contributed some ideas and between us we have sped the program by another factor of 6 or 8. However, we are still short of the speed needed to solve worst cases.

An ABC Sliding Block Puzzle

By Jerry GORDON

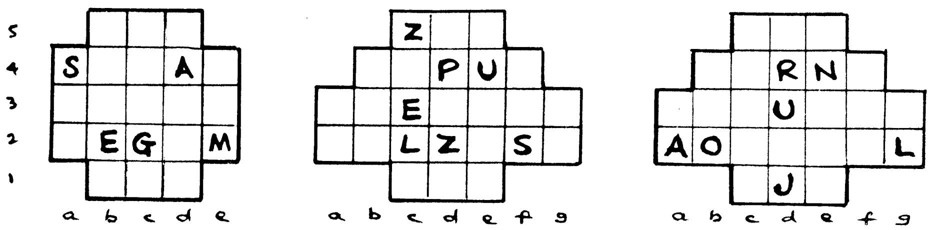


This puzzle, invented in May this year, may interest your readers. It has 8 square pieces on a 3 x 3 base. Three pairs of the pieces combine to show the letters A, B, C. The case illustrated is one of 23 positions solved and has a unique solution in fewest moves.

Letter Zig-Zags

By Vladimir PRIBYLINEC

Here are three puzzles with letters. The pieces carrying the letters move along the rows, columns and diagonals to the last free square in the chosen course. You must find the shortest way to spell out the determined word on any row of the diagram in fewest moves.



PUZZLE QUESTIONS AND ANSWERS

Pages 87-88 consist of corrections and extensions to questions that appeared in the earlier issues. Pages 89-90 deal with the mathematical topics relating to permutations and number theory that were introduced last time, followed by some new mathematical questions on page 91. Word and letter puzzles then follow on pages 92 - 95.

Probable Inequality

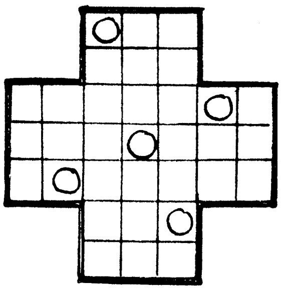
Both Clive PALMER and Gareth SUGGETT pointed out that the question as posed requires further assumptions for an explicit formula, the simplest being: If P,Q,R are the probabilities of $A \neq B, B \neq C, A \neq C$, suppose that there are n possible values other than B that each of A or C can take, and that these values are equiprobable. Then the probability that A=C= any particular one of these values is (P/n)(Q/n). So the probability that A=C with A and C not equal to B is n times this, i.e. PQ/n. Thus:

$$R = P + Q - PQ(1 + 1/n)$$

If n is large, so that 1/n can be ignored, this simplifies to the formula given before.

Cross-Point

Angela NEWING sent a bibliography of the general 'Crossed Ladders' problem, which goes back at least to Bhaskara, 1150AD. There is a chapter (No.8) on it in Hugh ApSimon's book Mathematical Byways which is No.1 in the Oxford 'Recreations in Mathematics' series. One is usually given the lengths of the ladders and the height of the crosspoint and asked for the width of the street. This requires the solution of a 4th-degree equation. This is also pointed out by G.J.SUGGETT, who mentions an even more difficult version given by 'Dipole', IEE News, 1980, in which the feet of the ladders do not coincide with the bases of the walls.



Solitaire Queens

Michael KELLER notes an error in the diagrams of the arrangements of 5 Queens in unguard on the solitaire board, page 43. The third diagram in the last row is a rotated duplicate of row 2 number 6. The correct diagram is inset. The enumeration total is unaltered. I'm somewhat baffled as to how this error occurred.

Cryptarithms

Michael KELLER points out that the cryptarithms (**8**) and (**9**), page 60, do not have unique solutions since the values of the letters N,V and W in (**8**) and N and O in (**9**) are interchangeable. Evidently the authors Madachy and Hunter, from whom I quoted these examples without properly checking them, have a different interpretation of uniqueness of solution from us (in chess problem terminology, they do not regard 'duals' as flaws).

In the solution to cryptarithm (**10**) the fifth line is misprinted and should read as follows: $P = AN + (A-1)$, $Q = 10^r(10-A) + AR$, $K = A(10^r-R) - 1$, $L = 10^{n+1} - A - AN$.

Multiplication Table

The formula in the eighth line should of course be $(x_1+1)(x_2+1)(x_3+1)$...not as typed.

The Four Rs

A simpler expression for 12 given by T.H.WILLCOCKS is 12=(RR+R)/R. John DAWES points out two typing errors: 13=R/·R+√(R/·R˙), 17=(R+R-·R˙)/·R˙ and adds the following cases to the list: 33=RR/√(Rx·R˙), 80=(R-·R˙)/(·R˙-·R), 98=(RR-·R˙)/·R˙, 108=(RR+R)/·R˙, 109=(RR-·R)/·R, 110=RR/√(·Rx·R), 120=(RR+R)/·R, 180=(R+R)/(·R˙-·R), 900=R/(·R˙-·RR), 990=RR/(·R˙-·R). For those not familiar with the recurrence dot notation it may be as well to explain that ·R=R/10 and ·R˙=·RRR...=R/9, so that ·R˙-·R=R/90.

PUZZLES

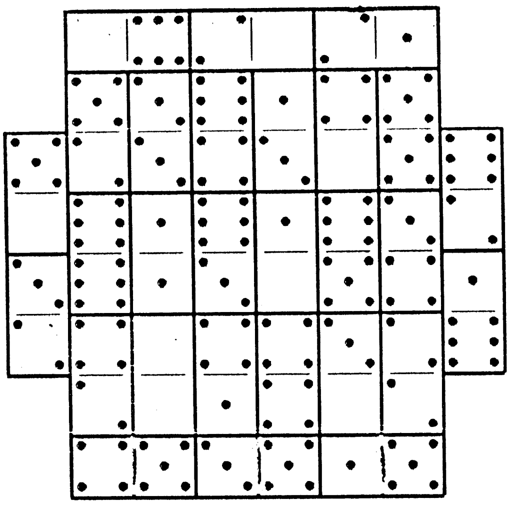
Domino Quadrilles

Michael KELLER provides another English Quadrille to go with that on page 21. He has even provided the artwork. He comments: "Though the solutions are not as striking visually as French Quadrilles, the problem is interesting in a different way - it requires a trial-and -error combinatorial approach rather than the analytical approach used in the French Quadrille problem" (e.g. as treated in Fred Shuh's Master Book of Mathematical Recreations). The ends of the dominoes form squares-of-four, each containing four different digits, no two sets of four being the same. It improves on my version by having the sets of four combinatorially different (i.e. regardless of order), whereas my one counts

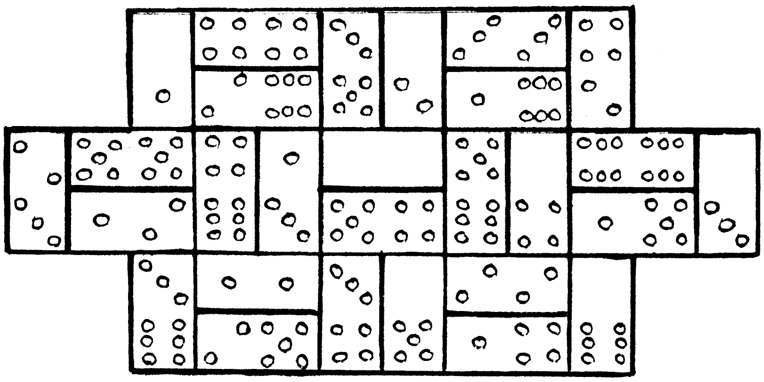
[3 4 / 6 5] , [4 3 / 6 5] and [0 3 / 2 1] , [1 0 / 3 2] as different.

My solution can be improved to the same standard by three interchanges, as shown.

A. M.Keller



B. G.P.Jelliss



Balanced and Unbalanced Magic Squares

A sequence of n of the numbers 1, 2, ..., n^2 adding to the magic constant $n(n^2+1)/2$, arranged in order of magnitude, is called a magic series. Any rank file or magic diagonal of a magic square is composed of n numbers that form a magic series. There are just two magic series for n=2, namely (1,4) and (2,3). These will form a square whose rows add to the same sum, but not the columns also. There are 8 magic series for n=3 and they are all used in the ranks, files and diagonals of the 3x3 magic square. This is a feature of completeness that is not possessed by magic squares of larger sizes. The series are of course: (1,5,9) (1,6,8) (2,4,9) (2,5,8) (2,6,7) (3,4,8) (3,5,7) (4,5,6).

There are 86 magic series of 4 of the numbers 1, 2, ..., 16 adding to the magic sum 34. Of these, 28 are combinations of pairs of complements: (1,16) (2,15) (3,14) (4,13) (5,12) (6,11) (7,10) (8,9). These 28 magic series are self-complementary, meaning that if we replace each number by its complement and rearrange them in order of magnitude then we are left with the same series. The other 58 cases occur in 29 complementary pairs.

Every self-complementary series contains two small numbers (i.e. 1, ..., 8) and two large numbers (i.e. 9, ..., 16) also two even and two odd numbers. Any series with these properties may be termed balanced. Of the 29 pairs, 16 are balanced.

Of the 13 pairs that remain, 6 are balanced high-low but not even-odd, and 6 are balanced even-odd but not high-low. This leaves one pair that is unbalanced. **Problem:** Find these unbalanced series and construct a magic square using them in the diagonals.

This line of research can also be extended to higher squares (n = 5, 6, etc).

Counting Derangements

The number of derangements of n objects is given by $d_n = nd_{n-1} \pm 1$ (+ or - according as n is even or odd). Starting from $d_1=0$ we can use this recurrence relation to calculate the successive values: $d_2=1$, $d_3=2$, $d_4=9$, $d_5=44$, $d_6=265$, $d_7=1854$, $d_8=14833$, $d_9=133496$, $d_{10}=1334961$, and so on.

By coincidence a note on 'The worst postman problem' by John ANSTICE appears in the latest issue of The Mathematical Gazette (dated October 1988) where a rather round-about proof of the recurrence is given. The author asks for a 'neat direct argument' to prove it. An outline of one is as follows.

If we remove the nth letter and nth envelope from a derangement of n and put the loose letter in the loose envelope we get either a derangement of the remaining n-1 letters or an arrangement with one letter in its correct envelope. The number of derangements of the first type is $(n-1)d_{n-1}$. We have thus to show that the number of derangements of the second type is $d_{n-1} \pm 1$. In other words it is a matter of showing a (nearly) 1-to-1 correspondence between these derangements and those of n-1 letters. A process that does this begins as follows:

$$\begin{bmatrix} 1...t...n-1 \\ a...k...\ t \end{bmatrix} \leftrightarrow \begin{bmatrix} 1...t...n-1\ n \\ a...n...\ k\ \ t \end{bmatrix} \quad \text{or if } k = n-1 \quad \begin{bmatrix} 1...t...n-2\ n-1\ n \\ a...s...\ t\ \ \ n\ \ n-1 \end{bmatrix}$$

and further steps are necessary if t=n-2, and so on.

For example, in the case of 3 or 4 letters we get:

$$\begin{bmatrix} 123 \\ 231 \end{bmatrix} \leftrightarrow \begin{bmatrix} 1234 \\ 4321 \end{bmatrix}, \quad \begin{bmatrix} 123 \\ 312 \end{bmatrix} \leftrightarrow \begin{bmatrix} 1234 \\ 3412 \end{bmatrix}, \quad \text{and} \begin{bmatrix} 1234 \\ 2143 \end{bmatrix} \text{over}$$

and for 4 or 5 letters, omitting the upper lines 1234, 12345, we get:

2143 ↔ 31254, 2341 ↔ 53421, 2413 ↔ 24513, 3142 ↔ 35412
3412 ↔ 23154, 3421 ↔ 54231, 4123 ↔ 41523, 4312 ↔ 45132
with 4321 over.

Probably Deranged

An 'explicit' formula for the number of derangements d_n is the 'subfactorial':

$$d_n = n![1/2! - 1/3! + \cdots + (-1)^n 1/n!]$$

This is quite useless for calculation, but the the series in the brackets is interesting because of its evident relationship to the series for the exponential function. The probability of making a derangement, assuming that all arrangements are equally likely, is $d_n/n!$, i.e. the expression in square brackets. Leonard J. Gordon has applied his computer to calculate these values and finds as follows:

n	$d_n/n!$	n	$d_n/n!$
n=2	$d_n/n!$=0.5	n=7	$d_n/n!$=0.3678571
3	0.3333333	8	0.367882
4	0.375	9	0.3678792
5	0.3666667	10	0.3678795
6	0.3680556	11	0.3678795
		12	0.3678795

The probability for large n is thus 1/e, the reciprocal of the exponential number.

Exponentiation

In the Introduction to R.D.Carmichael's Diophantine Analysis (1915) one finds the following infuriating passage: "Thus, for example, one may ask what integers x and y can satisfy the relation $x^y - y^x = 0$. This more extended problem is all but untreated in the literature. It seems to be of no particular importance, and therefore will be left almost entirely out of account in the following pages." And he does not mention it again! What is the answer? In other words, for which numbers x and y ($x \neq y$) is the operation of raising to a power (exponentiation) commutative: $x^y = y^x$?

It is easy to see what the answer must be, but less so to prove that there are no other answers. When I put the problem to Clive Grimstone in April 1983 he provided a nice graphical proof (simple when you know how) which also answers another question I put to him - to provide a geometrical representation of the number e. All this shows that e is very appropriately named the exponential number.

PUZZLES

Permutable Primes

A permutable prime is one that remains prime if its digits are permuted in any other order. The single-digit primes 2,3,5,7 are trivially permutable. The digits 0,2,4, 5, 6, 8 cannot occur in any permutable prime of two or more digits since they can be permuted to the units place to produce a number divisible by 2 or 5. All higher permutable primes are therefore formed from the digits 1,3,7,9.

The two-digit permutable primes are: 11;13,31;17,71;37,73;79,97, and the three-digit cases are: 113,131,311;337,373,733;199,919,991.

There are, perhaps surprisingly, no four-digit permutable primes, since:
(a) 1379 = 7 x 197, eliminating the form abcd,
(b) abab = 101 x ab, eliminating the forms abab and aaaa,
(c) 1711 = 29 x 59, 1333 = 31 x 43, 3337 = 47 x 71, 7771 = 19 x 409, 9919 = 7 x 1417, and 9997 = 13 x 769, eliminating form aaab (other cases are divisible by 3),
(d) 1139 = 17 x 67, 1339 = 13 x 103, 1337 = 7 x 191, 9373 = 7 x 1339, 7973 = 7 x 1139, 1939 = 7 x 277, 1799 = 7 x 257, 7399 = 7 x 1057, eliminating the form aabc.

Whether there is a largest permutable prime is unknown. The only others known are those consisting of repeated ones (rep-unit primes), i.e. numbers of the form $(10^n-1)/9$. The latest information I have is that primes occur when n = 2, 19, 23, 317, 1031.

In the binary system, the permutable primes are: 10 (2), 11 (3), 111 (7), 11111 (31) and so on, that is (above 2) primes of the form 2^n-1. These are known as Mersenne's numbers, and 29 cases are now known: n=2,3,5,7,13,17,19,31,61,81,107,127,521,607,1279,2203, 2281, 3217, 4253, 4423, 9689, 9941, 11213, 19937, 21701, 23209, 44497, 86243, 132049 (as listed in W.W.Rouse Ball & H.S.M.Coxeter Mathematical Recreations & Essays, 13th edn, p 65).

In the ternary system, the permutable primes are: 2, 10 (3), 12 (5), 21 (7), 111 (13), no more of three or four digits.

Readers may like to investigate other cases.

Circular Primes

G.J.SUGGETT proposes the problem of finding primes that remain prime when their digits are permuted circularly (i.e. digits are taken from one end and put at the other). Besides the permutable primes there is an extra case of 3 digits and two extra cases each of 4, 5 and 6 digits (counting a number and its permutes as one case).

Incirculable Sets

While we are on the subject of circular permutations, it may be of interest to note that a set that cannot be circularly permuted must have more than an infinity of members!

A permutation is a one-to-one correspondence of a set with itself. A finite set is one that cannot be placed in one-to-one correspondence with a proper subset of itself. Any other set is infinite. [All ordinary everyday sets are finite - infinite sets only exist in the fevered imaginations of mathematicians!]

A circular permutation of a set X may be defined as a permutation of X that leaves no proper subset of X invariant. If we consider an element e then the elements e, Pe, P^2e, P^3e, ..., P^re, ... must all be different until, in the finite case, we reach the number n of elements in X, when Pe^n=e. In the case of an infinite set a 'circular' permutation orders the set into an endless succession: ...$(-P)^re$,...,(-P)e,e,Pe,...,P^re,...(where -P is the opposite permutation: -Pa=b if and only if a=Pb). The 'number' of elements in an infinite circularly permutable set is thus the first infinity (denoted ∞). Sets of higher infinity (like all the 'real' numbers) cannot be circularly permuted.

Show that the rational numbers can be circularly permuted.

It may be as well to note here that a cyclic permutation differs from a circular permutation in that it circularly permutes some elements but leaves the rest, if any, invariant.

We can now return full circle to the topic at the top of the preceding page and note that a derangement is a permutation that leaves no single element invariant. A circular permutation is always a derangement, except when n=1. In fact a derangement is a union of circular permutations of mutually exclusive sets of two or more elements.

Golden Sequences

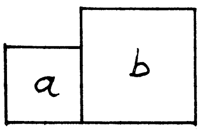
We can construct diagrams similar to the dissections of the golden rectangle shown on an earlier page by beginning with any small rectangle (shown black) and adding a square to the long side of the rectangle at each stage. If we start with a golden rectangle we will end with one of course, but if we start with any other rectangle the successive outer rectangles will become progressively more nearly golden as we proceed. What is the sequence of lengths of the sides of the rectangles if we start with one a x b?

A Gnomonic Question

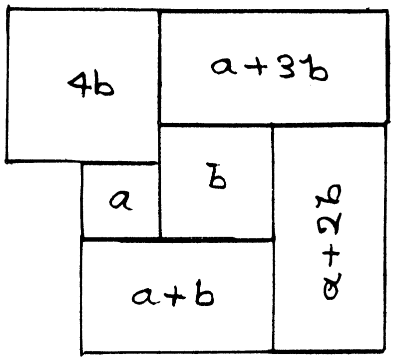
What is the name for a shape formed by placing two squares of different sizes side by side on the same base? If the ratio of the sides of the two squares is the golden ratio then the shape is a 'gnomon' since it then consists of a rectangle with a 'similar' rectangle removed from one corner. Perhaps 'gnomoid' will serve. Anyway, this comment just serves to link the preceding question to the following: from Mr T.H.WILLCOCKS.

Starting from a hexagon formed by two squares a and b, b>a (diagram A), we form new hexagons by adding squares to the sides, the only conditions being that no two squares of the same size may be adjacent and the square must be of size ma+nb where the coefficients m and n must be positive or zero. (In drawing diagrams I find it convenient to represent squares as rectangles - provided they are properly labelled). If we continue from the initial squares as in diagram B we find that the 6th square is a function of a single variable - i.e. its side is a multiple of the side of one of the two original squares. What other multiples of the original squares are possible?

A



B



Readers might like to try solutions for n<20 (say). I have found solutions for any finite value of n with two exceptions - both less than 20. You will understand that I should be interested to see this gap plugged. (I hope I have not been unduly blind!) Some values of n are very easy to find and can be solved in many different ways. Where this is so it is reasonable to define the optimal solution as the one which requires the smallest number of squares and if there is still a tie, the one with the smallest greatest coefficient of a and b.

A Jigsaw Puzzle Puzzle

K.A.L.ANDERSSON (Sweden) poses this question. Pretend you have 400 pieces in a puzzle and every piece looks like all the other pieces, and even the result of putting all the pieces together looks like a piece but bigger. You then use 'application stuff' to hold the pieces together and around every internal piece you have 1 gram of application stuff. How much must you have for the whole construction?

There's a Hole in my Bucket...

This is another puzzle from Kåre ANDERSSON. One fine day I poured water in my swimming pool. When the water level went up one unit I stopped filling, but an hour after I had begun the water level had sunk back again to the earlier level. If the sink-back-process had happened twice as fast, the whole procedure would still have taken the same time. How long would it have taken if the sink-back rate was only half as fast?

WORDPLAY

A Self-Documenting Sentence

Tom MARLOW writes that the idea of a self-documenting sentence seems to have originated in Holland (in Dutch) and was taken up in the USA under the name of "pangrams", usually beginning "This pangram includes ..." and using every letter of the alphabet at least once. Finding a solution involves analysis to find likely ranges of values for the variables and then an exhaustive search within those ranges. The difficulty is that with so many variables the possibilities multiply up to huge numbers, like 10^{12}. Nevertheless he has succeeded in finding a personalised sentence, as follows:

"This sentence created for GPJ contains four A's, one B, four C's, three D's, thirty-three E's, eight F's, four G's, nine H's, eleven I's, two J's, one K, two L's, one M, twenty N's, sixteen O's, two P's, one Q, thirteen R's, twenty-eight S's, twenty-four T's, six U's, four V's, seven W's, three X's, five Y's and one Z."

Alpha Pairs

The solution to the problem by Loretta BRUCE is: **JEANS, FLAIR, GNASH, KVASS, CZECH, DWELL, THINK, SMIRK, QUITS, OXIDE, PROWL, AIRED, BYWAY.**

Philip COHEN finds **PRINK** and **THOWL** as an alternative answer.
Gareth SUGGETT gives another: **QUIRK, GAITS, SNASH, MIRED.**

Code Words

The number of letters of the alphabet between successive letters is the same throughout each of the words given. For GNU the number is 6 (GHIJKLMNOPQRSTU), for EMU it is 7, for FOX it is 8 and for APE it is 14. In the latter case the alphabet is considered to be arranged in circular order, so that upon reaching Z one starts again at A.

The longest word possible unfortunately appears to be MUCKS (separation 7). Other four letter words are BUNG (18), DINS (4), KEYS (19) and three letter words ACE (1), JOT (4), NUB (6), OWE (7), RAJ (8), PAL (10), PET (14), JAR (16), ASK (17), MEW (17), HAT (18) and FED (24). The longest pronounceable sequence of letters I have found is FILORUX (2), which might be a suitable name for a cryptographer.

Permutable Words

Philip COHEN answers: There are several 3-sets, such as AER, for which all permutes are lexical items (names or words). Only one 4 qualifies - AEST. For AELST and AERST over half the 120 possibilities have been found, as attested in a 1987 issue of Word Ways.

Prime Words

Paul VALOIS was the first off the mark with the answer SYZYGY which scores 21 in Scrabble (a blank has to be used for the third Y). Several others found the same word. The next best offering, from Gareth SUGGETT, was CRWTHS scoring 14 (and spellable without a blank). Michael KINDRED, who set the problem, offered KIKUYU scoring 12 (with a blank for the second K). Mr Suggett also considered the thunderclap from Joyce's Finnegan's Wake: PTHWNDXRCLZP only to find that, besides not being in 'Chambers' it contains CRWTH as a factor! A seven letter prime word might be possible with a more restrictive dictionary as arbiter (say the Pocket Oxford). The values attributed to letters in Scrabble are as follows: Blank=0, AEILNORSTU=1, DG=2, BCMP=3, FHVWY=4, K=5, JX=8, QZ=10.

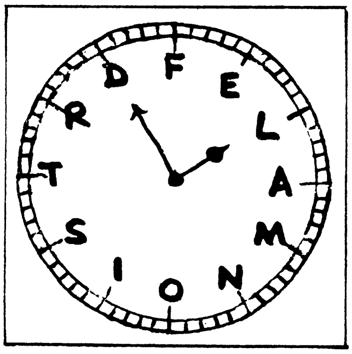
Un-Scrabbleable Words

Following on from 'Prime Words' the question arises: Are there any words that cannot occur in Scrabble? If so what is the shortest such word? The board is 15 by 15 thus we have 16 as an incontrovertible upper limit to our search. The frequency of occurrence of letters in a Scrabble set is as follows: E=12, AI=9, O=8, NRT=6, DLSU=4, G=3, BCFHMPVWY and Blank=2, JKQXZ=1 (Total 100 tiles). Of course, the rules specifically prohibit proper names, foreign words, abbreviations, and words with hyphens or apostrophes. We require a word that is not excluded by these conventions. 'Chambers' is the arbiter once again.

WORDPLAY

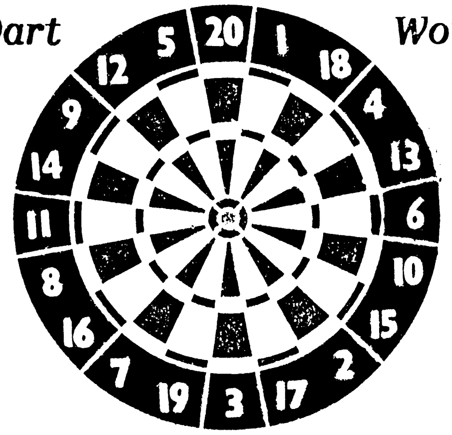
Clock Words

By R.C.McWILLIAM



You are required to form a sequence of at least 40 words by reading round the clock clockwise, starting with the F at 12 o'clock. You can jump any number of letters at each step, but the letter you begin with cannot be used or passed again in the same word. e.g. you can begin with FEN, FEAST, FAR but not with FAME, FEE, or FAIL. The first letter of each succeeding word must be the next after the last letter of the previous word, e.g. FEAR,DAM,NOISE, LOST,REAM,...No two-letter words - otherwise the usual Scrabble conditions apply.

Dart Words



By Loretta BRUCE

The 20 numbers round the dartboard each represent a different letter. Reading clockwise from the top '20' four words can be spelt out: one of 6 letters, two of 5 letters, and one of 4 letters. To solve the code, study the score sheet below. 16 throws of 3 darts have been needed to reach the winning score of 501. The score for each word is the sum of the numbers represented by the letters. A double letter means a double score. For example the score for the word BEET would be 61, which is 14 + 18(twice) + 11. By comparing the scores for each throw, you should be able to work out the values of the other letters. G,J,K,M,V,Z are not used.

Scoresheet

BALL = 47	INN = 24
BULL = 39	TOSS = 51
QUA = 30	WHY = 16
COX = 32	YOU = 6
BOX = 33	WEE = 43
PURR = 41	PUFF = 27
TOFF = 29	QUO = 23
DELL = 47	NOW = 13
298	203

Elementary Words

The journal Chemistry & Industry used to have a crossword at Christmas in which each square contained the symbol for a chemical element. For example, 1 across on one occasion was HeLiCoPtEr. What is the longest word that can be spelt in this elemental fashion?

HALF-LIFE

When asked if he had helped to find Nobelium,
My scientific friend replied "Like Helium!
Why waste one's time when half of what is in its
Make-up disappears in less than thirty minutes!"

Colin VAUGHAN

Are You Game?

By Julian COURTLAND-SMITH

S	C	R	A	B	B	L	E
M	O	N	O	P	O	L	Y
G	U	E	R	I	L	L	A
O	V	E	R	L	O	R	D
M	A	N	E	A	T	E	R
T	R	E	S	P	A	S	S
C	O	N	Q	U	E	S	T
S	O	R	C	E	R	E	R

On a chessboard I have listed eight games of eight letters. Place eight Rooks on the board, none guarding or attacking any other, so that the letters they cover, when read down the diagram, spell out another well-known eight-lettered game.

WORDPLAY

Mnemonics for Pi

Further to the examples A, B, C quoted in issue 2, G.J.SUGGETT sends the following two. D which he learnt 20 or so years ago, was the winning entry in a competition for Pi-mnemonics. E was quoted in The Guardian in a letter from Dr S.Roy, and is also quoted in Martin Gardner's Mathematical Puzzles and Diversions, but in both sources the spelling of 'rivaled' is Anglicised to 'rivalled' - thus spoiling the mnemonic! It was first published by Adam C. Orr, Literary Digest, 20 January 1906. Both these mnemonics are one digit shorter than W.Stead's example B.

D. *Now I will a rhyme construct*
By chosen words the young instruct
Cunningly devised endeavour,
Con it and remember ever
Widths in circle here you see
Sketched out in strange obscurity.

E. *Now I - even I - would celebrate*
In rhymes unapt the great
Immortal Syracusan rivaled nevermore,
Who in his wondrous lore,
Passed on before,
Left men his guidance
How to circles mensurate.

Rhyme Schemes

Philip COHEN writes in connection with terms for metric feet (mentioned on p14 of the first issue) that: "All feet of lengths 2 to 4 have names taken from classical prosody, where they refer to long/short rather than stressed/unstressed." He cites Shapiro and Benn, A Prosody Handbook (Harper & Row, 1965) for most of them. Tabulated they are:

00 pyrrhic	000 tribrach	0000 proceleusmatic
01 iamb	001 anapaest	1000 - 0001 first - fourth paean
10 trochee	010 amphibrach	1100 major ionic
11 spondee	100 dactyl	0110 antispast
	011 bacchic	0011 minor ionic
	101 cretic/amphimacer	1010 ditrochee
	110 antibacchic	0101 diiamb
	111 molossus	1001 choriamb
		0111 - 1110 first - fourth epitrite
		1111 dispondee

"Unstrictly interpreted, the number of rhyme schemes is given by the Catalan numbers (e.g. 5 for 3 lines: aaa, aab, aba, abb, abc) as explained in Martin Gardner's Time Travel and Other Mathematical Perplexities, ch 20 (from a column of 1977 or so)."

Pronounceable Codes

The idea of a pronounceable code is that the letters or sounds of the message be replaced by other letters or sounds, but in such a way that the resulting sequence of letters or sounds is not just a jumble, as is usual with traditional codes, but can be pronounced, as if it were some alien language. The simplest way to do this would seem to be to change vowels to vowels and consonants to consonants. Obvously there are many ways in which this might be done. Readers are invited to experiment to find a method that produces a good result - ideally something that does not look as if it is in code at all, but merely in some strange language. To begin with I offer the following example for decoding:

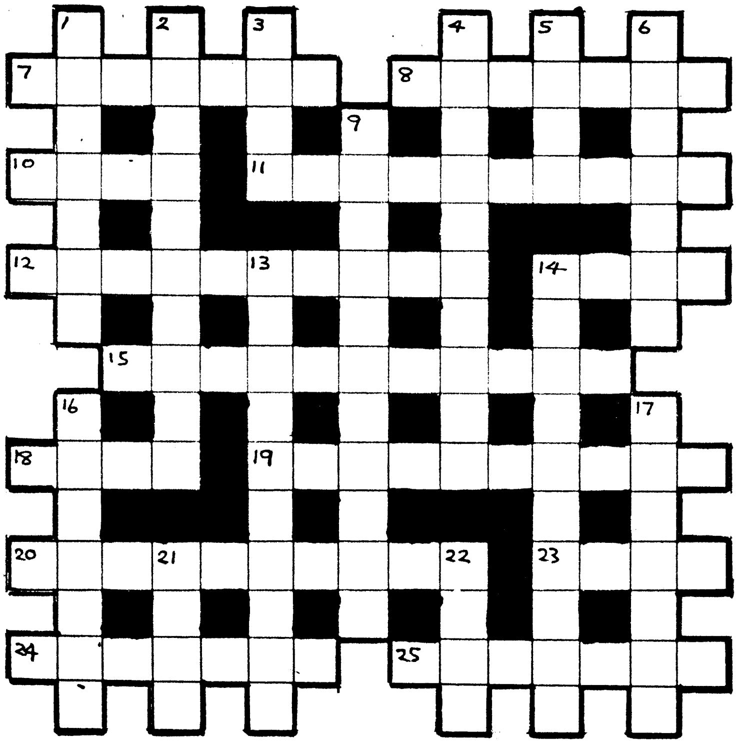
Op Yepefa fof Lacme Ljep
E tvevimz qmietasi funi fidii
Xjisi Emqj vji tedsif sowis sep
Vjsuahj dewispt nietasimitt vu nep
Fuxp vu e tapmitt tie.

It shouldn't be difficult to do better than that! It is just about pronounceable? Please try something yourself.

The Alphabetical Cube and **Crossword Puzzle Patterns** are held over to the next issue.

WORDPLAY

Cryptic Crossword 5 By QUERCULUS

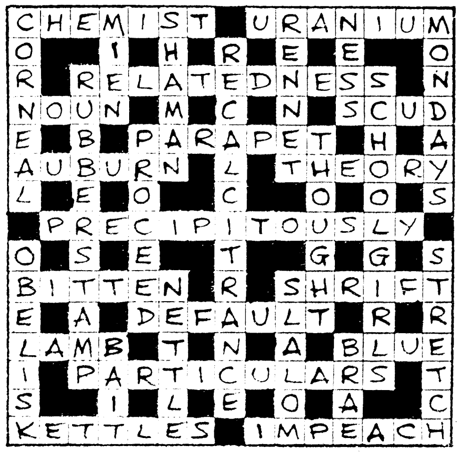


ACROSS
07. Good man throws the dice and walks it. (7)
08. Fish or Rummy player? (7)
10. Break biscuit spell. (4)
11. Roman Brits' sudden inspiration. (10)
12. Minute sari for Indians perhaps. (10)
14. Neckwear to turn the trick. (4)
15. A wag's burden maybe 52 in 18. (4,2,5)
18. Plucky lame grouse detective? (4)
19. Silly PC had order easy to do. (6,4)
20. Released from prison - underworld blamed. (10)
23. One less is company. (4)
24. To madden - try to pun H in German. (7)
25. Different view of a taller order. (7)

DOWN
01. Substitute or spectator? (5-2)
02. Entangle comical pet at play. (10)
03. A 5 that is not 14. (4)
04. Magnificent partnership from spectator's point of view (10)
05. Will do as a quarter of 15 (4)
06. Copy socialist floater concerning conscription (7)
09. Combination essential in Chess (4,2,5)
13. Absorb attack and assume command. (4,6)
14. Measly snooker with only the colours left? (3-7)
16. Being out 1G entails shake-up. (7)
17. Dismay or alarm shows leadership attributes. (7)
21. Copy with little credit for same again. (4)
22. Lead out to start 18. (4)

The four-letter words, and some others, are loosely connected.

Crossword 4. SOLUTION



Uncommon English

Bishopship. The state or domain of a bishop, or the art of being a bishop.

Deteriation. Poor speech marked by omission of letters or syllables (e.g. Auxil(i)ary, Mete(or)ological).

Flangle. A combination of flange and angle.

Ingled. Possessing numerous small sharp indentations.

Oddysey. An epic journey in search of the eccentric.

Orbled. Possessing numerous small round protrusions.

Uncularity. Uncle-like behaviour. ('Avuncular' applies apparently only to maternal uncles. What is the corresponding term, if any, for paternal uncles?)

Verbiary. A collection of strange or special words exhibited or classified (by analogy with beastiary, aviary). The art of using strange or special words (cf. topiary, embroidery).

Many interesting-sounding words arise due to misprints. Definitions of the likely meanings of the following are invited (and any others you may have come across): **Clearified** - **Circule** - **Conduction** - **Unindation** - **Irrelevalent** -

BINARY ARRAYS

Chessboard Mosaics

The numberings from which the mosaics on page 64 were derived are these: A, natural order; B, diagonal order; C and F, the two magic numberings of the same 'three-leaper plus knight' tour given in Chessics 26, p119; D, the diagonally magic 'emperor' (knight plus wazir) tour given on the same page; E, spiral (or spider?) order.

A

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

B

1	2	6	7
3	5	8	13
4	9	12	14
10	11	15	16

C

1	12	5	16
14	7	10	3
11	2	15	6
8	13	4	9

D

1	8	9	16
14	13	4	3
7	2	15	10
12	11	6	5

E

1	2	3	4
12	13	14	5
11	16	15	6
10	9	8	7

F

12	15	2	5
1	6	11	16
8	3	14	9
13	10	7	4

The numbering that results from rotating the first diagram is (in one of its arrangements) the Durer magic square (diagram A')! This was an unexpected discovery. This effect shows the close relationship of this magic square to the natural order (see for example in Rouse Ball's Mathematical Recreations, 11th edn, p 199, where it is shown that by reversing the sequences in the diagonals of the natural order produces a magic square that is a permute of Durer's). The process of rotating the mosaic means that where before the rotation it indicated a move x to y, now it indicates a move 17-y to x.

Rotation of the second diagram gives the numbering B'.

A'

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

B'

1	10	13	14
11	2	12	8
7	16	3	4
15	6	9	5

G

1	2	9	10
3	4	11	12
13	14	5	6
15	16	7	8

G'

1	2	7	8
4	3	6	5
15	16	9	10
14	13	12	11

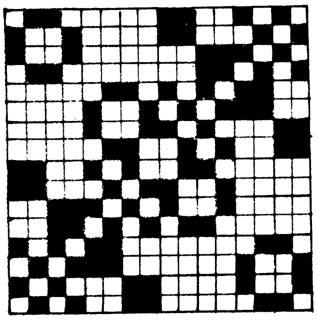
H

1	2	9	10
3	4	11	12
5	6	13	14
7	8	15	16

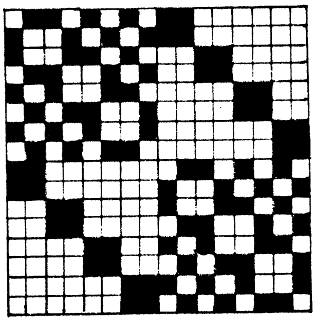
H'

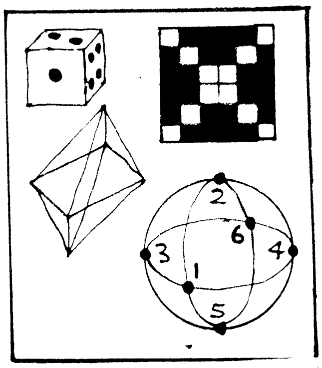
1	7	14	12
8	2	11	13
10	16	5	3
15	9	4	6

G



H

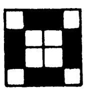




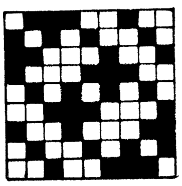
Len Gordon has devised a computer program for converting numberings to patterns and vice versa. He offers the patterns G and H and their associated numberings as examples.

The other question asked was whether the numbering that results from the rotated pattern can be the same as the original. This requires the pattern to have all four quarters the same. It is easily seen that this is possible only in the simple 2 by 2 case shown below. Two 3 by 3 numberings and their derived mosaics are also shown. Denoting the complement (n^2+1-x) of x by x' if 1 is in line with x then (in the rotatable case) it must also be in line with x', and 1' must also be in line with x and x' - i.e. 1, x, 1', x' form a square on the nxn board. If y is also in line with 1 then 1, y, 1', y' also need to form a square - an impossibility.

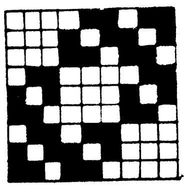
1	2
3	4



1	2	3
4	5	6
7	8	9



6	7	2
1	5	9
8	3	4



A 'board' on which this does become possible is provided by the example of a fly moving along the edges of an octahedron - or if you prefer, a plane flying along quadrants of great circles round the globe. The resulting mosaic is shown inset above. It can also be interpreted as showing the rolls of a single die about a single edge at a time.