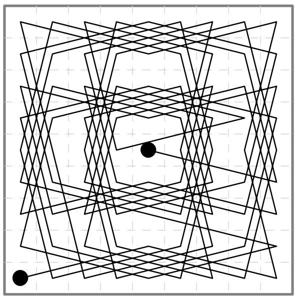
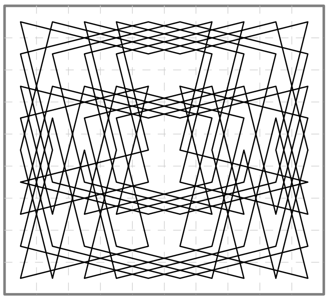
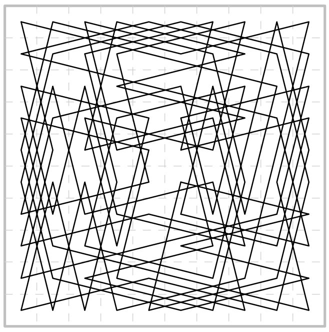
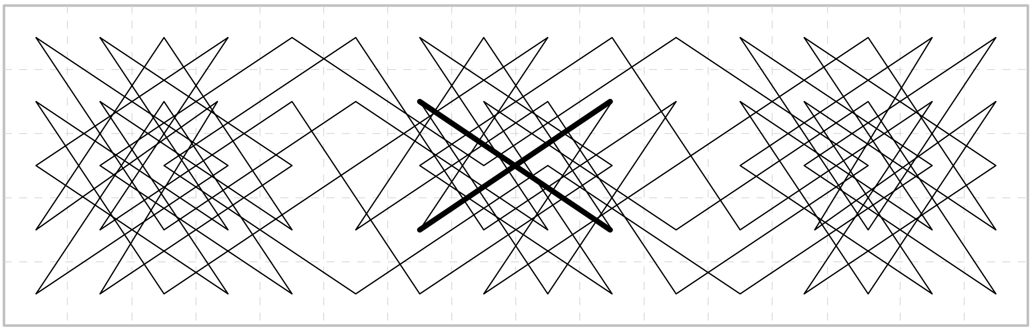
⌘ 10

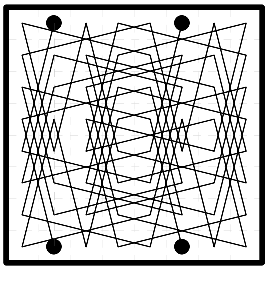
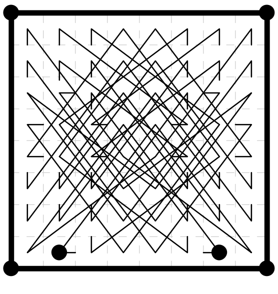
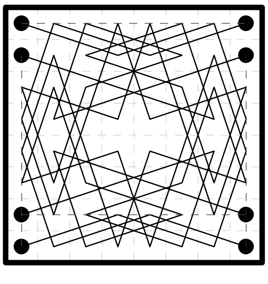
Augmented Knight & Leaper Tours

by G. P. Jelliss



2019

==

Title Page Illustrations:
Giraffe Tours 9×9, 9×10, 10×10.
Zebra Tour 5×16.
Magic Two-Giraffe Tour, Diagonally Magic Fiveleaper Tour, Magic Four-Camel Tour.

Contents

Knight's Tour Notes, Volume 10, Augmented Knight & Leaper Tours.

Augmented Knights

We look at tours by pieces that combine the knight move with one or more lateral or diagonal moves. At the end of each section we also include pieces with extended ‘nightrider’ moves such as the two-move {2,4} and three-move {3,6}

All such pieces can be regarded as Knight + Queen, though some particular cases with restricted queen moves have their own individual names. In chess variants the Queen + Knight, Rook + Knight and Bishop + Knight are quite common and have been given various names in different games. Problemists favour the names Amazon, Empress and Princess for these ‘knighted’ pieces.

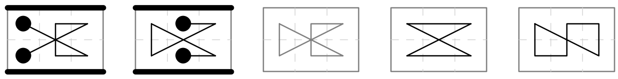
The tours are arranged according to the number of different moves employed, usually including the closure move that joins the end cell to the initial cell.

Two-Move Knighted Pieces

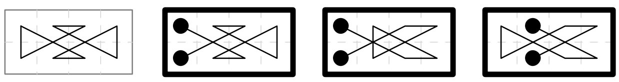
{1,2}+{0,1} Emperor

The Emperor is the simplest piece in the Empress (R+N) family. Being a combination of two free-leapers, wazir {0,1} and knight {1,2}, it has considerable freedom and ability to make tours.

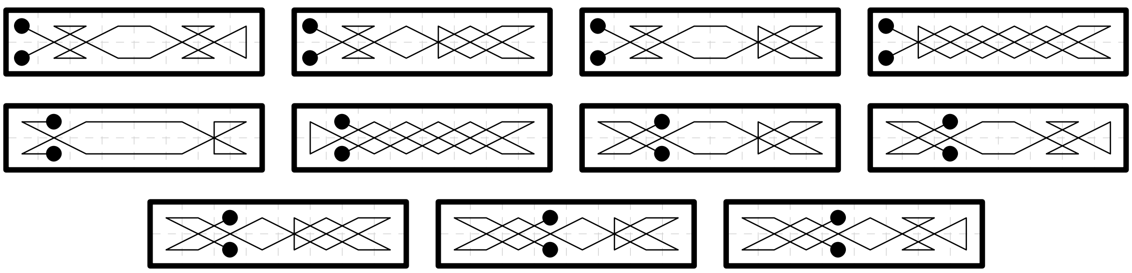
2×3 Board. It has two semi-magic tours 2×3. When closed these open tours give only one geometrically distinct tour. There are also single tours with biaxial and rotary symmetry.



2×4 Board. There are three magic emperor tours, one giving a biaxially symmetric closed tour.

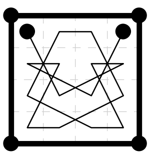


2×8 Board. There are 11 magic emperor tours, three of which are biaxial when closed.

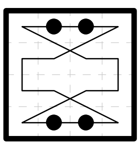


4×4 Board. A diagonally magic emperor <u>open</u> tour is found among the catalogue of magic squares drawn up by Frénicle de Bessy around 1660. The magic constant is 34. That this is the only two-pattern solution does not seem to have been noticed until I reported it in *Chessics* #26 1986.

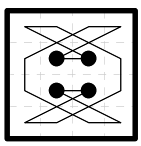
1	8	9	16
14	13	4	3
7	2	15	10
12	11	6	5



2	1	16	15
13	14	3	4
12	11	6	5
7	8	9	10

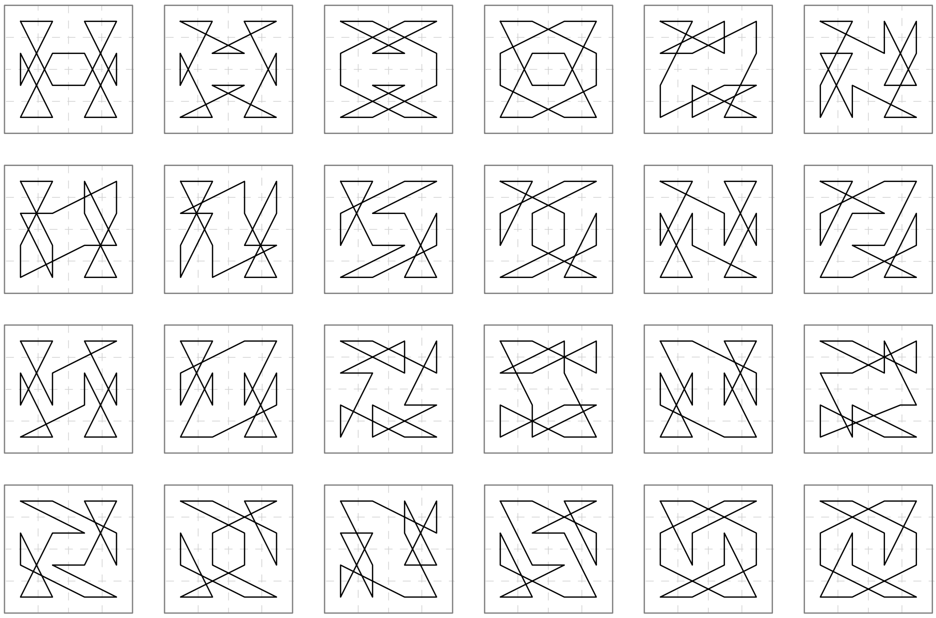


2	3	14	15
13	16	1	4
12	9	8	5
7	6	11	10

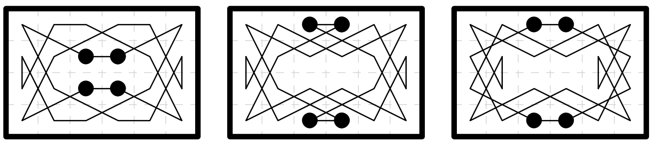


Also in *Chessics* #26 I noted that there are two other emperor nondiagonal magic tours that show biaxial symmetry when the two end-points of the path are joined up to give a closed path.

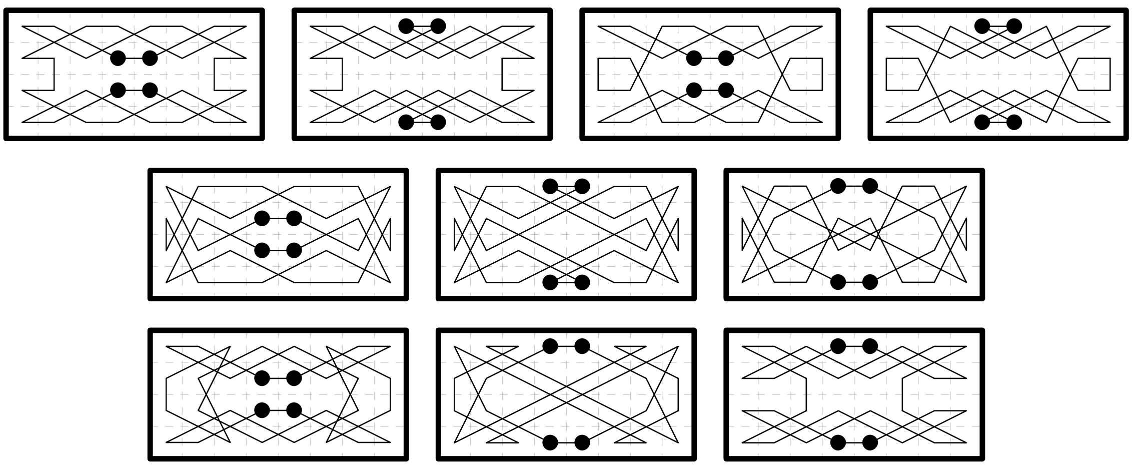
We show here 24 symmetric emperor tours with alternating wazir and knight moves. The first 4 are biaxial and the other 20 ae rotary.



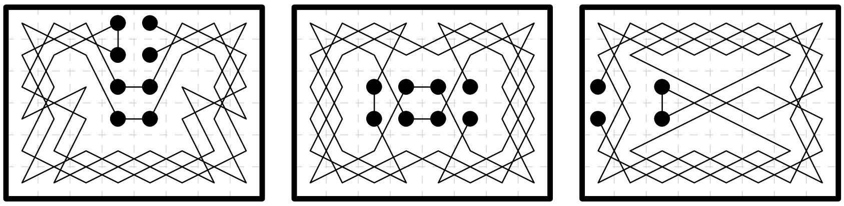
4×6 Board On this the knight half-tours (see ⌘ 4) can be paired with themselves to give semi-magic emperor tours adding to 50 in the files or 75 in the ranks. I found 15 of the first and 4 of the second type. By using more wazir moves it is possible to form fully magic tours on the 4×6. I found the following three biaxially symmetric examples on 10 Aug 1991. The first of these is also given by Trenkler (1999). The ranks consist of three pairs of complements adding to 25. Each quarter-path covers six cells, one in each file. If numbered from the other ends of the quarters the results are semi-magic with files adding to 50, but the ranks having two different sums. Since the diagrams are clear it is left to the reader to put in the numbers if desired.



4×8 Board Here are ten magic emperor tours with biaxial symmetry found 16 Oct 2014. The ranks consist of four pairs of complements adding to 29 (sum 116). Each quarter-path covers eight cells one in each file (sum 58).

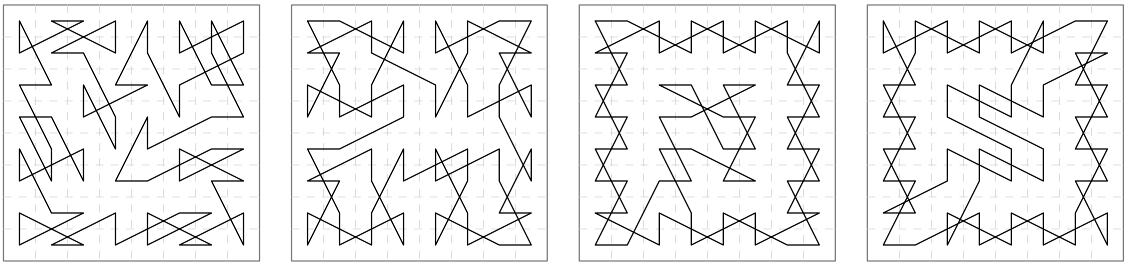


6×8 Board (Jelliss 2003) The first was consciously constructed using ‘contiguous contraparallel chains’, but the others were constructed for their visual symmetry. In the first tour if the files are divided into three pairs then all these pairs add to 49 (file total therefore 3×49 = 147. In the ranks pairs related by reflection in the horizontal median add to 25 or 73 which together equal 98 (twice 49) thus ensuring the rank sum is 4×49 = 196. This is the same in the second tour, but in the third tour the constants in the ranks are 37 and 61. [‘Emperor Magic Tours’ *The Games and Puzzles Journal* #26]

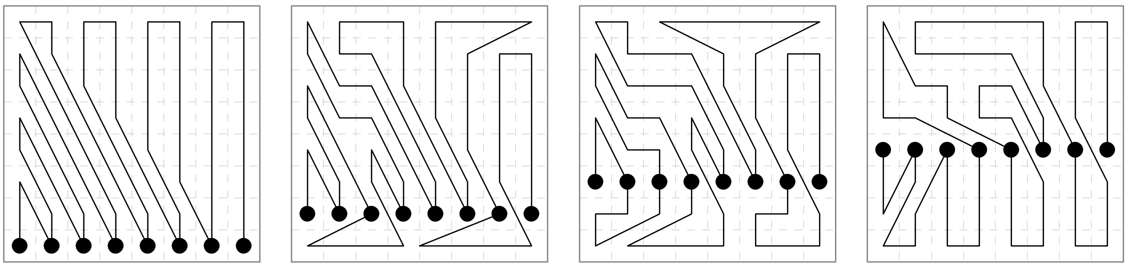


8×8 Board For numerous open knight tours with ends a wazir move apart see ⌘ 6 & 7.

Alternating wazir-knight tours interested Indian composers. One by the Rajah of Mysore, from Harikrishna (1871). Twofrom Naidu (1922). The symmetric solution is my own (Jelliss 1984).



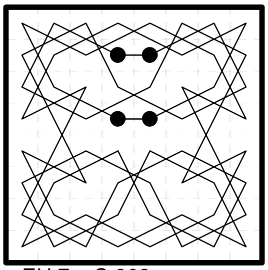
Figured Emperor Tours. W. E. Lester (*Fairy Chess Review* vol.3 #8 Oct 1937 p.86 ¶2930-31, #9 Dec 1937 p.99 and #10 Feb 1938 p.110 ¶3035-36). Emperor open tours with square numbers along the ranks, without intersection of move lines. The first generalises to work on n×n boards.

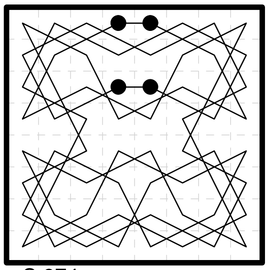


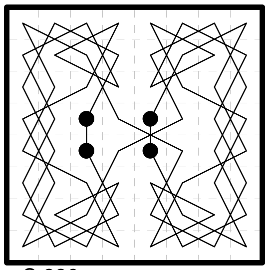
Magic ‘One-Knight’ Emperor Tours. These are magic knight tours where the ends are a wazir move apart. There are nine: (01a) to (01i) shown in ⌘ 9. Three were found by Ligondes 1883-4, one by Grossetaite 1896, two by Murray 1939, two by Marlow 1987 and the last by Roberts 2003.

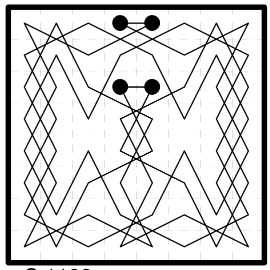
Magic ‘Two-Knight’ Emperor Tours. These consist of two paths of knight moves connected by two wazir moves. (For cases with longer closure move 64-1 see the triple-move Empress tours). Most of these tours are of the biaxial type described in the Magic Theory in ⌘ 1. They are magic when numbered 1-64 with either of the wazir moves being the closure move. The examples shown here are mainly from the early years of *Le Siècle*. Later years I have not yet fully explored.

The first examples here are not diagonally magic. They are shown in their original orientation. (1) is from the *Knightly Peripatatics* column by E. H. in the *Glasgow Weekly Herald* ¶7 (12 Apr/3 May 1873) diagonals 260±20 [also *Le Siècle* ¶862 (8 Aug 1879) puzzle by Clerville]. (2) *Le Siècle* ¶874 (22 Aug 1879) 260±12 (3) *Le Siècle* ¶886 (5 Sep 1879) 260±20. (4) *Le Siècle* ¶1198 (3 Sep 1880) 260±4 by M. A. D. of Rouvres.

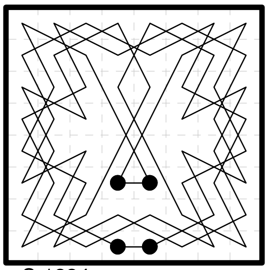

EH 7 = S 862

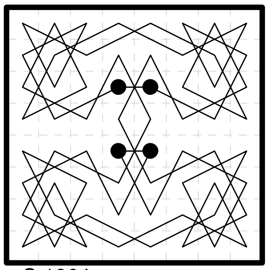

S 874

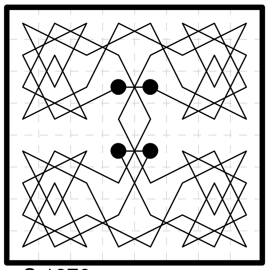

S 886

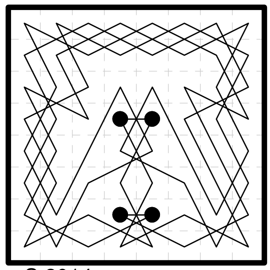

S 1198

(5) *Le Siècle* ¶1234 (15 Oct 1880) 260±32. (6) *Le Siècle* ¶1864 (20 Oct 1882) 260±4. (7) *Le Siècle* ¶1876 (3 Nov 1882) 260±4. (8) *Le Siècle* ¶2014 (13 Apr 1883) with approximate axial symmetry 260±24 by Adsum (Bouvier).

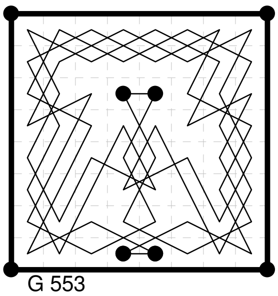

S 1234

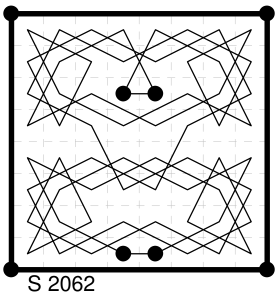

S 1864

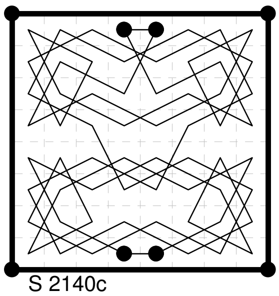

S 1876

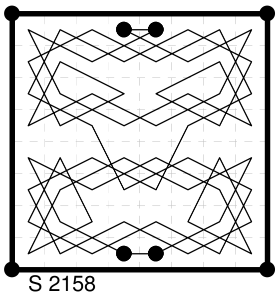

S 2014

The next examples are diagonally magic. (1) Asymmetric published by 'Adsum' (Bouvier) in the 'Passe-Temps Hebdomadaire' column in *Le Galois* ¶553 (16 Apr 1883). (2) *Le Siècle* ¶2062 (8 Jun 1883). (3) *Le Siècle* ¶2140c (7 Sep 1883) found independently by both Palamede (Ligondes) and Adsum (Bouvier) published in a three-part problem with similar Empress tours. (4) *Le Siècle* ¶2158 (28 Sep 1883) by 'M. E. Reuss à Strasbourg'. Three very similar of double half-board type.

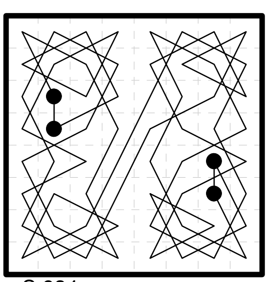

G 553

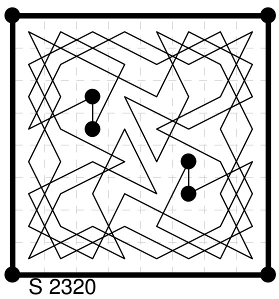

S 2062

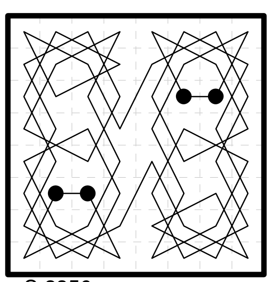

S 2140c

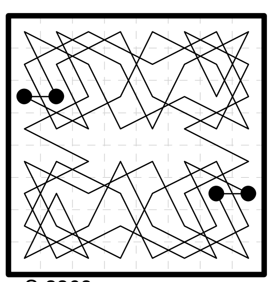

S 2158

The above are mainly axial. Some examples are more rotary (described as 'angulaire' in the French text): (1) *Le Siècle* ¶934 (31 Oct 1879) irregular with non-complementary diagonals 296 and 272. (2) *Le Siècle* ¶2320 (4 Apr 1884) by Palamede (Ligondes) diagonals 260. (3) *Le Siècle* ¶2356 (16 May 1884) by Adsum (Bouvier) diagonals 296 and 256. (4) *Le Siècle* ¶2368 (30 May 1884) by Reuss diagonals 236 and 240.

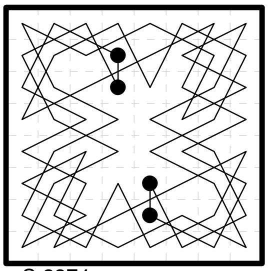

S 934

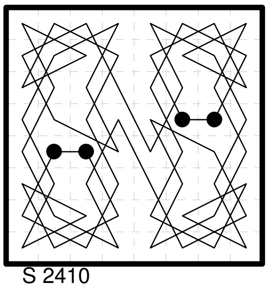

S 2320

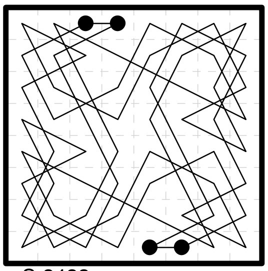

S 2356

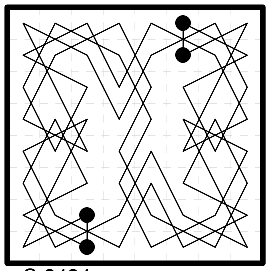

S 2368

Further rotary (5) *Le Siècle* ¶2374 (6 Jun 1884) by Béligne, diagonals 292 and 240. (6) *Le Siècle* ¶2410 (18 Jul 1884) by Béligne, diagonals 272 and 280. (7) *Le Siècle* ¶2422 (1 Aug 1884) by Palamede (Ligondes) diagonals 208 and 300. (8) *Le Siècle* ¶2434 (15 Aug 1884) by Adsum (Bouvier) diagonals 260±24.


S 2374

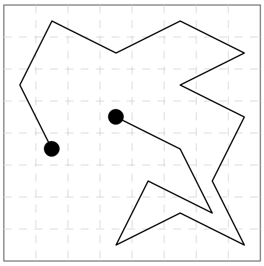

S 2410

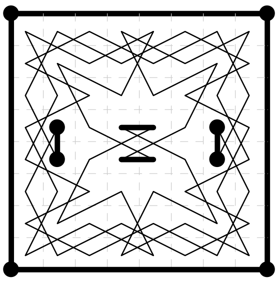

S 2422


S 2434

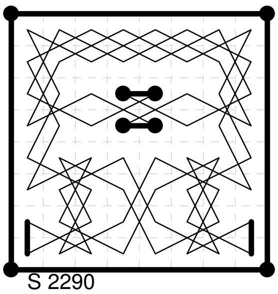
Magic 'Four-Knight' Emperor Tours. The first example here (Jelliss 1986) consisting of four identical paths of knight moves connected by four wazir moves appeared in *The Problemist* (vol.12 #10 p.196 Jul) where I reported examining all possible magic emperor closed tours derived from four identical 16-cell knight paths with wazir links, finding only one was diagonally magic. It so happens that the path is one that is non-intersecting. The ranks are formed of pairs adding to 33 and 97.

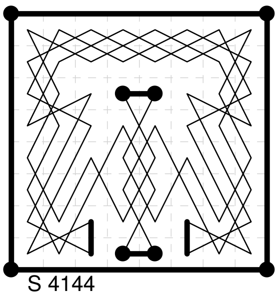
The other examples each use two different 16-cell paths and are from *Le Siècle* ¶2290 (29 Feb 1884) by 'M. D. Zibetta à St Ouen', which includes two double-Beverley quads, and *Le Siècle* ¶4144 (21 Feb 1890) by 'M. A. F' (i.e. Feisthamel himself). The paths used in these are also non-crossing. The non-diagonal example (Jelliss unpublished) is based on the Beverley pattern.

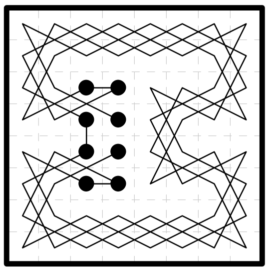




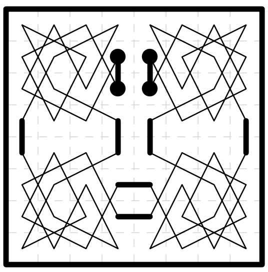
10	35	60	21	12	37	62	23
59	14	11	36	61	22	19	38
34	9	58	13	20	39	24	63
57	32	15	48	49	18	1	40
8	33	50	17	16	47	64	25
31	56	7	52	45	26	41	2
6	51	54	29	4	43	46	27
55	30	5	44	53	28	3	42


S 2290


S 4144



Magic 'Eight-Knight' Emperor Tour. This tour constructed 27 Apr 2018 while writing this note is axial but is not of the complemented type. Ranks consist of pairs adding to 33 and 97. Files are of pairs adding to 17 and 113 on the left and 49 and 81 on the right. The diagonals add to 260±4.



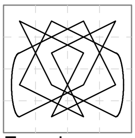
59	4	63	6	27	34	29	38
62	7	60	1	32	37	26	35
3	58	5	64	33	28	39	39
8	61	2	57	40	31	36	25
9	52	15	56	41	18	45	24
14	55	12	49	48	21	42	19
51	10	53	16	17	44	23	46
54	13	50	11	22	47	20	43

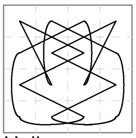
Tours of ‘16-knight’ type can easily be constructed but I haven't found a magic example. The alternating tours can be regarded as ‘32-knight’ tours. Emperor tours of other types can of course also be constructed, for example formed mainly of wazir paths with knight-move connections.

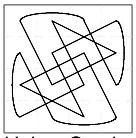
{1,2}+{0,2} Templar

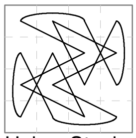
This augmented knight is another pece in the Empress (Rook + Knight) group.

4×4 Board Some tours with this pair of moves, showing various different symmetries were constructed by R. J. French, E. Huber-Stockar, S. H. Hall and others in *Fairy Chess Review* 1941 (¶4772, ¶4842, ¶4892, ¶5042). Other solvers contributed 17 further tours that were not diagrammed.


French


Hall

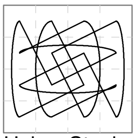

Huber-Stockar

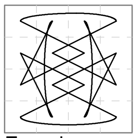

Huber-Stockar

{1,2}+{0,3} Empress

Another piece from the Empress Group of Rook+Knight pieces, but not having its own name.

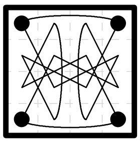
4×4 Board Some tours with these pairs of moves, showing various different types of symmetry were constructed by R. J. French and E. Huber-Stockar in *Fairy Chess Review* 1941.


Huber-Stockar

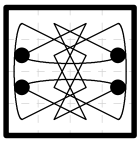

French

Magic tours 4×4 (nondiagonal) are possible with biaxial symmetry when the two end-points of the path are joined up to give a closed path (Jelliss *Chessics* #26 1986).

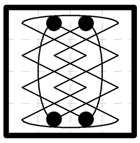
1	12	5	16
14	7	10	3
11	2	15	6
8	13	4	9



4	7	10	13
9	14	3	8
16	11	6	1
5	2	15	12



4	9	16	5
15	6	3	10
2	11	14	7
13	8	1	12

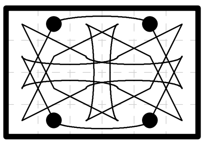


The black dots mark 1, 8, 9, 16 The first two diagrams show the same closed tour numbered from different points; such a magic tour is termed **cyclic**.

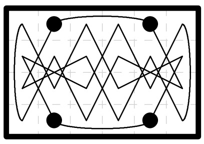
John Beasley (*Variant Chess* #64 2010 p.233) notes that the {0,3}{1,2} piece on the 4×4 board has four moves at each cell, and that the tours above are self-complementary in the sense that when rotated a quarter turn they use the other two moves at each cell. He also notes that they have the alternative magic sums 32 and 36 in all the odd and even diagonals, including the broken diagonals.

4×6 Board. More {0,3}{1,2} biaxial magic tours (Jelliss 10 Aug 1991) 50 in files 75 in ranks.

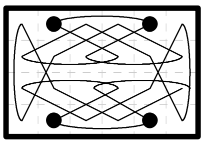
4	13	18	7	12	21
23	8	3	22	17	2
14	5	10	15	20	11
9	24	19	6	1	16



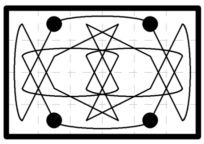
6	13	4	21	12	19
23	8	15	10	17	2
14	5	22	3	20	11
7	24	9	16	1	18



6	13	16	9	12	19
15	8	11	14	17	10
22	5	2	23	20	3
7	24	21	4	1	18



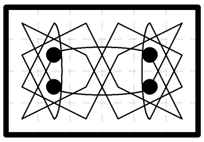
6	13	16	9	12	19
23	8	3	22	17	2
14	5	10	15	20	11
7	24	21	4	1	18



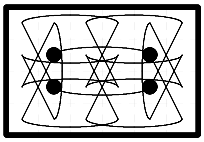
Four with origin in the outer ranks.

Six with origin in the inner ranks.

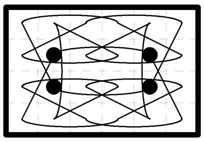
2	19	16	9	6	23
17	12	3	22	13	8
20	1	10	15	24	5
11	18	21	4	7	14



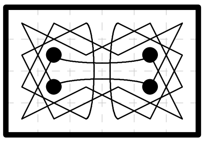
2	19	22	3	6	23
17	12	9	16	13	8
20	1	4	21	24	5
11	18	15	10	7	14



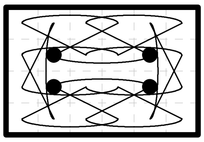
2	19	22	3	6	23
21	12	5	20	13	4
16	1	8	17	24	9
11	18	15	10	7	14



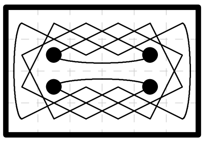
2	21	18	7	4	23
17	12	3	22	13	8
20	1	10	15	24	5
11	16	19	6	9	14



3	18	23	2	7	22
20	1	4	21	24	5
17	12	9	16	13	8
10	19	14	11	6	15



6	21	14	11	4	19
15	12	5	20	13	10
22	1	8	17	24	3
7	16	23	2	9	18

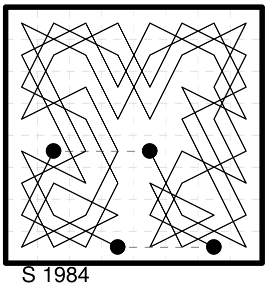


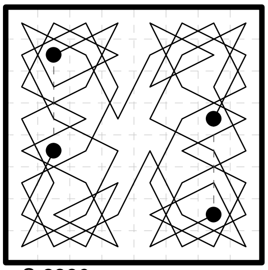
The arithmetical forms are shown since where curved {0,3} moves cross may be unclear.

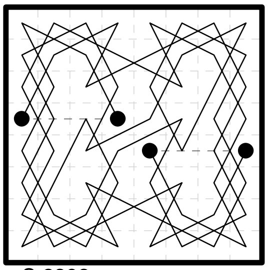
8×8 Board. Magic one-knight tours with {0,3} link. There are seven magic one-knight tours with end-points a {0,3} move apart. In our catalogue ⌘ 9 they are (03a) to (03g). Five were found by Ligondes 1883/4, one by Murray 1940 and one by Marlow 1988.

8×8 Board. Magic two-knight tours with {0,3} links.

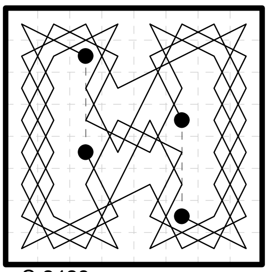
(1) *Le Siècle* ¶1984 (2 Mar 1883) by Palamède (Ligondes) asymmetric, diagonals 352 & 216 from b4=1, or 344 & 224 from g1=1. (2) *Le Siècle* ¶2386 (20 Jun 1884) by Palamède, rotary, diagonals 312 and 256 from b4=1. (3) *Le Siècle* ¶2392 (27 Jun 1884) by Adsum (Bouvier), rotary, diagonals 232 and 256 from d5=1.

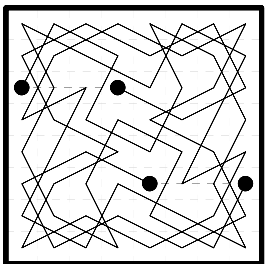

S 1984

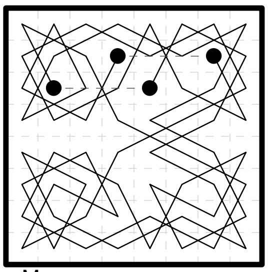

S 2386


S 2392

(4) *Le Siècle* ¶2428 (8 Aug 1884) by 'M. A. D. à Rouvres', rotary, diagonals 224 and 232 from c7=1. (5) *Le Siècle* ¶2560 (9 Jan 1885) by Adsum (Bouvier), rotary, diagonals 248 and 232 from a6=1. (6) Example from Murray (1951) nearly axial, diagonals 208 and 312.


S 2428

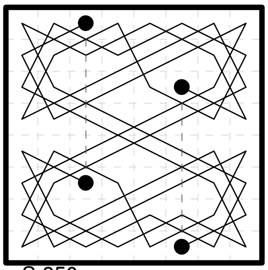

S 2560

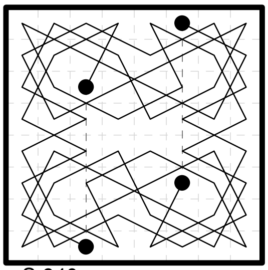

Murray

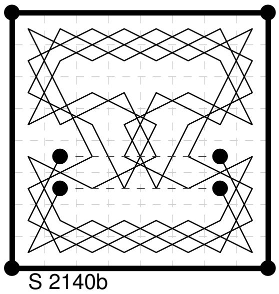
{1,2}+{0,5}

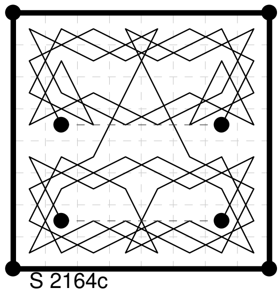
8×8 Board. Magic one-knight tours with {0,5} link. There are seven magic one-knight tours with end-points a {0,5} move apart (05a) to (05g) in the catalogue in ⌘ 9. Two by Bouvier 1876/82, one by Caldwell 1879, three by Francony 1881/2, and one by Ligondes 1883.

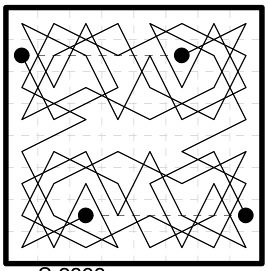
8×8 Board. Magic two-knight tours with {0,5} liks. Here are eight magic tours of this type from the earlier years of the Feisthamel column. (1) *Le Siècle* ¶850 (25 Jul 1879) rotary, diagonals 232 and 240. (2) *Le Siècle* ¶940 (7 Nov 1879) rotary, diagonals 260±20. (3) *Le Siècle* ¶2140b (7 Sep 1883) by Adsum (Bouvier), axial, diagonally magic (4) *Le Siècle* ¶2164c (5 Oct 1883) by Reuss, axial, diagonals magic.

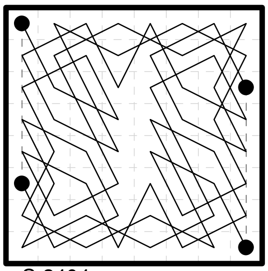

S 850

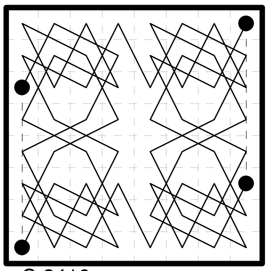

S 940

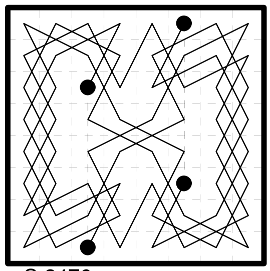

S 2140b


S 2164c


S 2398


S 2404


S 2416


S 2476

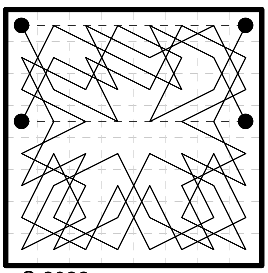
(5) *Le Siècle* ¶2398 (4 Jul 1884) by M. A. D. rotary diagonals 216 and 256. (6) *Le Siècle* ¶2404 (11 Jul 1884) by Reuss, rotary, diagonals 280 and 204. (7) *Le Siècle* ¶2416 (25 Jul 1884), rotary, diagonals 220 and 276. (8) *Le Siècle* ¶2476 (3 Oct 1884) by Reuss, rotary, diagonals 280 and 268.

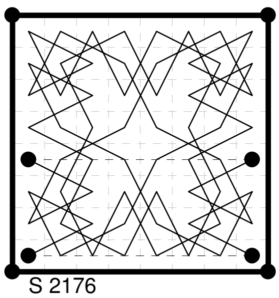
{1,2}+{0,7}

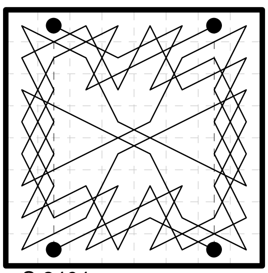
8×8 Board. Magic one-knight tour with {0,7} link. There is just one magic one-knight tour with end-points a {0,7} move apart (07a) found by Mackay, Meyrignac and Stertenbrink in 2003.

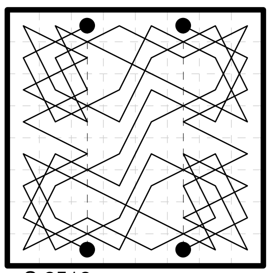
8×8 Board. Magic two-knight tours with {0,7} links. Here are some from Feisthamel.

(1) *Le Siècle* ¶2032 (4 May 1883) asymmetric, diags 320 and 220. (2) *Le Siècle* ¶2176 (19 Oct 1883) by Reuss, axial, diags magic. (3) *Le Siècle* ¶2464 (19 Sep 1884) by Adsum (Bouvier), rotary, diagonals 280 and 264. (4) *Le Siècle* ¶2512 (14 Nov 1884), rotary, diagonals 264 and 268.


S 2032

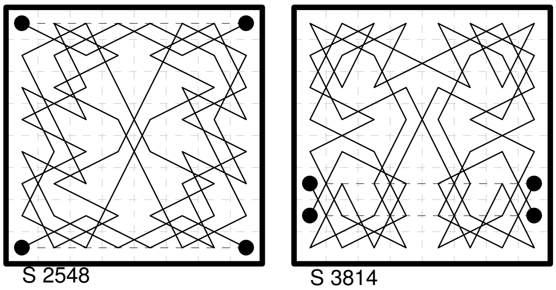

S 2176


S 2464


S 2512

continued

(5) *Le Siècle* ¶2548 (26 Dec 1884) by Adsum (Bouvier), rotary, diagonals 280 and 264. (6) *Le Siècle* ¶3814 (1 Feb 1889), axial, diagonals 260±44.



S 2548 S 3814

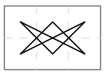
Further magic tours of the Empress family will be found in the notes on Triple movers, combining the knight with two rook moves of odd length {0,1}, {0,3}, {0,5}, {0,7}.

==

{1,2}+{1,1} Prince

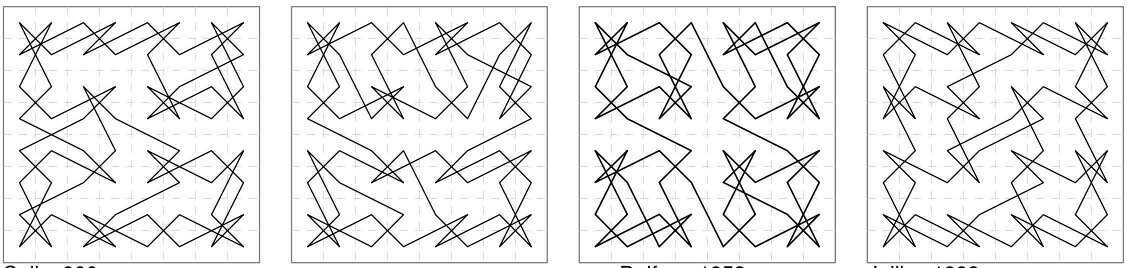
This is the simplest piece from the Bishop + Knight family.

2×3 board There is an arithmic prince tour with biaxial symmetry. Whatever way it is numbered the files add to 5, 7, 9 (cd=2) and the ranks to 9 and 12.



1	5	3
4	2	6

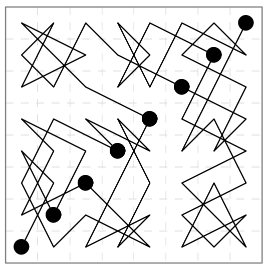
8×8 board A knight+fers tour was first given by as Suli (c.900). It is reproduced in numerical form in H. J. R. Murray *A History of Chess* (1913) p.336. I give it in diagram form here. Similar tours also appear in other Indian and Turkish sources of later date, as in the one by the Rajah of Mysore.



Suli c.900 Mysore before 1871 van Dalfsen 1953 Jelliss 1993

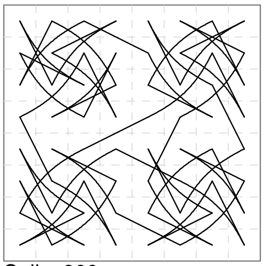
This subject was rediscovered by P. B. van Dalfsen who published a symmetric solution in *Fairy Chess Review* 1953. My example is symmetric and uses the same pattern of knight moves as in my Hospitaller tour shown below. It is easy to prove that the fers moves must form a fixed pattern of crosses: Just start at a1 and put in the only possible fers move, then go to c1 (which cannot join to b2) then go to e1 and so on; only the knight's moves vary.

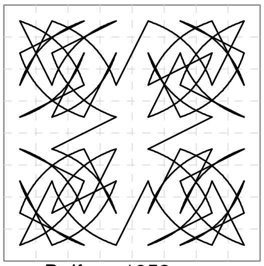
Figured Prince Tour. W. E. Lester *Fairy Chess Review* (vol.3 #10 Feb 1938 p110 ¶3109). Prince (Fers+Knight) open tour of 5×5 and 8×8 with squares in sequence along diagonal.

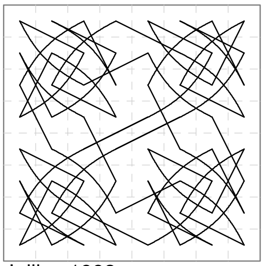


{1,2}+{2,2} Hospitaller

8×8 board. As noted in the Prince section above a Kight+Afil tour was first given by as-Suli (c.900), as shown in Murray's *History of Chess* (1913) p.336. In any alternating tour of this type the diagonal moves form the same fixed pattern of crosses; only the knight's moves vary. The topic was rediscovered by P. B. van Dalfsen who found a symmetric solution. My example is also symmetric and uses the same pattern of knight moves as in my Prince tour shown above.


Suli c.900


van Dalfsen 1953

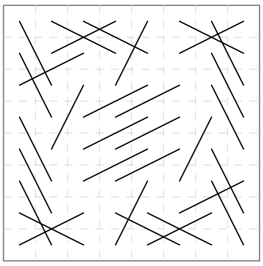

Jelliss 1993

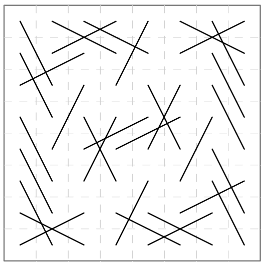
To avoid ambiguity it has been necessary to curve the alfil moves in the above diagrams.

{1,2}+{4,4} Night-Commuter

8×8 board.I sent this and the above Prince and Hospitaller results to Stefanos Pantazis for the *US Probem Bulletin* on 14 Jun 1993, but I don't think they were ever published there. The alternating Kight+Commuter tour here is in numerical form because of difficulty in showing it graphically. As with the Knight+Fers and Knight+Alfil tours above the diagonal commuter {4,4} moves similarly form a fixed pattern in alternating tours.

1	50	12	59	37	30	8	23
46	16	2	53	11	24	38	29
20	35	49	15	60	7	57	42
26	45	19	32	54	41	63	4
36	31	9	22	64	51	13	58
10	25	39	28	47	17	3	52
61	6	56	43	21	34	48	14
55	40	62	5	27	44	18	33





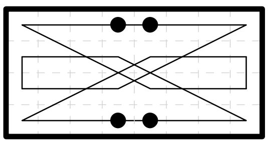
As reported in the first online issue of the *Games and Puzzles Journal* (#19, Jan-Apr 2001) Prof. D. E. Knuth found these two symmetric knight formations that will combine with all three cases: alternating Fers, Alfil and Commuter to give tours (and with other properties).

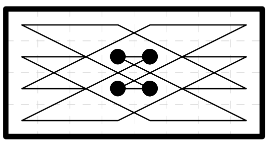
{2,4}+{0,1} Lancelot

We now look at some two-move pieces where the knight-move component is extended. Lancelot is a proposed name for this little studied piece, combining Wazir and Lancer. The Lancer move is a double-length Knight move passing over a cell centre. I show such moves 'unbolded'.

4×8 Two biaxially symmetric 4×8 magic tours from a partial enumeration. When numbered from the quarter points these remain magic in the files but take two different sums in the ranks.

4	3	2	1	32	31	30	29
24	23	22	21	12	11	10	9
25	26	27	28	5	6	7	8
13	14	15	16	17	18	19	20

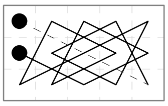


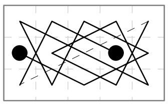


2	3	4	5	28	29	30	31
22	23	24	17	16	9	10	11
27	26	25	32	1	8	7	6
15	14	13	12	21	20	19	18

{1,2}{2,4}

Extending the knight move in a straight line leads to the **Nightrider**, analogous to the Rook along lateral lines or the Bishop along diagonals. Little work seems to have been done on Nightrider tours. Using its {2,4} move the nightrider can make <u>open</u> tours of the 3×5 board.

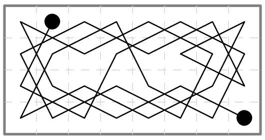


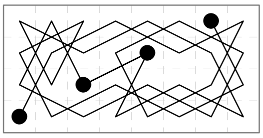


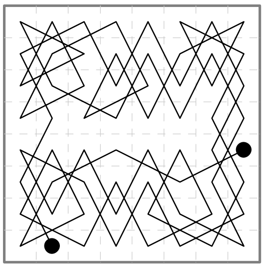
A knight tour with ends a {2,4} leap apart has both end cells the same shade so requires an odd board. In the notes on oblong boards ⌘ 4 we show the ten tours of this type on the 3×9 board.

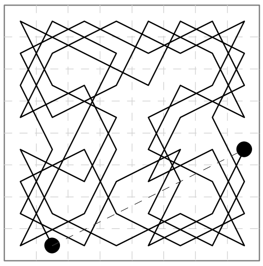
{1,2}{3,6}

All 4×8 knight tours are open, since the end cells have to be in the top and bottom edges. A nightrider however can make a closed tour. Two of the edge-hugging tours can be regarded as nightrider tours, since their end points are a {3,6} move apart. Here are two, not quite of the edge hugging type. One from a Turkish ms (c.1850) and one in which the central link is also in the nightrider line (Jelliss 2014).







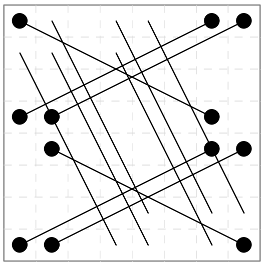


The first 8×8 tour is also from the Turmish ms, the other (Jelliss Sep 2014) is of squares and diamonds type, and both have the {3,6} closure move.

{2,4}{3,6}

A symmetric 8×8 tour using {2,4}{3,6} moves was sent to me by T. W. Marlow on 4 Apr 2001. It uses only moves in the knight direction, though no actual single knight moves. In the diagram I show only the twelve {3,6} moves (including the closure move 64-1). The dots indicate points where switchbacks occur, for instance 13-14-15-16 are all in the same nightrider line.

61	18	21	25	41	38	46	33
8	56	59	51	48	31	3	12
42	37	45	34	62	17	22	26
47	32	4	11	7	55	60	52
20	28	23	39	43	36	64	15
58	54	49	30	2	13	5	10
44	35	63	16	19	27	24	40
1	14	6	9	57	53	50	29



18	26	15	21	29	6	9	1
60	52	55	45	40	63	35	43
30	5	12	2	17	27	14	24
39	63	36	42	57	51	54	48
16	22	19	25	10	4	32	7
56	46	59	49	34	44	37	62
11	3	31	8	13	23	20	28
33	41	38	61	53	47	58	50

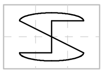
The second example above is by Juha Saukkola.'Nightrider Tours without Knight Moves' *The Games and Puzzles Journal* #20 (online) May-Aug 2001. The tour is symmetric, the first half 1-32 using the cells of the even ranks, the second half the odd ranks. The eight triple knight moves {3,6} are drawn in, the others are all double knight moves {2,4}.

Three-Move Knighted Pieces

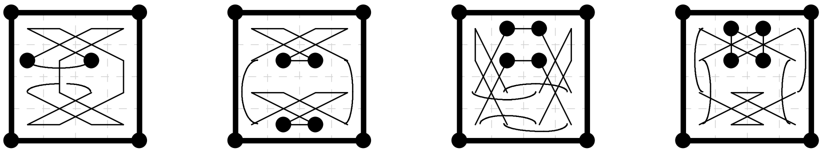
{1,2}+{0,1}{0,2} Warden

Pieces combining Rook and Knight form the Empress class. This is the simplest three-pattern augmented knight in this class, with a newly proposed name, with moves WDN.

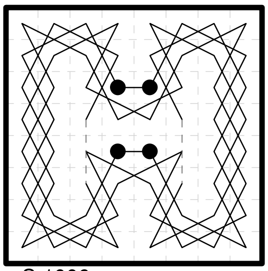
2×3 board There is one tour using wazir, dabbaba and knight moves. It is rotary.

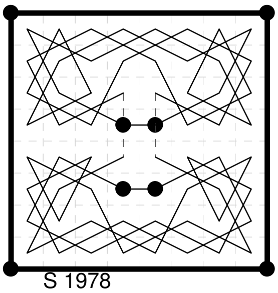


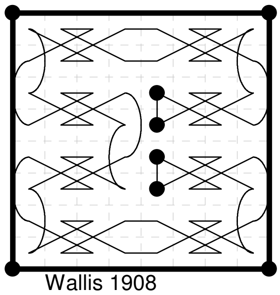
4×4 board There are 8 WDN tours in the Frénicle list of 4×4 diagonally magic squares but they have only four geometrical forms. The asymmetric tour can be numbered from either end (giving magic squares 231 + 424 in the list). The three symmetric closed tours can be numbered from two different origins (giving 224 + 603 and 298 + 638 and 350 + 549 in the list).



8×8 board. Here are two four-knight magic tours from the Feisthamel column using wazir and dabbaba links. (1) *Le Siècle* ¶1006 (23 Jan 1880), axial, diagonals 260±20. (2) *Le Siècle* ¶1978 (2 Mar 1883), axial, diagonals 260.


S 1006

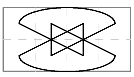

S 1978

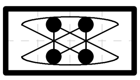

Wallis 1908

The other diamagic tour by J. Wallis was submitted to the *Strand Magazine* 1908 (vol.36, p.400 of the American edition) as reported to me by D. E. Knuth (16 Sep 2000) in *Games and Puzzles Journal* #19 (online) Jan-Apr 2001. The knight and rook moves alternate. See p.25 for a variant.

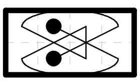
{1,2}+{0,1}{0,3}

2×4 board The Wazir + Threeleaper + Knight piece gives a biaxial tour that is magic in ranks and files when numbered about the long axis but only magic in the files when numbered about the short axis. It is again a permute of the magic king tour.

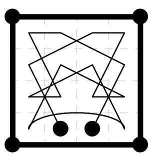




2	5	8	3
7	4	1	6



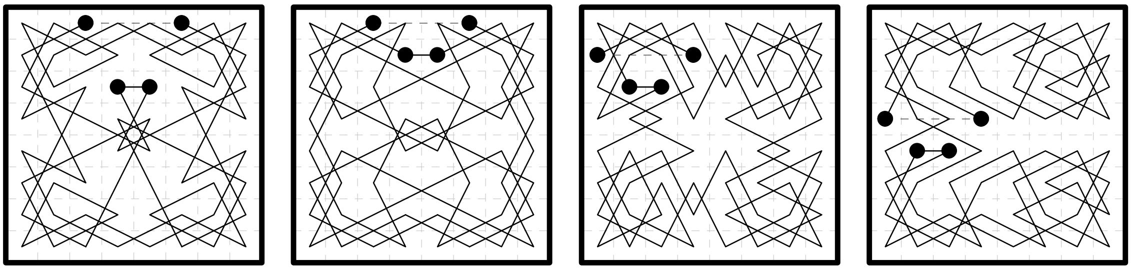
1	6	3	8
4	7	2	5



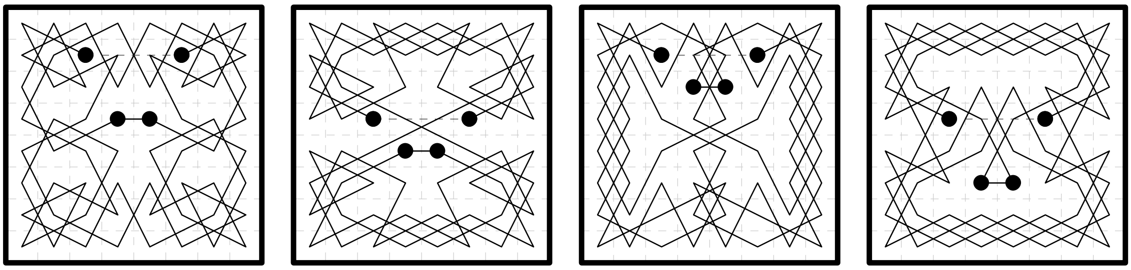
4×4 board There is also a WTN tour among the Frénicle 4×4 diagonally magic squares (diagram above). This is a closed version of the WN (Emperor) tour, the ends being joined and numbered from a new origin (It is 617 in the list of magic squares).

8×8 Magic Two-Knight tours with {0,1} and {0,3} links .

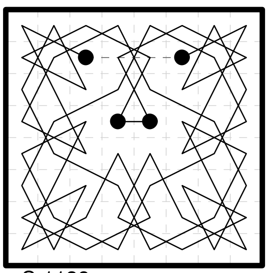
(1) E. H. *GWH* ¶13 (16/30 Aug 1873) 260 ±28. (2) E. H. *GWH* ¶24 (20 Dec 1873/3 Jan 1874) 260±4. (3) E. H. *GWH* ¶9 (24/31 May 1873) 260±4. This uses two different 16-move paths. (4) is from Lehman (1932) 260±4.

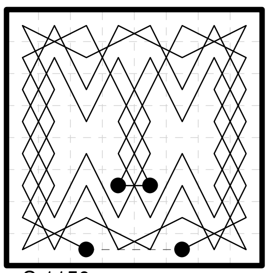


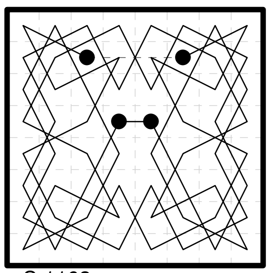
(5) and (6) are from Murray (1951) 260±52 and 260±20. (7) Celina (Francony) *Le Siècle* ¶982 (26 Dec 1879) 260±8. (8) Celina *Le Siècle* ¶952 (21 Nov 1879) 260±20.

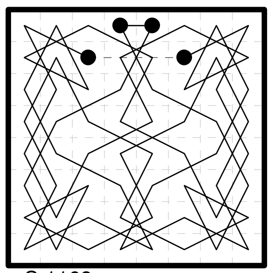


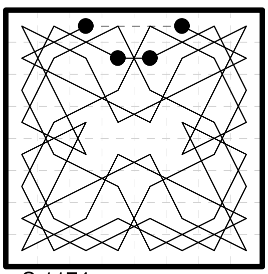
Here are another eight from the early years of *Le Siècle*. (9) ¶1138 (25 Jun 1880) 260±124. (10) ¶1150 (9 Jul 1880) 260±12. (11) ¶1162 (23 Jul 1880) 260±8. (12) ¶1168 (30 Jul 1880) 260±56. (13) ¶1174 (6 Aug 1880) 260±64. (14) ¶1186 (20 Aug 1880) 260±80. (15) ¶1384 (8 Apr 1881) 260±60. (16) ¶1450 (24 Jun 1881) 260±84 by L. de Croze à Marseille. There are many others of this type in *Le Siècle* in later years.

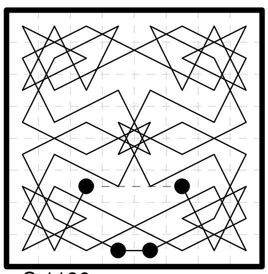

S 1138

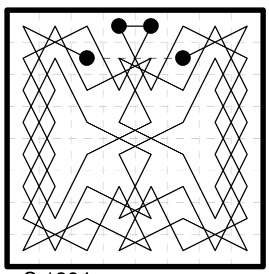

S 1150

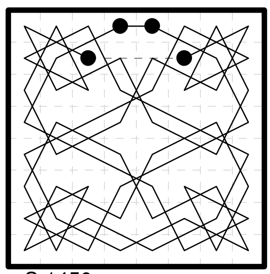

S 1162


S 1168


S 1174


S 1186

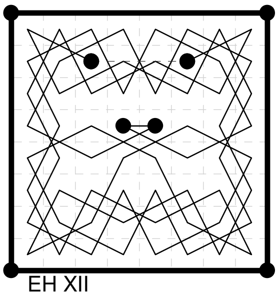

S 1384

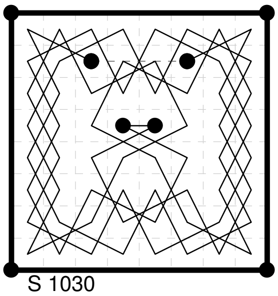

S 1450

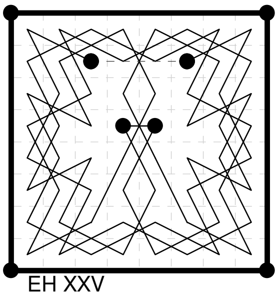
Diagonally Magic 'Two-Knight' with {0,1} and {0,3} links.

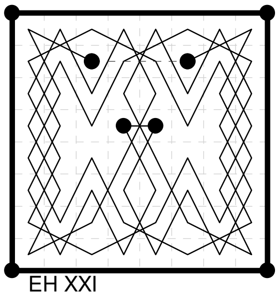
Murray (1951) gives the following five tours (A, B, G, H, K in his list), attributing them to Count Ligondes and other French composers, but at least four appear in the chess column by E. H. in *Glasgow Weekly Herald* 1873-4. I have not seen the whole series.

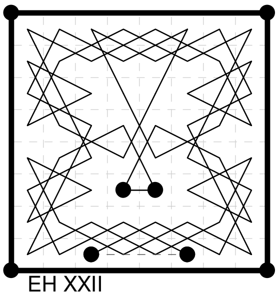
(1) EH *GWH* ¶XII (19 July/2 Aug 1873) = *Le Siècle* ¶1054 (19 Mar 1880). (2) *Le Siècle* ¶1030 (20 Feb 1880). (3) EH *GWH* ¶XXV (10/24 Jan 1874) = *Le Siècle* ¶1090 (30 Apr 1880). (4) EH *GWH* ¶XXI (18 Oct/1 Nov 1873) = *Le Siècle* ¶1078 (16 Apr 1880).(5) EH *GWH* ¶XXII (8/27 Nov 1873) = *Le Siècle* ¶1102 (14 May 1880). The last three in *Le Siècle* are by X a Belfort (Reuss). The first four have the quarter points all in the same pattern, and the path 1-16 is non-intersecting.

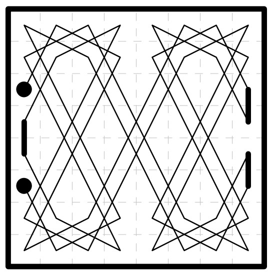

EH XII

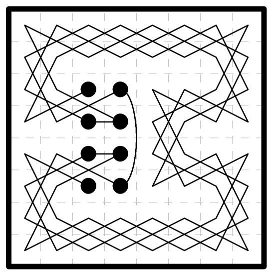

S 1030


EH XXV


EH XXI


EH XXII





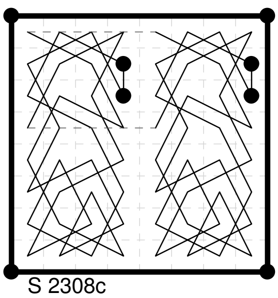
Magic 'Four-Knight' tours with {0,1} and {0,3} links.

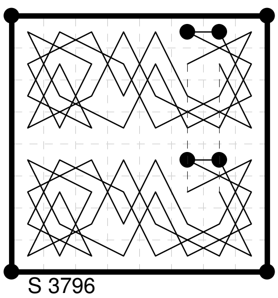
Two tours are shown above, the first of these consists of four paths of knight moves connected by three wazir moves and a three-leap closure. This example (Jelliss undated, but probably not original). The second, based on Beverley, is almost identical to the Emperor tour shown earlier.

{1,2}+{0,1}{0,4}

8×8 Diagonally Magic Four-Knight 1-4 Empress Tours.

These, of double half-board type, are by A. Béligne in *Le Siècle* (1) ¶2308 (21 Mar 1884). (2) ¶3796 (11 Jan 1889).

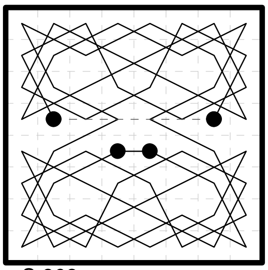

S 2308c

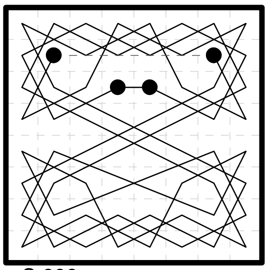

S 3796

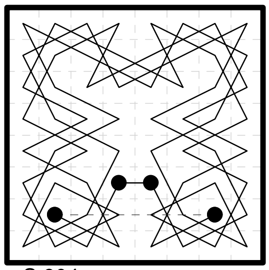
{1,2}+{0,1}{0,5}

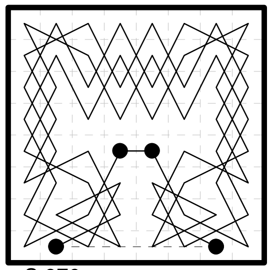
8×8 Magic Two-Knight 1-5 Empress Tours.

(1) *Le Siècle* ¶868 (15 Aug 1879) 260±20. (2) *Le Siècle* ¶898 (19 Sep 1879) 260±20. (3) by Béligne *Le Siecle* ¶904 (26 Sep 1879) 260±68. (4) by Béligne *Le Siècle* ¶976 (19 Dec 1879) 260±54.

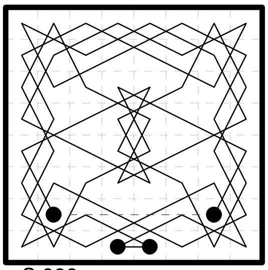

S 868

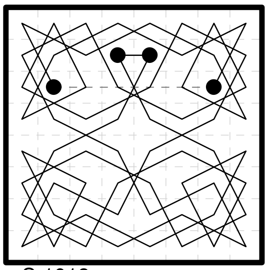

S 898

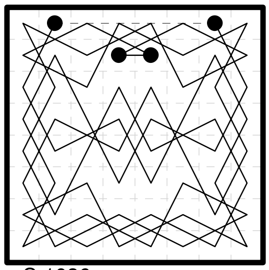

S 904

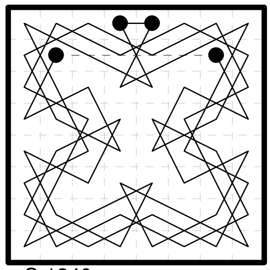

S 976

(5) *Le Siècle* ¶988 (3 Jan 1880) 260±28. (6) *Le Siècle* ¶1018 (6 Feb 1880) 260±80. (7) *Le Siècle* ¶1036 (27 Feb 1880) 260±12. (8) *Le Siècle* ¶1240 (22 Oct 1880) 260±56. Many more are possible.


S 988

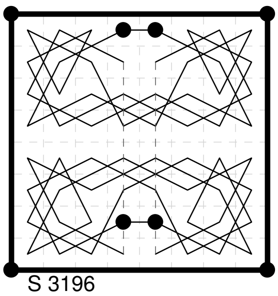

S 1018


S 1036

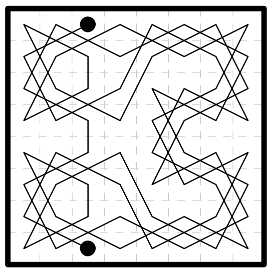

S 1240

{1,2}+{0,1}{0,6}

8×8 Diagonally Magic Four-Knight 1/6 Empress Tour. Reuss *Le Siècle* ¶3196 (4 Feb 1887).


S 3196

47	30	1	52	5	28	43	54
2	51	48	29	44	53	6	27
31	46	49	4	25	8	55	42
50	3	32	45	56	41	26	7
15	62	33	20	9	24	39	58
34	19	16	61	40	57	10	23
63	14	17	36	21	12	59	38
18	35	64	13	60	37	22	11

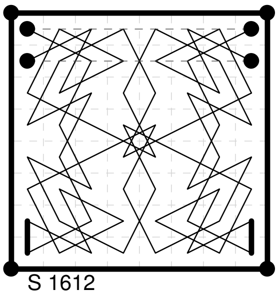
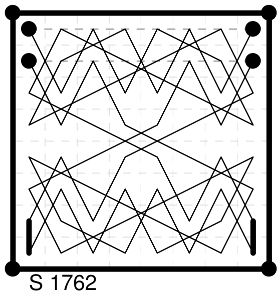
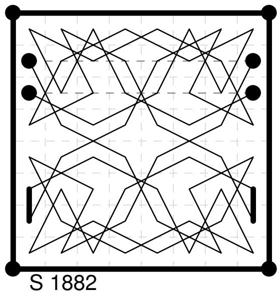
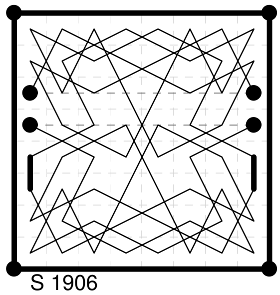


{1,2}+{0,1}{0,7}

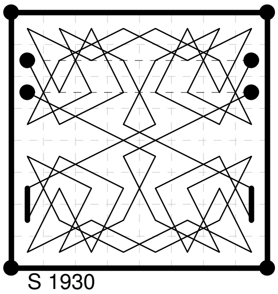
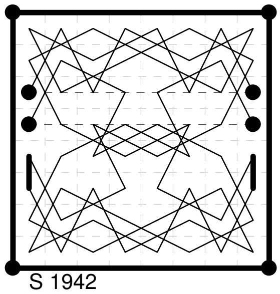
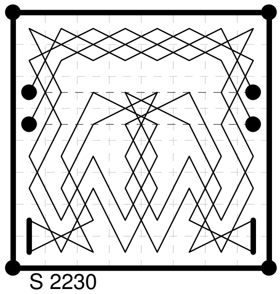
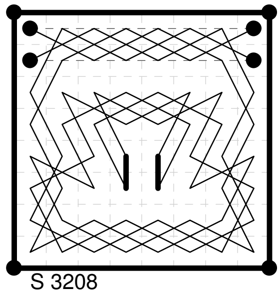
8×8 Magic Four-Knight 1/7 Empress Tour. Derived from Beverley's tour by four interchanges. Diagram inserted above.

8×8 Diagonally Magic Four-Knight 1/7 Empress Tours. Eight examples from *Le Siècle* :

(1) ¶1612 (30 Dec 1881) by Béligne, (2) ¶1762 (23 Jun 1882) by L. de Croze, (3) ¶1882 (10 Nov 1882) by M. A. F. (i.e. Feisthamel), (4) ¶1906 (8 Dec 1882) by Béligne.

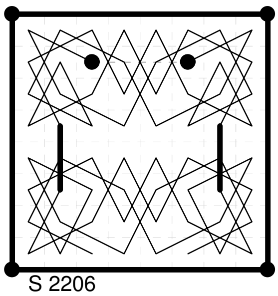
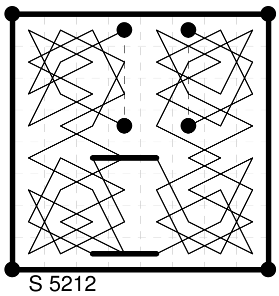
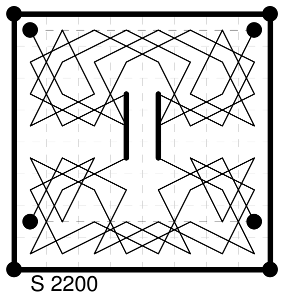

S 1612

S 1762

S 1882

S 1906

(5) ¶1930 (5 Jan 1883) by Béligne, (6) ¶1942 (19 Jan 1883), (7) ¶2230 (21 Dec 1883) by Adsum (Bouvier), (8) ¶3208 (18 Feb 1887) by E. Grossetaite.


S 1930

S 1942

S 2230

S 3208

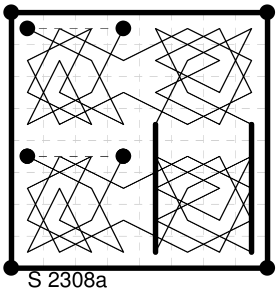
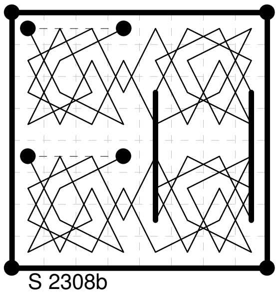
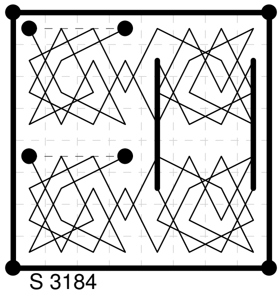
{1,2}+{0,2}{0,3} **and {1,2}+**{0,2}{07}

Diamagic tours *Le Siècle* ¶2206 (23 Nov 1883) axial by R. Bricard and ¶5212 (10 Feb 1893) asymmetric by M. A. F. Also, on right, *Le Siècle* ¶2200 (16 Nov 1883) by M. A. F. (Feisthamel).


S 2206

S 5212

S 2200

{1,2}+{0,3}{0,4}

Three diagonally magic tours by A. Béligne from *Le Siècle*: ¶2308a and b (21 Mar 1884) ¶3184 (21 Jan 1887). Two reflected in the a8-h1 and the a1-h8 diagonals for comparison with the third.

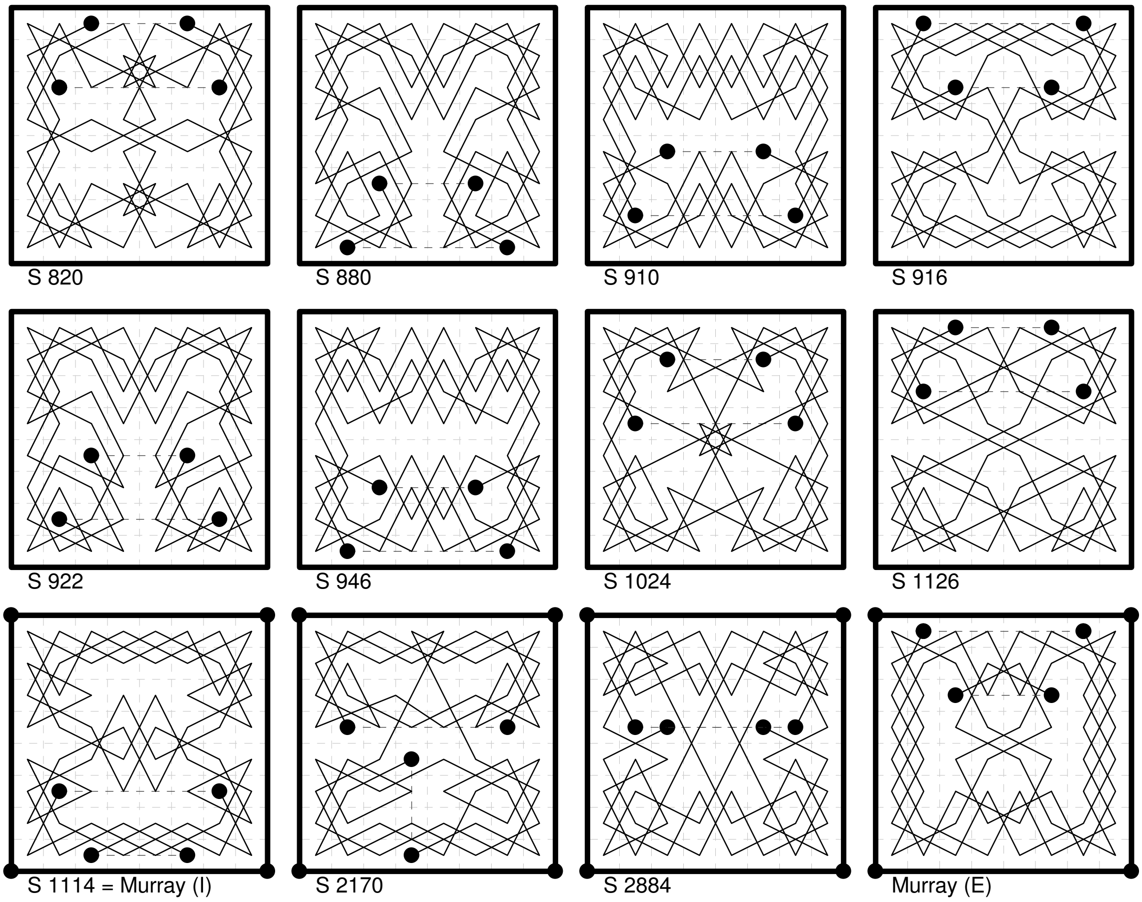

S 2308a

S 2308b

S 3184

{1,2}+{0,3}{0,5}

8×8 Magic & Diamagic Two-Knight 3-5 Empress Tours.

From *Le Siècle* (1) ¶820 (20 Jun 1879) 260±24 with two complete octangles. (2) by Béligne ¶880 (29 Aug 1879) 260±4. (3) ¶910 (3 Oct 1879) 260±54 (4) ¶916 (10 Oct 1879) 260±20.

(5) by Béligne ¶922 (17 Oct 1879) 260±4. (6) by Béligne ¶946 (14 Nov 1879) 260±54. (7) ¶1024 (13 Feb 1880) 260±4. (8) ¶1126 (11 Jun 1880) 260±28.



(9) *Le Siècle* ¶1114 (28 May 1880) = Murray tour I. (10) by Reuss *Le Siècle* ¶2170 (12 Oct 1883) asymmetric, near axial. (11) by de Hijo *Le Siècle* ¶2884 (29 Jan 1886). The two rook moves overlap. (12) This is E in Murray's list.

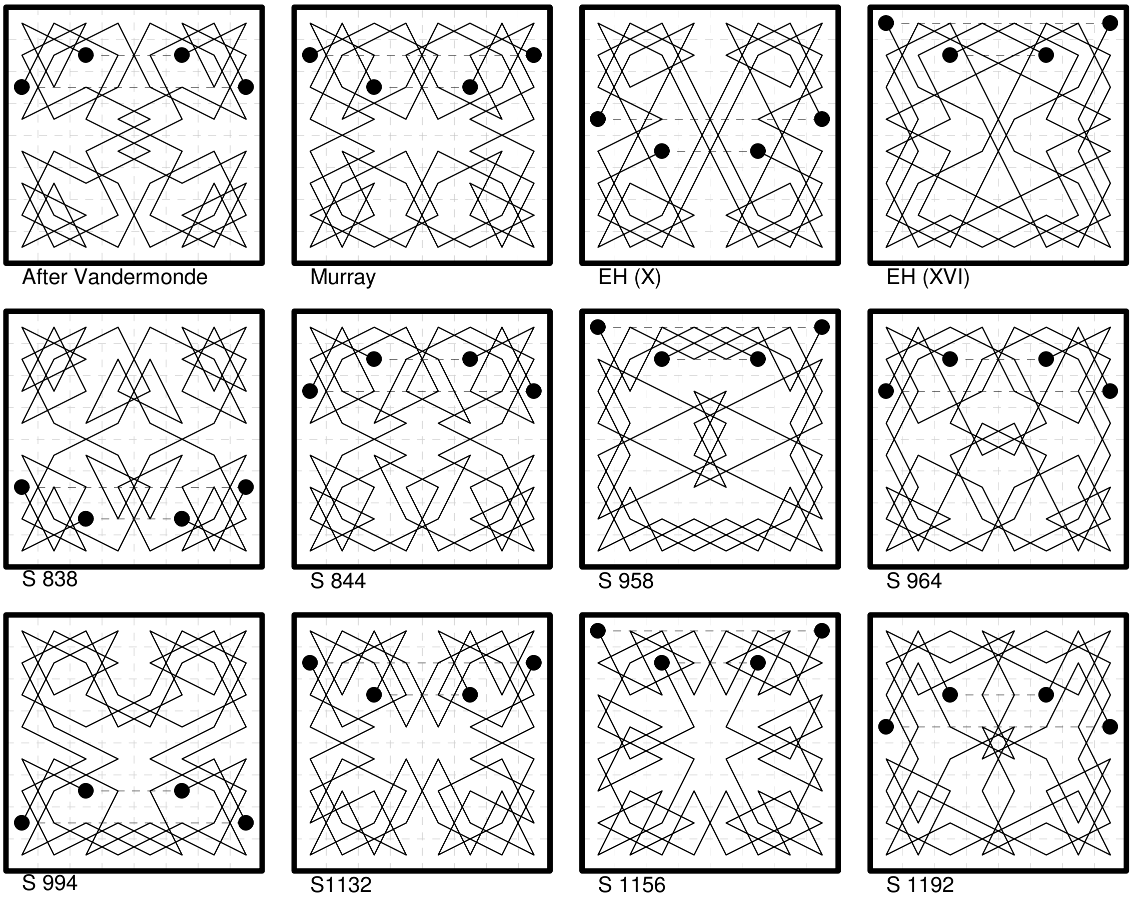
{1,2}+{0,3}{0,7}

8×8 Magic Two-Knight 3-7 Empress Tours.

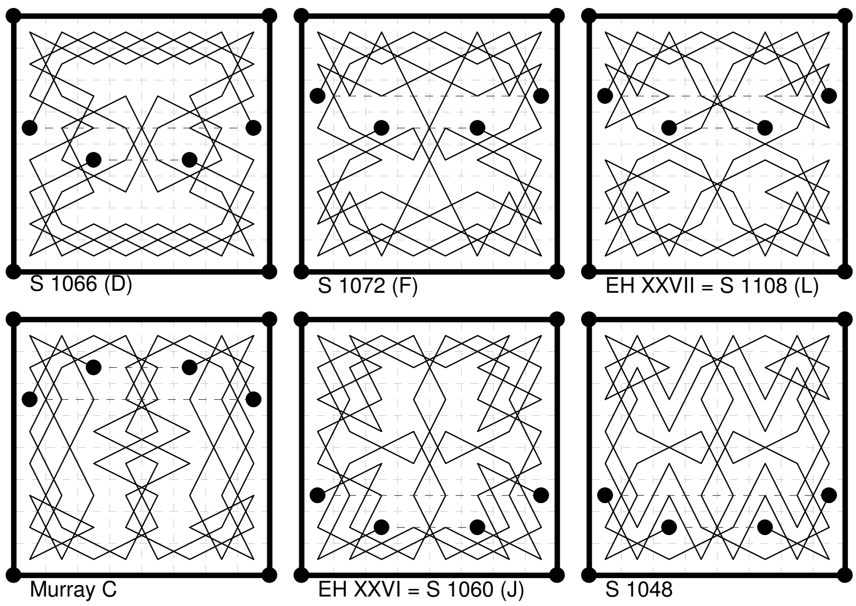
Twelve diagrams follow. (1) Derived from Vandermonde's circuit, diagonals 260±12. (2) Murray example, diagonals 260±68. (3) and (4) E. H. in *Glasgow Weekly Herald* ¶X (14 Jun 1873) diagonals 260±28.and ¶XVI (20 Sep/4 Oct 1873) diagonals 260±96.

The 3-7 Empress seems to have generated the most magic tours in *Le Siècle*. These are a sample. (5) by Clerville *Le Siècle* ¶838 (11 Jul 1879) 260±56. (6) by Celina (Francony) *Le Siècle* ¶844 (18 Jul 1879) 260±132. (7) *Le Siècle* ¶958 (28 Nov 1879) 260±28. (8) *Le Siècle* ¶964 (5 Dec 1879) 260±28.

(9) *Le Siècle* ¶994 (9 Jan 1880) 260±56. (10) *Le Siècle* ¶1132 (18 Jun 1880) 260±148. (11) *Le Siècle* ¶1156 (17 Jul 1880) 260±76. (12) *Le Siècle* ¶1192 (27 Aug 1880) 260±136.



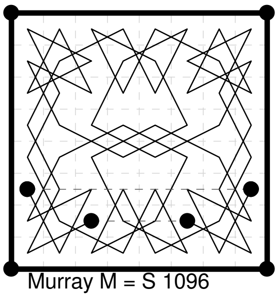
8×8 Diamagic Two-Knight 3-7 Empress Tours.
Five of these are D, F, L, C and J in Murray's list (1951).

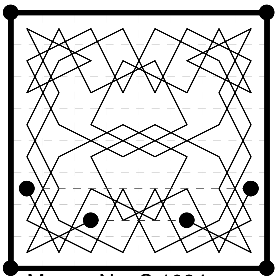


However I have since found D in *Le Siècle* ¶1066 (2 Apr 1880) and F ¶1072 (9 Apr 1880) by X a Belfort (Reuss). And L ¶1108 (21 May 1880) and earlier in *Glasgow Weekly Herald* as ¶XXVII (1874 p.88). Also J is in the E. H. column in *Glasgow Weekly Herald* as ¶XXVI (7/21 Feb 1874) and in *Le Siècle* ¶1060 (26 Mar 1880). The sixth is one I found when I made an attempt (c.1988) to check Murray's enumeration, but have since found that it is also in *Le Siècle* ¶1048 (12 Mar 1880).

The next three are M, N and O in Murray's list. N and M appear as by X a Belfort (Reuss) in *Le Siècle* ¶1084 (23 Apr 1880) and ¶1096 (7 May 1880). Tour O which has only binary symmetry (when 1-32 and 33-64 are joined to make a pseudotour) is formed by combining the paths used in the quaternary tours M and N, which cover the same set of cells.

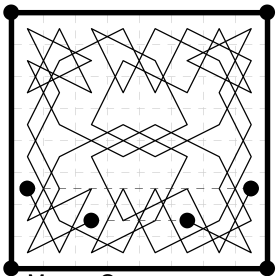
15	18	25	38	27	40	47	50
20	37	16	41	24	49	28	45
17	14	19	26	39	46	51	48
36	21	42	59	6	23	44	29
13	60	7	22	43	58	5	52
64	35	62	55	10	3	30	1
61	12	33	8	57	32	53	4
34	63	56	11	54	9	2	31


Murray M = S 1096

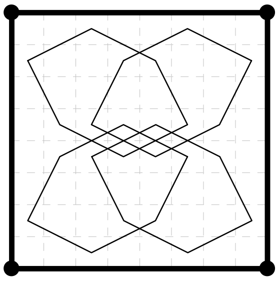

Murray N = S 1084

15	18	25	38	27	40	47	50
24	37	16	45	20	49	28	41
17	14	19	26	39	46	51	48
36	23	44	59	6	21	42	29
13	58	5	22	43	60	7	52
63	35	62	55	10	3	30	1
57	12	33	4	61	32	53	8
34	63	56	11	54	9	2	31

15	18	25	38	27	40	47	50
20	37	16	45	24	49	28	41
17	14	19	26	39	46	51	48
36	21	44	59	6	23	42	29
13	60	5	22	43	58	7	52
64	35	62	55	10	3	30	1
61	12	33	4	57	32	53	8
34	63	56	11	54	9	2	31


Murray O

15	18	25	38	27	40	47	50
24	37	16	41	20	49	28	45
17	14	19	26	39	46	51	48
36	23	42	59	6	21	44	29
13	58	7	22	43	60	5	52
64	35	62	55	10	3	30	1
57	12	33	8	61	32	53	4
34	63	56	11	54	9	2	31

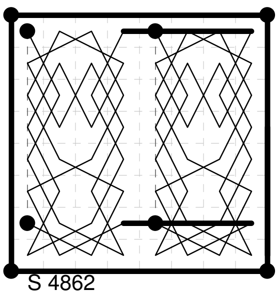


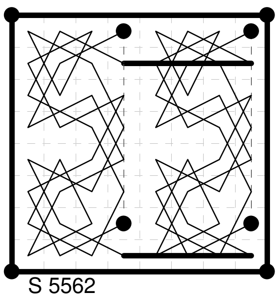
The forward and reverse numberings of tour O are both shown since these numberings are very similar and can be easily confused. This was pointed out to me in an email from Jaime Gutierrez Salazar in Dec 2016, who traces the tour back to General Parmentier (1890s).

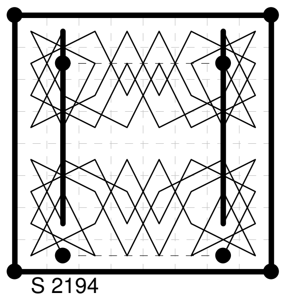
In Rouse Ball (1939/1956 p.184) tour O is shown in the first numbering here (transposed) and is attributed to A. Rilly (1905). The four numberings in M, N, and O and its reverse, differ only in the sequence of numbers round the four hexagons shown in the last diagram.

{1,2}+{0,4}{0,6}

Diamagic four knight tours *Le Siècle* ¶4862 (26 Feb 1892) by Adsum *Le Siècle* ¶5562 (26 Jan 1894) by Grossetaite.


S 4862


S 5562


S 2194

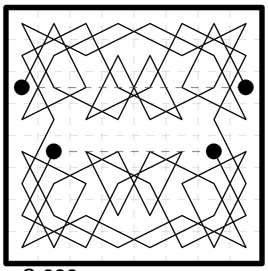
{1,2}+{0,5}{0,6}

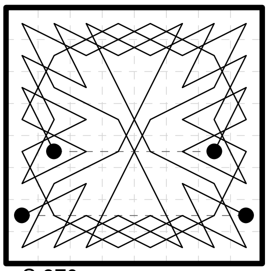
Diamagic four knight tour from *Le Siècle* ¶2194 (9 Nov 1883) by Adsum. Above right.

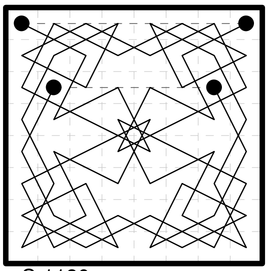
{1,2}+{0,5}{0,7}

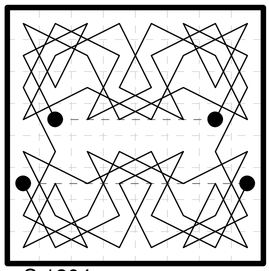
8×8 Magic Two-Knight 5-7 Empress Tours.

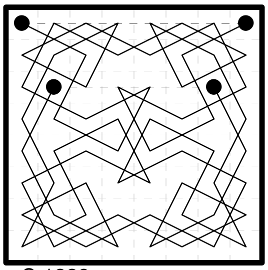
Here are some nondiagonally magic tours from *Le Siècle*. (1) A. Béligne ¶832 (4 Jul 1879) 260±20. (2) ¶970 (12 Dec 1879) 260±4. (3) ¶1180 (13 Aug 1880) 260±40. (4) ¶1204 (10 Sep 1880) 260±12. (5) ¶1228 (8 Oct 1880) 260±88. (6) ¶1414 (13 May 1881) 260±124. (7) by Adsum ¶1468 (15 Jul 1881) 260±44. (8) by L. de Croze ¶1492 (12 Aug 1881) 260±84.

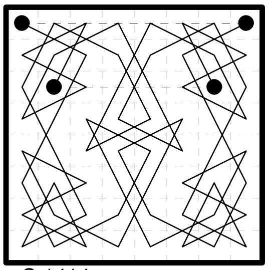

S 832

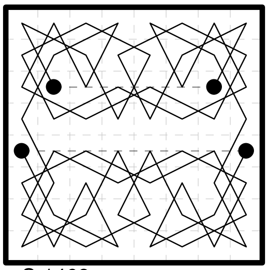

S 970

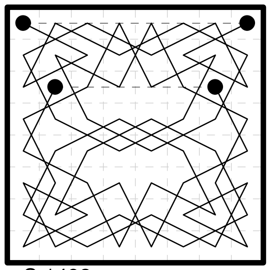

S 1180


S 1204


S 1228


S 1414


S 1468

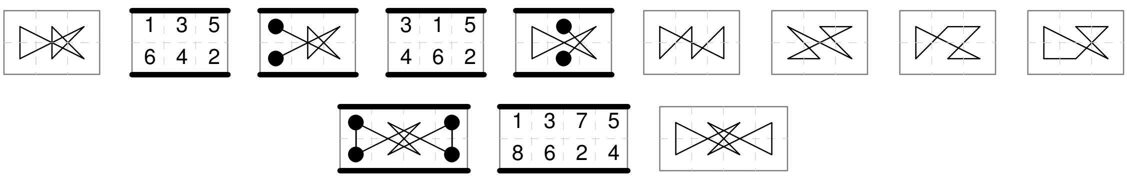

S 1492

{1,2}+{1,1}+{0,1} Centaur

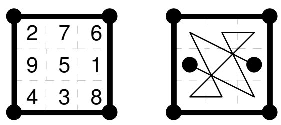
We tend to apply names from Greek Myth to pieces that combine moves in all the three directions used in chess, namely those of rook, bishop and knight (RBN). They all form part of the Q+N or Amazon family. The combined wazir-fers-knight (WFN) is the simplest piece of this type, making a single step of each type, making it King+Knight, and has the individual name of Centaur.

2×3 board There is an axially symmetric closed centaur tour that can be numbered in two ways to be magic in the files (magic in the ranks of course is impossible). There are also four other closed tours. Two rotary and two asymmetric.

2×4 board There is a biaxial centaur tour that is semi-magic.

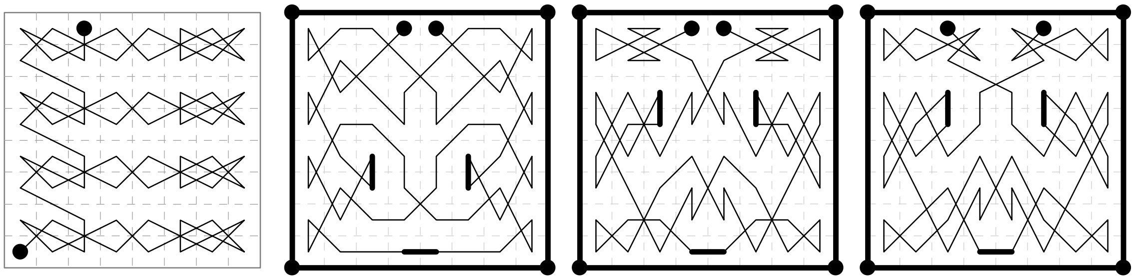


3×3 board. Some history of the 3×3 magic square is outlined in ⌘ 1. It is an <u>open</u> centaur tour.



32×32 board. Robert Bosch has published a series of 32×32 centaur tours on Twitter which are supposedly impressionistic representations of classic works of art, such as the Mona Lisa, darker cells being those where more knight moves congregate.

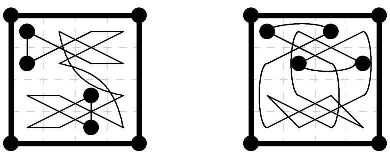
8×8 board Alternating Knight+King open tour (¶50) by the Rajah of Mysore (d.1868). Three diagonal magic tours using king and knight moves by a Monsieur Galtier of La Mastre published in the 'Jeux D'Esprit' column in *Le Gaulois* (¶309 14 Apr 1882, ¶448 2 Mar 1885, ¶468 30 Mar 1885). The dark links connect the quarters of the tour.



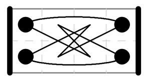
{1,2}+{2,2}**+{0,1}** and **{1,2}+{1,1}+**{0,2} and **{1,2}+{1,1}+**{0,3}

4×4 board There is one WAN asymmetric closed tour among the Frénicle 4×4 diamagic tours, with four magic numberings (31, 237, 420, 582). See also tours wirh {0,3} closure move.

There is also one DFN tour, with four magic numberings (148, 344, 553, 720).



1	3	6	8
5	7	2	4



2×4 board A semi-magic biaxial 2×4 closed tour by TFN (threeleaper+fers+knight) mover.

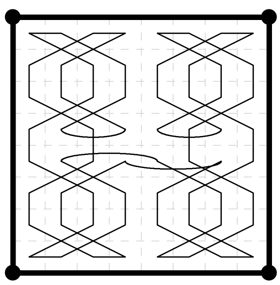
{1,2}{2,4}{3,6} **Nightrider**

Might it be possible with the nightrider's extra powers to construct a diagonal magic 8×8 tour? Little seems to have been done on Nightrider tours, as noted in the double-move section above.

Four-Move Knighted Pieces

{1,2}+{0,1}{0,2}{0,3}

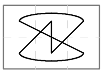
8×8 board. A four-move piece Rook+Knight Magic Empress Alternating tour (Jelliss 2001). Derived from Wallis (1908), see p.14, by interchange of the 2-3 and 6-7 ranks and files. Also rotated 90 degrees to conform to the Frénicle convention.



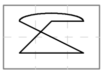
5	4	60	61	29	28	36	37
59	62	6	3	35	38	30	27
58	63	7	2	34	39	31	26
8	1	57	64	32	25	33	40
9	16	56	49	17	24	48	41
55	50	10	15	47	42	18	23
54	51	11	14	46	43	19	22
12	13	53	52	20	21	45	44

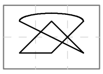
{1,2}+{1,1}+{0,1}{0,2}

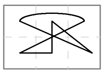
2×3 board. These are the four moves possible on the 2×3 board. Tours using all four moves, one rotary, three asymmetric:



4	1	5
2	6	3







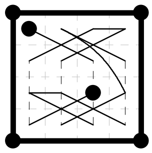
The symmetric tour is arithmic, the files adding to 6, 7, 8 (cd=1) and ranks 10, 11.

3×3 board. The arrangement of the first nine numbers in a square so that all eight lines of three add to the total 15 is the oldest known magic square See p.22 and ⌘ 1. This is an open centaur tour requiring a fourth type of move {0,2} for closure.

{1,2}+{2,2}**+{0,1}**{0,2}

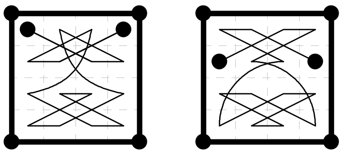
4×4 board. This magic square was reported to A. H. Frost (1819-1907) by Robert Shortrede (1800-1868) as having copied it in 1841 from an inscription in an old temple in the hill fort of Gwalior, said to bear the date 1483. It is 304 in the Frénicle list.

16	9	4	5
3	6	15	10
13	12	1	8
2	7	14	11



{1,2}+{2,2}**+{0,1}**{0,3}

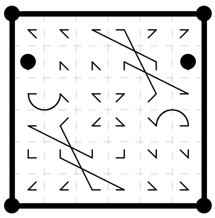
4×4 board There are two tours of this pattern among the Frénicle 4×4 diamagic tours both axially symmetric (17, 436) with {0,3} as the closure move.



{1,2}+{3,3}**+{0,1}**{0,3}

6×6 board. Tours of the Falkener type (based on 2×2 King paths) can easily be transformed to give other magic tours in which the {0,1} and {1,1} king moves are replaced by (0,3) and (3,3) moves. This transform applied to a tour using five move-types (shown later) produced this diamagic tour that uses only four types of move. Showing it in diagram form is difficult because of the overlap of moves. Only the ends of the three-unit moves are shown, and the {0,1} moves are curved.

7	26	21	8	27	22
36	18	3	33	20	1
13	12	31	14	9	32
5	28	23	6	25	24
35	17	4	34	19	2
15	10	29	16	11	30



==

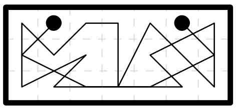
Five-Move Knighted Pieces

==

{1,2}+{1,1}+{0,1}{0,2}{0,3}

3×7 board. This pattern, which fancifully resembles two chess players, was derived from the tour using {0,4} move below by interchange of two files at each end. The {0,3} moves overlap. The reverse numbering is also shown.

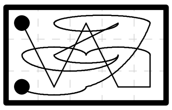
15	1	13	12	9	21	6
2	14	17	11	5	8	20
16	18	3	10	19	4	7



16	1	13	10	9	21	7
2	14	17	11	5	8	20
15	18	3	12	19	4	6

{1,2}+{2,2}**+{0,1}**{0,2}{0,3}

3×5 board. Magic rectangle tour by Marian Trenkler (1999), asymmetric with middle row a permuted arithmetic progression (numbering reversed).

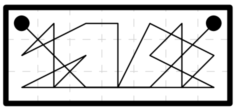


1	12	3	11	13
8	10	7	9	6
15	2	14	4	5

{1,2}+{1,1}+{0,1}{0,2}{0,4}

3×7 board. The second numbering shown will be seen to be related to the first by inversion of the second, fourth and sixth files. The tour however is the same as the reverse numbering of the diagram (then reoriented so the 1 is at the top left).

1	15	13	12	9	6	21
14	2	17	11	5	20	8
18	16	3	10	19	7	4



1	16	13	10	9	7	21
14	2	17	11	5	20	8
18	15	3	12	19	6	4

==

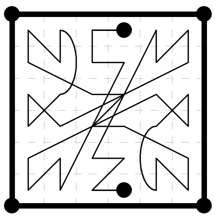
Six-Move Knighted Pieces

==

{1,2}{2,4}**+{1,1}+{0,1}**{0,2}{0,5}

6×6 board. Diamagic tour using six move types, after Falkener.
Nightrider+Queen (Queen of the Night).

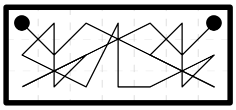
15	13	35	36	5	7
16	14	34	33	6	8
10	12	17	18	28	26
11	9	19	20	25	27
29	31	4	3	23	21
30	32	2	1	24	22



{1,2}{2,4}+**{1,1}**+**{0,1}**{0,2}{0,6}

3×7 board. Six-move type including the {0,6} closure move.

1	15	3	12	19	6	21
14	2	17	11	5	20	8
18	16	13	10	9	7	4



Seven-Move Knighted Pieces

{1,2}{2,4}+**{0,1}**{0,2}{0,3}{0,4}{0,5}

3×7 board. A Rook+Nightrider (Raven) tour with permuted arithmetical progression 8 - 14 in the middle rank. This is difficult to show graphically because of the multiple overlapping rook moves. This was published in my online *Jeepyjay Diary* 13 Oct 2014. The forward and reverse numberings shown, oriented by the Frénicle rule, look very different.

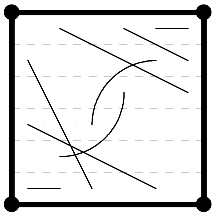
1	17	15	5	21	2	16
14	13	12	9	8	11	10
18	3	6	19	4	20	7

4	19	16	3	18	2	15
8	9	10	13	14	11	12
21	5	7	17	1	20	6

{1,2}{2,4}+{2,2}{3,3}+**{0,1}**{0,3}{0,5}

6×6 board. Transform of diamagic Falkener style tour; 6 move types. Only the links between the quartes are shown. The {0,3} and {3,3} moves overlap.

6	28	23	5	25	24
33	18	2	34	20	4
15	10	30	16	11	29
8	26	21	7	27	22
36	17	3	35	19	1
13	12	32	14	9	31



==

The Big Beasts

==

The ‘Big Beasts’ are the single-pattern {r,s} leapers that move in other directions than those of the regular chess pieces, that is not in rook, bishop and nightrider lines. The best known being the Camel {1,3}, Zebra {2.3}, Giraffe {1,4} and Antelope {3,4}. We arrange these according to their longest coordinate, which is why Zebra precedes Giraffe.

Following tours by these individual pieces we turn to tours by pieces that augment them with another move, which may be a rook, bishop or nightrider moves, an extended move, or another beast. Then we look at tours using three or more different move types.

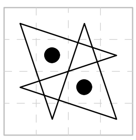
The Camel {1,3}

The shortest skew {r,s} leaper after the {1,2} knight is the {1,3} Camel whose move length is $\sqrt{10}$ = 3.16 approximately. Rouse Ball (1939, p.185) states that Euler applied his construction method “to find a reentrant route by which a piece that moved two cells forward like a castle and then one cell like a bishop would occupy in succession all the black cells on the board”, in other words a camel tour, but he gives no reference. The earliest tour using camel moves that I have found is a magic four-camel tour, where the ends of the camel paths are joined by {0,6} and {0,7} rook moves, by Bouvier 1887 (see the Three-Move Beasts p.53). The first purely camel tour I know of is that by T. R. Dawson in ‘Caissa’s Playthings’ in *Cheltenham Examiner* 1913, where he used the name.

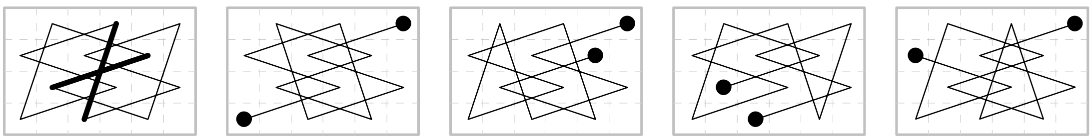
The camel has many attributes similar to the knight except that it is confined to cells of one colour. In particular the angles between its moves are the same as for the knight and it has the same facility for forming a wider variety of circuits than longer leapers. By a camel tour on any board we mean a tour of all the squares of one colour.

THEOREM: No camel tours are possible on 2×n, 3×n, 4×odd nor on 5×k boards with k > 7. *Proof*: (a) The 2×n and 3×n boards are too narrow to admit any ‘vertical’ moves. (b) The camel moves alternately to squares on the odd and even ranks of any board, and on a 4×odd board there are 2·h black squares on the even ranks and 2·h + 2 on the odd ranks, a difference of 2. The difference in a closed tour must be 0 and in an open tour 1. (c) On the 5×7 board there are 8 cells on the even ranks and 9 on the odd ranks, so the ends must be on the odd-rank cells. If we add two files to the right then this adds 2 cells on the even ranks and 3 on the odd ranks. It follows that no tour is possible on any longer 5-rank board.

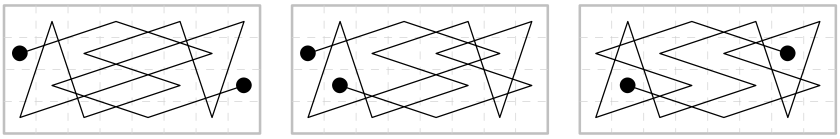
4×4: The six moves on one colour form a closed circuit, omitting the two central squares.



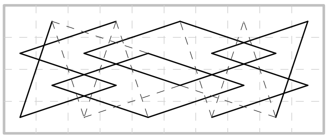
4×6: This is the smallest board on which the camel has a tour of all the cells of one colour. There is one closed camel tour (given by T. R. Dawson *L'Echiquier* 1928). This has Bergholtian symmetry (i.e. rotative, crossing at the centre) and by deletion of one move produces 7 reentrant tours (2 of which are symmetric) but in addition there are 4 non-reentrant open tours (1 symmetric):



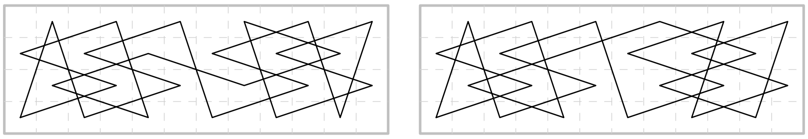
4×8. The 4×8 board has no closed camel tours since the moves at a3, g3 and at b2, h2 combine to form two 4-move circuits. Three open tours (2 symmetric) exist, having one end-point in each circuit.



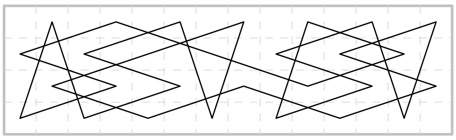
4×10: This board admits no camel tours since the moves at b2, h2 and c3, i3 form two 4-move circuits (P and S), preventing a closed tour, while the moves at a1, a3 and j2, j4 form two paths (Q and R) with no connection between them, the paths through d2 and g3 being fixed, and to form an open tour connections must be in the sequence PQRS or PRQS, the ends being in P and S.



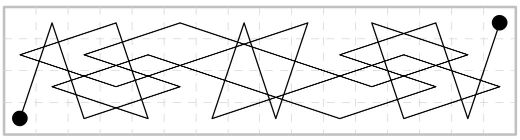
4×12:. The 4×12 admits 2 closed tours as shown (1 symmetric); open tours not studied.



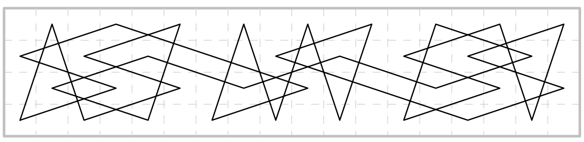
4×14. Admits only one closed tour (asymmetric), but 18 symmetric open tours (4 a1–n4, 5 a3–n2, 2 b2–m3, 1 c1–l4, 2 c3–l2, 1 e1–j4, 1 e3–j2, 2 g3–h2); of these, 13 have the middle move vertical and 5 have it horizontal; there are also asymmetric open tours, not counted.



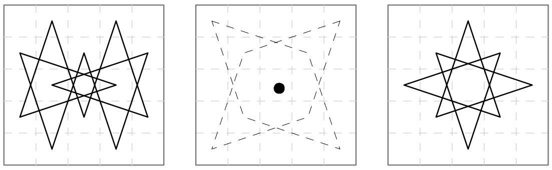
4×16: The 4×16 has no closed tour, but there are symmetric open tours.



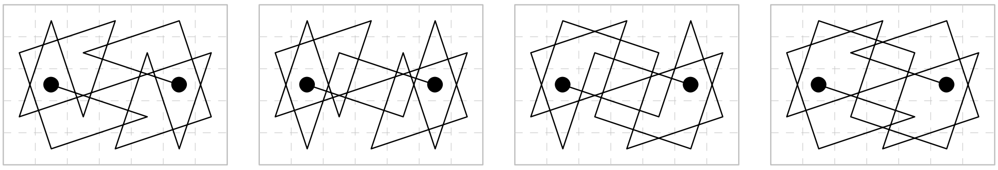
4×18: The 4×18 has some closed camel tours, symmetric and asymmetric 72 cells.



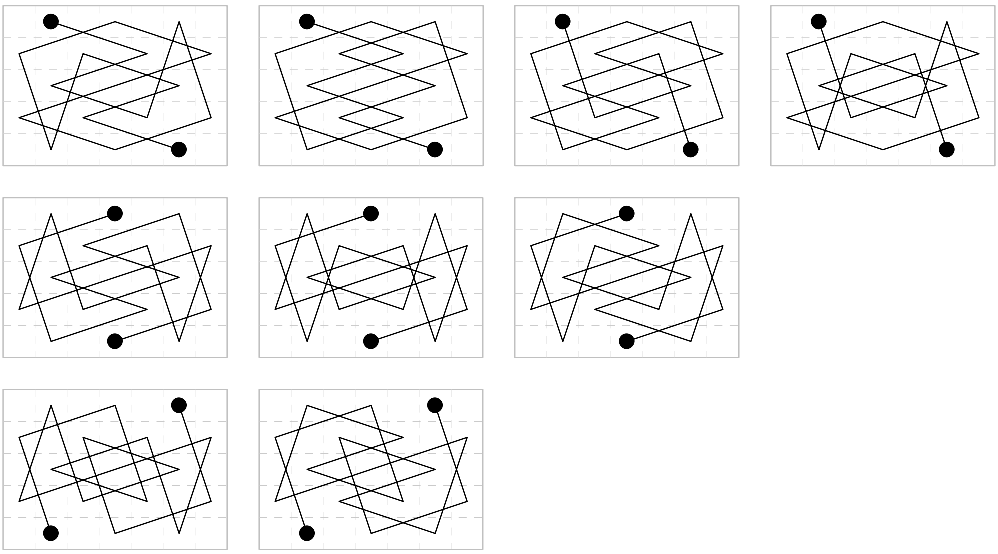
5×5: This board has a unique closed tour of the cells of the minority colour with biaxial symmetry; but on the majority colour there is no move at the centre cell and the moves on the eight outer cells form two circuits of 4 and one of 8 that snag at the four inner cells.



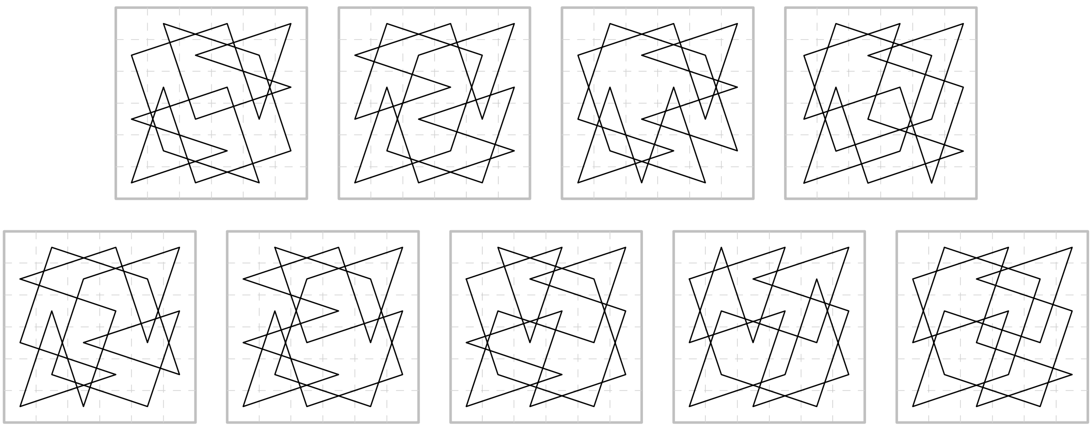
5×7: There are 13 symmetric open tours. They are oriented here with the central pair of moves all in the same direction. On this board there are 8 cells on the even ranks and 9 on the odd ranks, so the ends must be on the odd-rank cells. Four with ends in the middle rank:



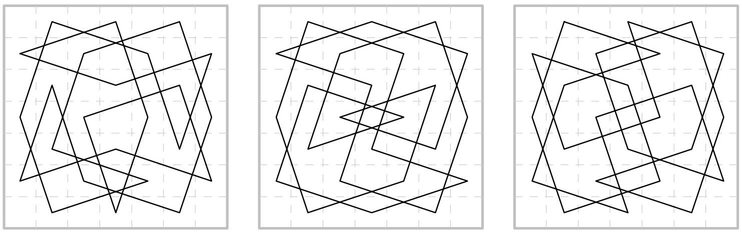
Nine with ends in the outer ranks:



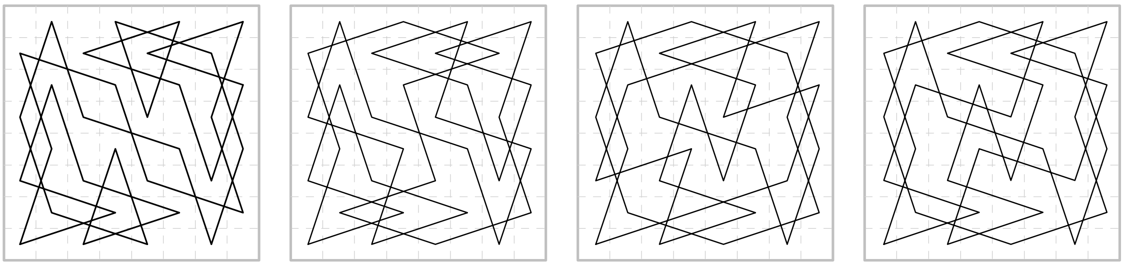
6×6: There are 9 distinct camel tours (2 rotative, Eulerian, and 2 reflective, Diagonal).



7×7:: Three tours 7×7 by Kraitchik (1927).



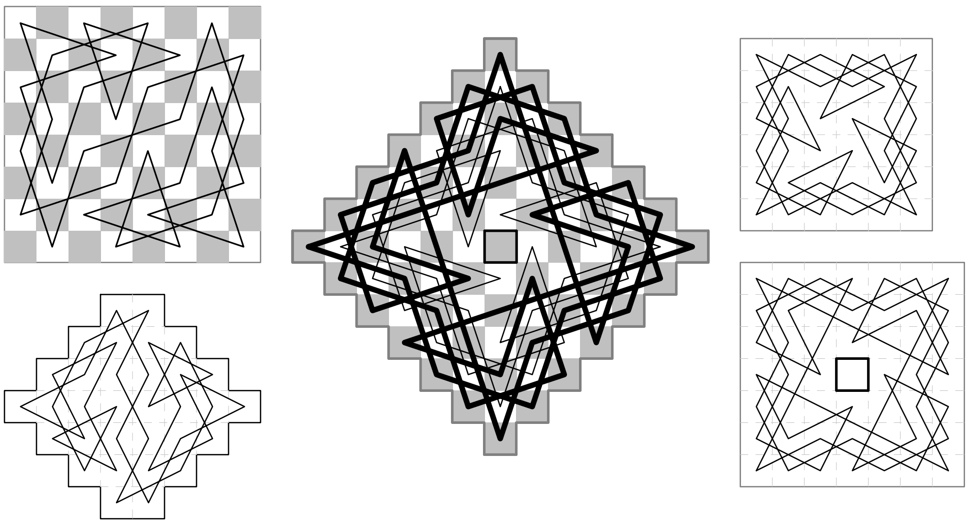
8×8: On the 8×8 board there are 4 symmetric camel tours, all in Bergholtian symmetry.



F. Hansson found these [*PFCS* Apr/Jun 1933 ¶715] and also made tours on the 8×8 cylinder. Dawson's 1913 tour mentioned earlier is asymmetric.

Camel on Shaped Boards

Tours on the cells of one colour in a rectangular board can be transformed into tours by related pieces on shaped boards. For example, moves of a bishop on the chessboard are equivalent to moves of a rook on a board of a serrated shape, representing the cells of one colour, rotated by 45 degrees. The same is true of other half-free movers, such as the camel, whose moves are equivalent to knight moves on this board, as pointed out by T. R. Dawson (*Problemist Fairy Chess Supplement* vol.1 #18 Jun 1933 p.125) and rediscovered by others from time to time since.

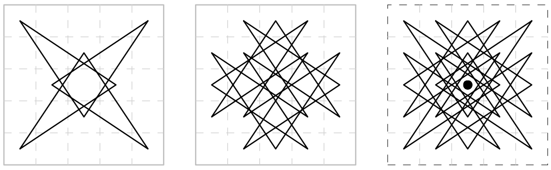


Conversely knight tours on square boards are equivalent to camel moves on serrated boards. Kraitchik (1927 p.70) constructed this remarkable double camel pseudotour on an 84-cell serrated board consisting of a 48-cell tour on the majority colour (except for the centre cell) and a 36-cell tour of the minority colour, both in 90° rotational symmetry. The equivalent knight tours are alongside on the right.

The Zebra {2,3}

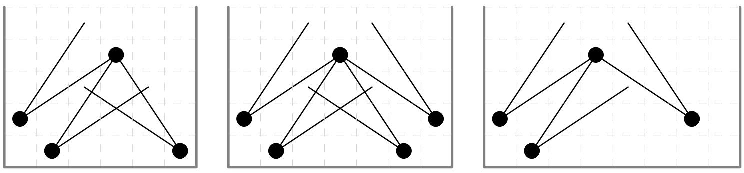
A zebra {2,3} move (length √13 ~ 3.61) is from corner to corner cells on a 3×4 board, so on a 3-rank board it has no moves at all on the middle rank, and on a 4-rank board it has only one horizontal move from the cells a2, a3, b2, b3 etc, and so no tours are possible. Tours only become feasible on 5-ranks.

5×5: has moves that form two octonary circuits, one of 8 moves through the corner cells and one of 16 moves, leaving only the centre cell unused.



5×6: The zebra is a free leaper on the 5×6 board and on any rectangle of larger dimensions (i.e. it can reach any cell from any other) however it cannot tour that board. In fact we can prove:

THEOREM: <u>The zebra cannot tour any board with sides 6, 7 or 8.</u> *Proof*: (a) No closed tours are possible since the path of the zebra through a2 and b1 is determinate and passes also through d4, but its paths through y1, y2, z1, z2 (where y is the penultimate and z the last file) are also determinate and at least one of these on a 6, 7 or 8 file board also uses d4, so we would have three moves impinging on d4, which is impossible in a tour. (b) In an open tour we have end cells, so we could avoid such a triple point by deleting one of the three moves and taking the cell at its other end as an end point. However, we only have two end-points, and we always have either more than two triple points or two points where at least four moves converge, or one point where at least five moves converge, so two deletions are insufficient. These interference points I call **snags**.



5×9: Since each zebra move is from one colour to another no closed tours are possible on odd×odd boards, and an open tour, if possible, must start and end on cells the same colour as the corner cells (which we take to be white). On the 5×9 board the moves through the black cells b3, h3 form a 4-move short circuit, so no open tour is possible.

5×10: The moves through b3, h3, c3, i3 form two 4-move circuits, so no closed tour is possible; and an open tour with one end in each circuit is impossible, since the moves through j2, j4, i1, i5 meet at f3, preventing moves c1-f3 or c5-f3, so that the moves through a2, a3, a4, b2, b4, c1, c5 form a 16-move circuit.

5×11: The moves forced at the black squares a2, a4, b1, b5 and k2, k4, j1, j5 fix the path through d2, d4, h2, h4 also; but now moves through f1, f5 must go to c3, i3, forming a 4-move short circuit.

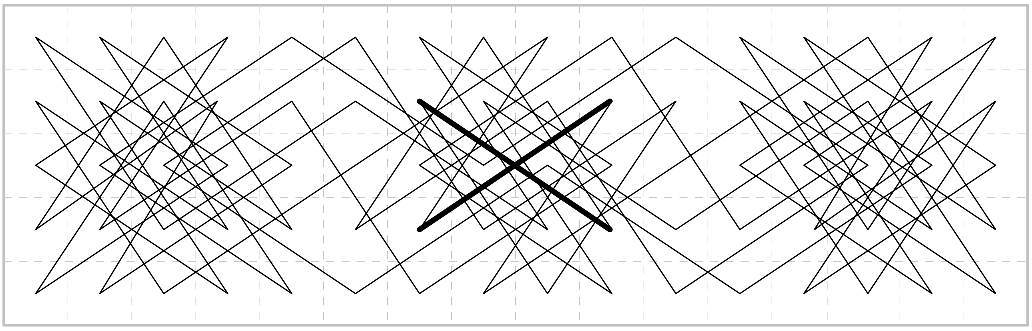
5×12: The moves through a2, a4, b1, b5 use the cells d2 d4. The moves through l1, l5 use the cell i3. Moves through l3, k2, k4 form a six-move path h2 to h4, and the moves through c3 connect f1 to f5, but now f1, f5 can only connect to h4, h2 forming a ten-move circuit, since the other cells they can reach d2, d4 and i3 are already used. (A further argument is needed to eliminate an open tour.)

5×13: The moves through the minority colour squares a2, b1, a4, b5 and m2, l1, m4, l5 fix the moves through d2, d4, j2, j4; but now the moves through g2, g4 must go to e1, e5, i1, i5, where they join with the moves from b3, l3 to form an 8-move short circuit.

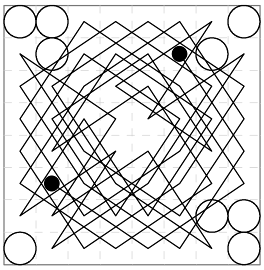
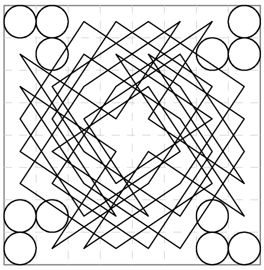
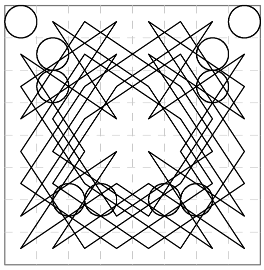
5×14: The moves through a3, b2, b4 use d1, d5, so the moves at g3, m3 form a 4-move circuit.

5×15: No closed tour is possible on an odd-sided board. Thus:

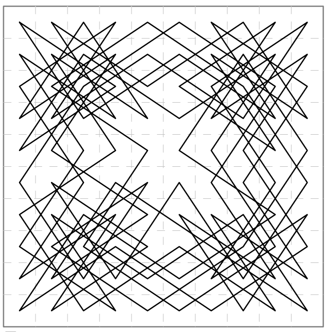
5×16: The smallest rectanglular zebra closed tour requires the surprisingly large 80-cell 5×16 board. This tour and two similar I found in 1992 and were published in *J. Rec.Math.* (1995).

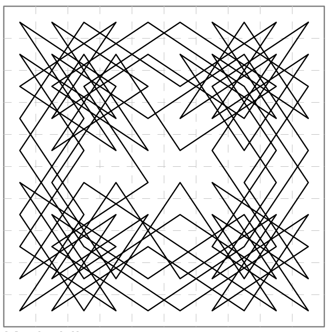


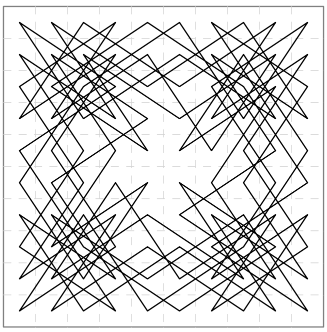
8×8 board. The snag theorem above shows, in particular, that no zebra tour is possible on the normal chessboard. The longest zebra paths achieved are 54 moves (55 cells) open, 52 closed symmetric rotary, and recently 54 closed symmetric axial this is by V. Kotesovec (2009 p.41).

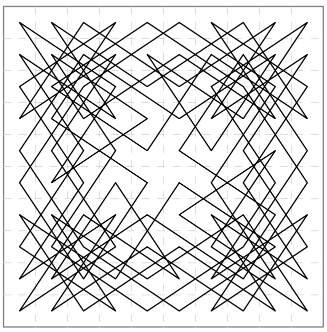



Kotesovec 2009

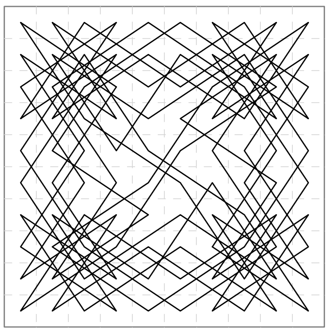
10×10: A zebra closed tour on this board was found by A. H. Frost as long ago as 1886. This subject was rediscovered by Kraitchik (1927 p.70-73) who gave two examples, open and closed, and details of how they were constructed by Euler's method.


Frost


Kraitchik

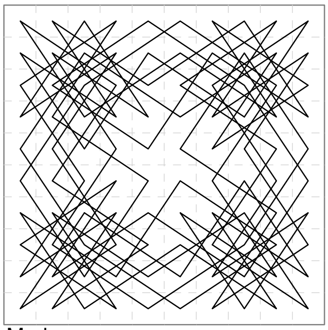

Cozens


Jelliss

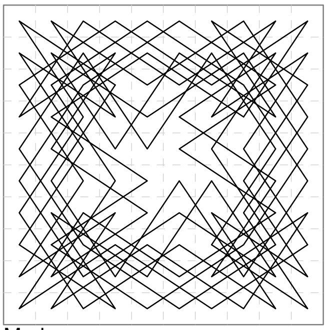

Marlow

H. H. Cross gave an open tour in *FCR* Feb 1941 (¶4709), which led T. H. Willcocks to compose a closed tour about the same time (but I have no diagram of it).

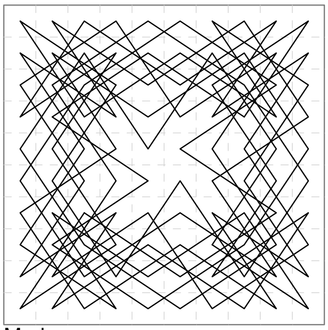
This led to a birotary example by W. H. Cozens in *Chessics* 1978 and one by myself, as above.



Marlow



Marlow



Marlow

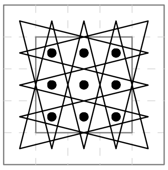
Tom Marlow (March 1998) applied the computer program he had used to count 10×10 knight tours with birotary symmetry to the zebra problem, with minor adaptations, and found there were only 6 geometrically distinct zebra tours of this type in all; these results were published in *The Games and Puzzles Journal* (vol.2 #16 1999 p.290).

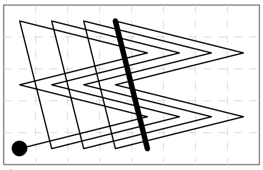
The Giraffe {1,4}

A giraffe {4,1} move (length √17 ~ 4.12) needs a 2×5 board, so the smallest board on which it can move on every cell is 2×8.

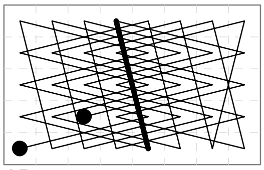
On the **5×5**:it has 16 moves that form a circuit that tours all the edge cells.

The **5×8** is the smallest rectangle on which the giraffe is a free leaper, and three distinct tours are possible, all given by T. R. Dawson (*L'Echiquier* Dec 1930). These are formed from two half-board tours A and B, joined AA, AB, BB with common central link e1-d5.

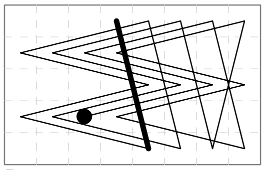




A



AB

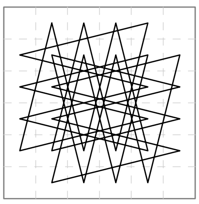


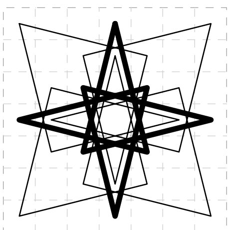
B

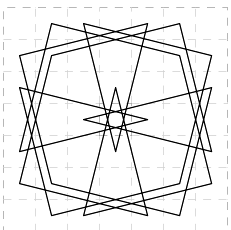
T. R. Dawson (*L`Echiquier*, Jul 1928 ¶278) also gave a general method for {1,2n}-mover touring a board (2n+1)×(4n). For n = 1 we get the corner-to-corner 3×4 knight's tour, and for n = 2 the similar 5×8 giraffe tour. The next case is a 7×12 tour by {1,6}-mover. Kraitchik noted that when numbered the numbers in the same rank are congruent mod 2n + 1.

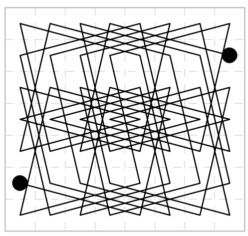
6×6: The giraffe cannot move on the centre cells and its moves from the corners meet to form 4-move circuits, but on the other cells it has a 28-move qaternary tour with diagonal axes.

7×7: Has no move at the centre cell, but forms octonary circuits of 8, 16 and 24 moves which together cover the other 48 cells. **7×8:** An open giraffe tour that I constructed at some time.



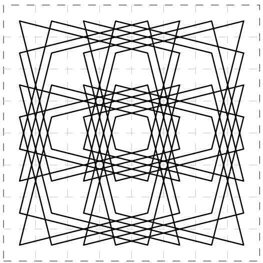


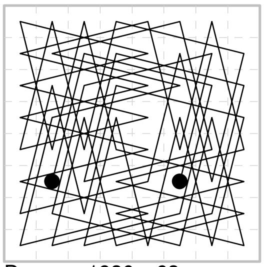


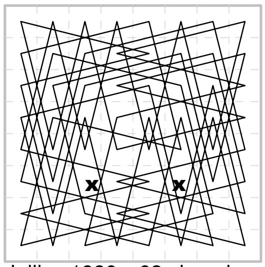


8×8 The board can be covered with 16 four-move giraffe circuits, analogous to the knights squares and diamonds, but the question of whether a giraffe tour on the 8×8 board is possible puzzled investigators for many years. T. R. Dawson wrote in *Cheltenham Examiner* 22 May 1913: “I see no reason why a giraffe tour of the board cannot be made, analogous to the knight's, but have not succeeded in working it out at the present.” Kraitchik in *L'Echiquier* (1927 p.257) says: “Mr T. R. Dawson believes that a tour of the 8×8 board by a (1,4) leaper is impossible”. In *Chessics* 1976 and 1980 I gave proofs that a Giraffe closed or open tour is impossible, as shown below.

In *L'Echiquier* (May 1928 problem 267) S. Vatriquant gave a 63-cell tour (omitting h8) that claims to be a giraffe tour, but his move 55–56 is a camel move! Dawson corrected this error (*L'Echiquier* Dec 1930 p.1085) with a 63-cell open tour omitting h8 (or c3 by diverting the route g4-h8-d7). Fifty years later I published a 62-cell closed tour (omitting c3 and f3).


Giraffe squares and diamonds


Dawson 1930—63 open


Jelliss 1980—62 closed

THEOREM: <u>A closed giraffe tour of the 8×8 board is impossible.</u> *Proof*: Letter the cells of the board ABCD as shown. Every A communicates by giraffe moves only with Bs, so in a closed tour every A must be preceded and followed by a B, but the number of Bs is the same as the number of As. This equality is possible only if the As and Bs form a closed circuit. But this would exclude the Cs and Ds. (Jelliss *Chessics* #2 1976).

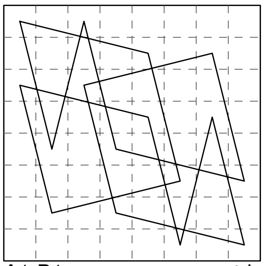
A	D	A	D	A	D	A	D
B	C	B	C	B	C	B	C
A	D	A	D	A	D	A	D
B	C	B	C	B	C	B	C
C	B	C	B	C	B	C	B
D	A	D	A	D	A	D	A
C	B	C	B	C	B	C	B
D	A	D	A	D	A	D	A

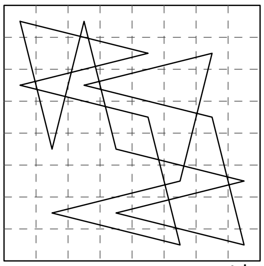
Proof Grid 1

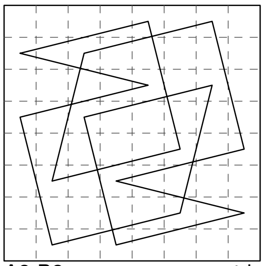
1	2	1	2	2	1	2	1
2	1	2	1	1	2	1	2
1	2	1	2	2	1	2	1
2	1	2	1	1	2	1	2
2	1	2	1	1	2	1	2
1	2	1	2	2	1	2	1
2	1	2	1	1	2	1	2
1	2	1	2	2	1	2	1

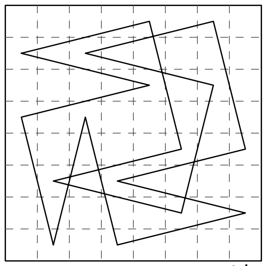
Proof Grid 2

THEOREM: <u>An open giraffe tour of the 8×8 board is impossible.</u> *Proof*: The previous argument to prove the impossibility of a closed tour shows that an open tour must be of the form ABABAB. ... AB–CD ... CDCDCD. Now combine the lettering with the numbering shown in the other diagram above (which chequers each quarter board, corners being the same colour 1). This has the property that cells A1 communicate only with cells B1. These cells are equal in number (8), so the A end of the tour must start on an A1. Similarly it must end on a D1. So the whole tour becomes A1B1...A1B1 – A2B2...A2B2 – C2D2...C2D2 – C1D1...C1D1. But the flaw in this scheme is that there is no move of the type B2 – C2 needed to link the two halves of the tour! (Jelliss *Chessics* #9 1980).


A1-B1 symmetric


asymmetric


A2-B2 symmetric

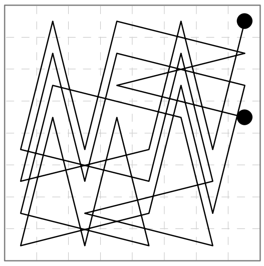

asymmetric

In October 2013, as reported in my online *Jeepyjay Diary*, I revisited the above proofs. This led me to find two 16-move closed tours of the A1-B1 cells and two 16-move closed tours of the A2-B2 cells, forming four closed quarter tours. In each case one tour is symmetric and the other asymmetric.

At the same time I combined two copies of a 32-cell path to form a Magic Two-Giraffe tour, where the link is a {0,7} rook move, but have since discovered that Prof C. E. Reuss constructed a Magic Four-Giraffe tour with {0,4} and {07} links as long ago as 1887 (see p.43 and 56).

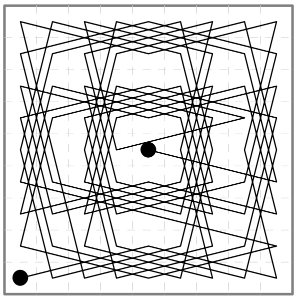
There are also 16-cell open paths of course. Two such paths can be joined to form a 32-cell open path as in this 32-cell giraffe open path given by E. Huber-Stockar (*FCR* 1933 ¶3938):

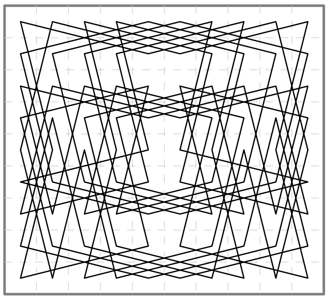
	27		29		3		1
	6		8		24		30
	19		31		11		9
	14		16		20		32
26		28		4		2	
5		7		25		23	
18		22		12		10	
13		15		17		21	

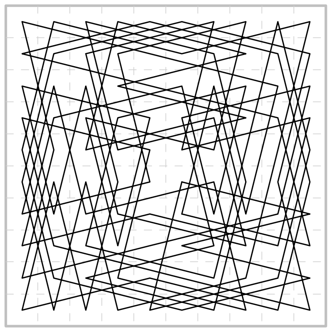


9×9 An open giraffe tour was given by T. R. Dawson (in *The Problemist* Jul 1926 ¶41) and the same tour is in Kraitchik (1927 p.73).

9×10 E. Huber-Stockar (*FCR* Feb/Apr 1945 ¶6304) unique 9×10 axial symmetric giraffe tour.

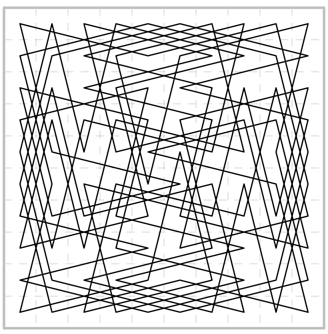


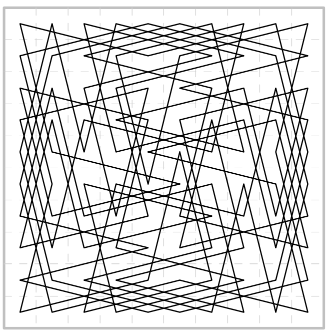


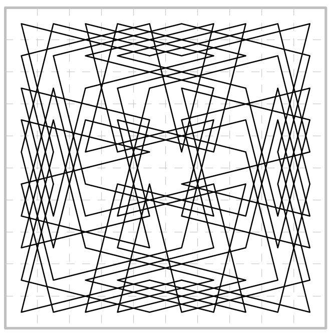


10×10 This case was solved with an asymmetric closed giraffe tour (above right) by A. H. Frost as long ago as 1886 (M. Frolow, *Les Carrés Magiques*, Paris 1886, Plate VII).

Quaternary examples have been shown by Kraitchik (1927 p.73) and by W. H. Cozens (*Chessics* 1978) and by T. W. Marlow (1998).

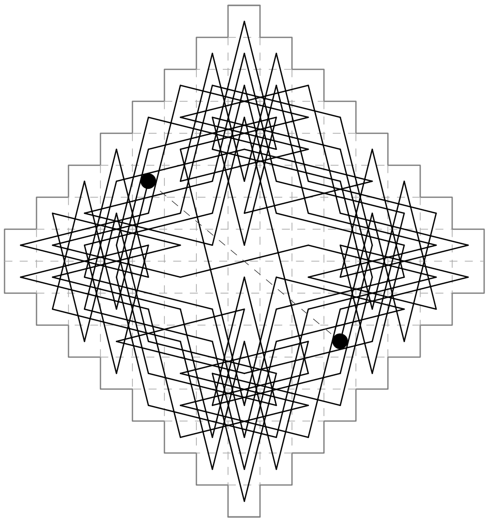






T. H. Willcocks conjectured (*Chessics* 1976) that any {r,s} free-leaper can make a closed tour on a square chessboard of side 2·(r+s). A proof of this was recently given by Nikolai Beluhov (2017).

A Giraffe symmetric open tour of 128 cell board, equivalent to a {3,5} tour on a board 16×16.

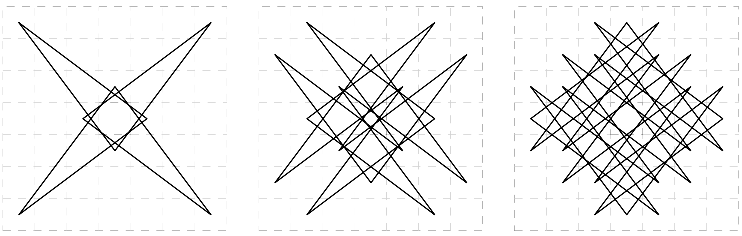


The Antelope {3,4}

The antelope or {3,4}-mover (move length √25 = 5) needs at least a 4×5 board to move.

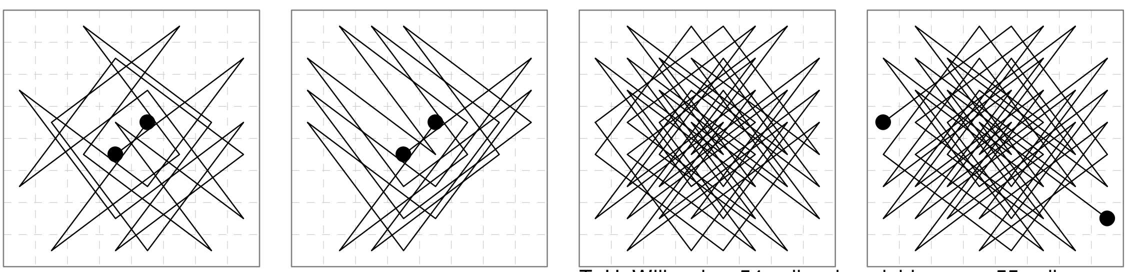
On a 6-rank board it has at most one move from the end four cells of central ranks, so no tours.

7×7: Has no move at the centre cell, but forms octonary circuits of 8, 16 and 24 moves which together cover the other 48 cells (similar to Giraffe).



7×8: It is a free leaper on this board but cannot tour it; W. Langstaff (*FCR* ¶5798, Dec 1943 p.69 and Feb 1944 p.78) gave a 47-move path on the 7×8.

8×8: F. Douglas *PFCS* 1932 showed the antelope "may describe two doubly symmetrical closed tours of 44 moves". Half-tours are shown, the other halves reflecting these in the a1-h8 diagonal. These have quaternary symmetry with diagonal axes.

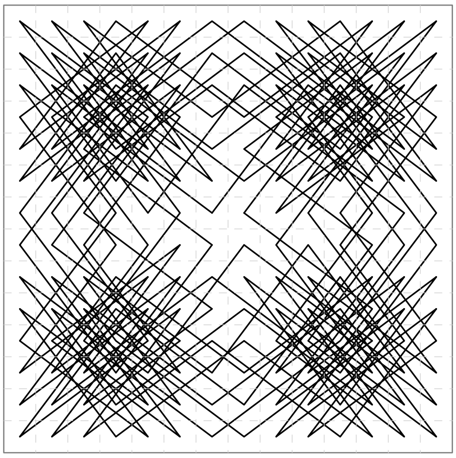


F. Douglas 44 cells closed quaternary, two ways T. H. Willcocks 54 cells, closed, binary 55 cells, open

W. Langstaff *FCR* 1943 gave a 50-move open path and later a 52-move solution, but these were surpassed by some 54-move examples by A. H. Haddy and T. H. Willcocks *FCR* 1944. Willcocks later gave a closed solution.

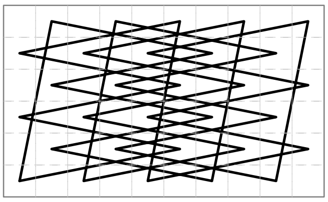
8 to 13 ranks: As for the zebra, we can prove, by considering the formation of ‘snags’ that no antelope tours are possible on boards 8×n to 12×n. On the 13×13 board its moves through a3, b2, b3, c3, and cognate cells form three short circuits, so no tours are possible.

14×14 Board. T. H. Willcocks gave a quatersymmetric {3,4} tour in *Chessics* 1978.



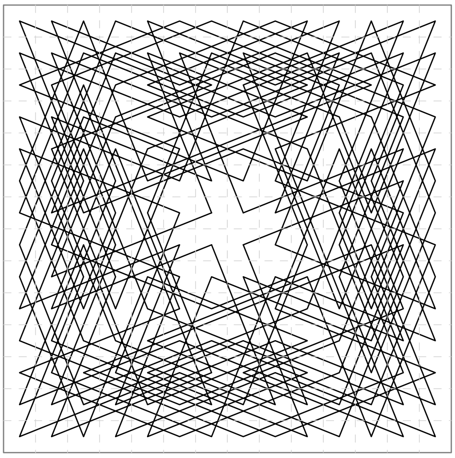
{1,5}

6×10 T. R. Dawson (*L`Echiquier* Aug 1928 ¶285) general method for a {1, 2·n + 1} mover on cells of one colour on a board (2·n + 2)×(4·n + 2). {1,5}-leaper on 6×10 Board, is the n = 2 case.



{2,5}

14×14 Board. T. H. Willcocks gave a quatersymmetric tour in *Chessics* 1978. For reasons of clarity, the hand-drawn diagram used there only showed one quarter of the path.



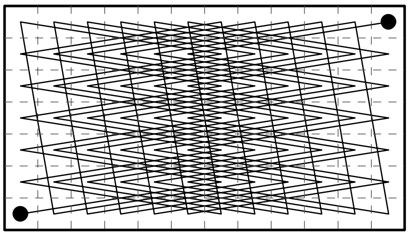
7-Rank Boards. F. Douglas *PFCS* ¶1415 (1934): “The root-29 leaper, moving a1-c6 and the like may play a 42-move closed tour on the 7×8 board. FD gave all possible closed paths of the leaper up to 42 moves. The longest tour on the 7×7 board is the 32-move one.”

{3,5}

A tour by a {3,5} leaper, √34 mover, on the cells of one colour on a 16×16 board transformed into an equivalent Giraffe tour on a serrated board is shown above, p.36.

{1,6} Flamingo

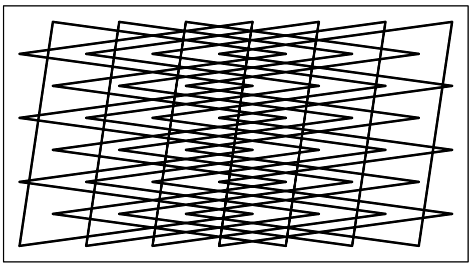
7×12 Board. T. R. Dawson (*L`Echiquier* Jul 1928 ¶278). This is a case of a general method for a {1, 2·n} mover touring a board (2·n + 1)×(4·n). Compare 3×4 knight and 5×8 giraffe tours.



{1,7}

8×14 T. R. Dawson (*L`Echiquier* Aug 1928 ¶285) noted a general method for a {1, 2·n + 1} mover touring cells of one colour on a board (2·n + 2)×(4·n + 2).

{1,7} = √50 leaper on 8×14 Board, is the n=3 case.



{4,7}

This is of length √(16 + 49) = √65. See {1,8}, which is also a √65 leaper.

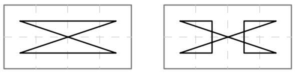
{6,7}

This is of length √(36 + 49) = √85. See {2,9}, which is also a √85 leaper.

Two-Move Beasts

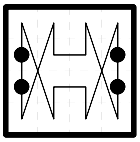
{1,3}+{0,1}

2×4 Two biaxial closed tours by Wazir + Camel

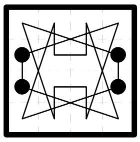


4×4 Biaxial magic tours (nondiagonal) with biaxial symmetry when the two end-points of the path are joined up to give a closed path. (Jelliss *Chessics* #26 1985).

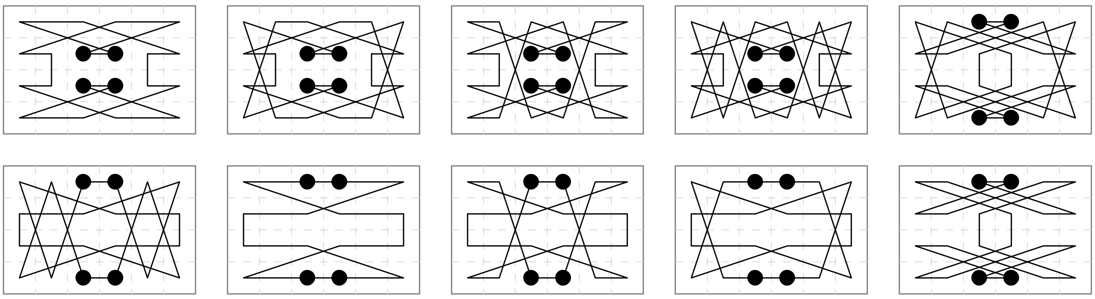
15	3	6	10
16	4	5	9
1	13	12	8
2	14	11	7



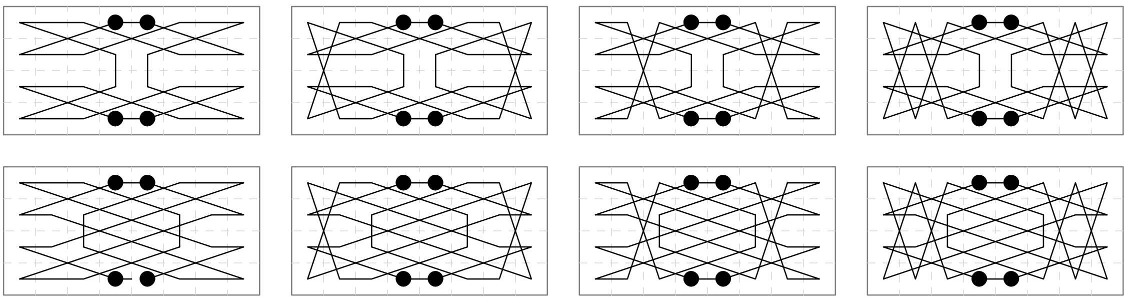
10	6	3	15
16	5	4	9
1	12	13	8
7	11	14	2



4×6 Biaxial magic rectangles (Jelliss 10 Aug 1991).
The first of these is also given by Trenkler (1999).

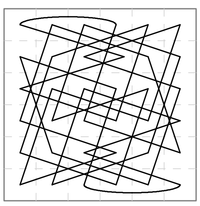


4×8 Biaxial magic rectangles (Jelliss 17 Oct 2014).



{1,3}+{0,3}

6×6 Board. The smallest possible square tour by this amphibian is 6×6. This symmetric example has the minimum of two (0,3) moves (Jelliss undated).



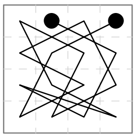
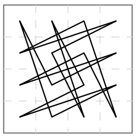
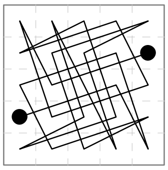
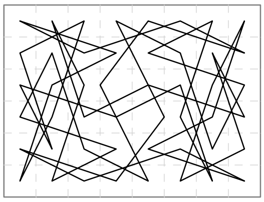
{1,3}{1,2} Gnu

This is our first combination of two skew movers.

4×4 The 4×4 open tour with one camel move is by S. H. Hall (*Fairy Chess Review* Aug 1944). The other is by E. Huber-Stockar (*FCR* Jun 1944 ¶6017) and has rotary symmetry (not quite birotary), and four can be joined to form an 8×8 quaternary tour.

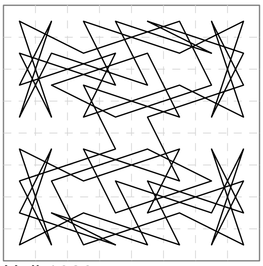
5×5 Symmetric open tour by E. Huber-Stockar (*FCR* 1941 ¶4710) with moves in only half of the possible directions. Four of these can be combined to form a closed quaternary 10×10 tour.

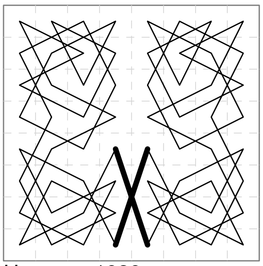
6×8 Rotary tour by E. Huber-Stockar (*FCR* 1942 ¶5085) alternating knight and camel moves.

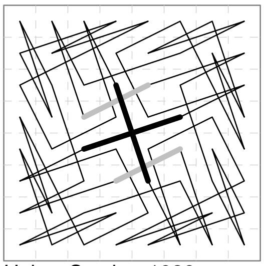





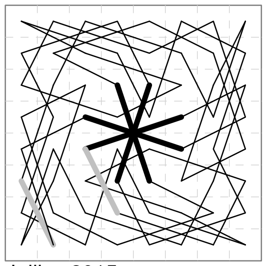
8×8 A symmetric gnu tour of alternating knight and camel moves joining two half-board tours was given by S. H. Hall (*Fairy Chess Review* Aug 1939).

F. Hansson (*FCR* Nov 1939 ¶3936) gave a gnu tour with axial symmetry and minimum two camel moves. This is just two half-board tours by knight joined by camel moves.


Hall 1939


Hansson 1939


Huber-Stockar 1939

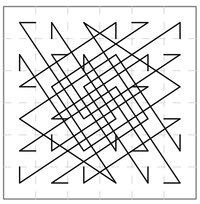

Jelliss 2015

E. Huber-Stockar (same issue ¶3937) gave an 8×8 gnu tour in approximate quaternary symmetry in which two camel moves cross at the centre (shown bold in our diagram). The four halves of these moves form part of the quaternary symmetry. The only deviation from quaternary are the two 1-1 knight moves shown in grey which have no corresponding pair.

Drawing this tour inspired me to try an alternating tour with four camel moves crossing (Jelliss Feb 2015), this is asymmetric, departing from symmetry with the two grey moves at the bottom left.

{2,3}+{0,4}

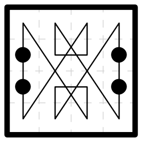
6×6 Board. E. Huber-Stockar (*FCR* ¶5395 Dec 1942 and Feb 1943) gave a 6×6 symmetric tour by this piece. For clarity the rook moves are only indicated partially in our diagram. This works since there is only one such move in each rank and file.



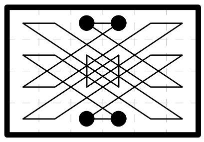
{2,3}{1,2} Okapi

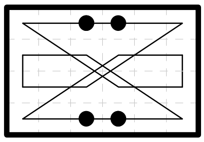
4×4 Magic tour (nondiagonal) with biaxial symmetry when the two end-points of the path are joined up to give a closed path. The black dots mark 1, 8, 9, 16. (Jelliss *Chessics* #26 1986).

15	6	3	10
16	5	4	9
1	12	13	8
2	11	14	7

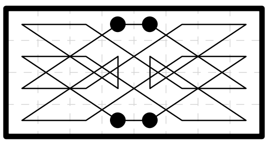


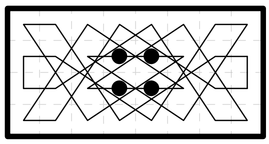
4×6 Biaxial (Jelliss 10 Aug 1991).

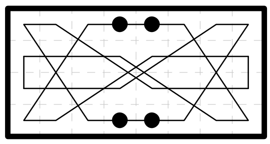


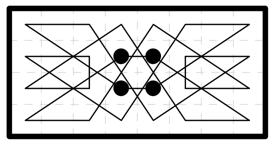


4×8 Biaxially symmetric magic tours. Results from a partial enumeration 18 Oct 2014.









{2,3}{1,3} Bison

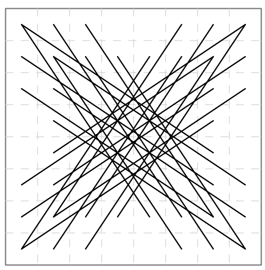
PUZZLE: Here is a cryptotour on the 6×6 board. It requires a reentrant tour by a ‘bison’ which moves in {1,3} and {2,3} leaps. The tour is comprised of camel {1,3} moves with only one zebra {2,3} move. The solution (see end page) is a verse I wrote on seeing the first Bison chess problem.

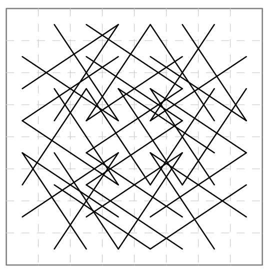
some	by	if	bison	may	've
or	chance	you	camel	ing	in
and	camel	chess	'swith	eye	or
son	son	'sgo	white	seen	you
stripe	bison	to	jumps	'swith	mak
lar	black	humps	son	zebra	set

{2,3}{4,6} Zebrarider

8×8 A closed tour by this piece was given by O. E. Vinje (*FCR* Nov 1939 and Feb 1940). It can be regarded as Zebra + double-length Zebra. In the diagrams the two types of move are shown separately since they overlap in places due to switchbacks.

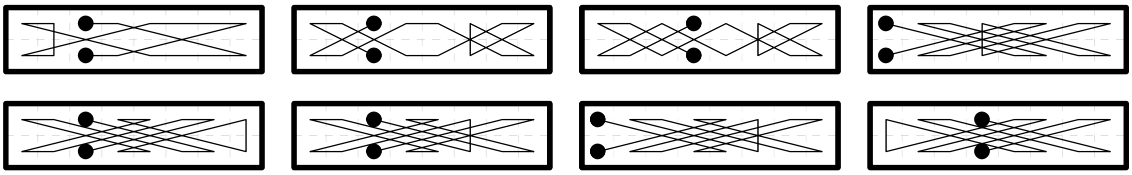
40	47	51	54	29	34	10	15
45	4	64	57	32	27	19	8
53	60	21	38	43	50	23	36
58	55	28	46	9	1	30	33
17	14	49	25	62	12	39	42
20	7	2	59	22	37	44	5
24	35	11	16	41	48	52	61
31	26	18	13	6	3	63	56



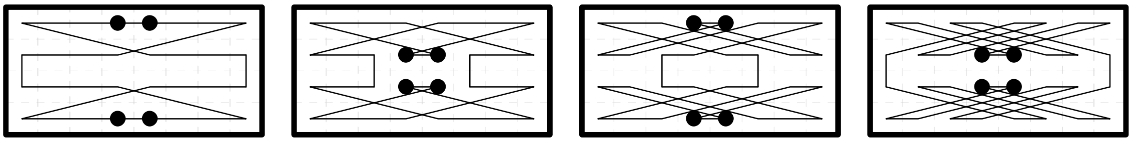


{1,4}+{0,1}

2×8 On the 2×8 board there are 8 magic tours by {0,1}+{1,4}, permuting magic king tours.

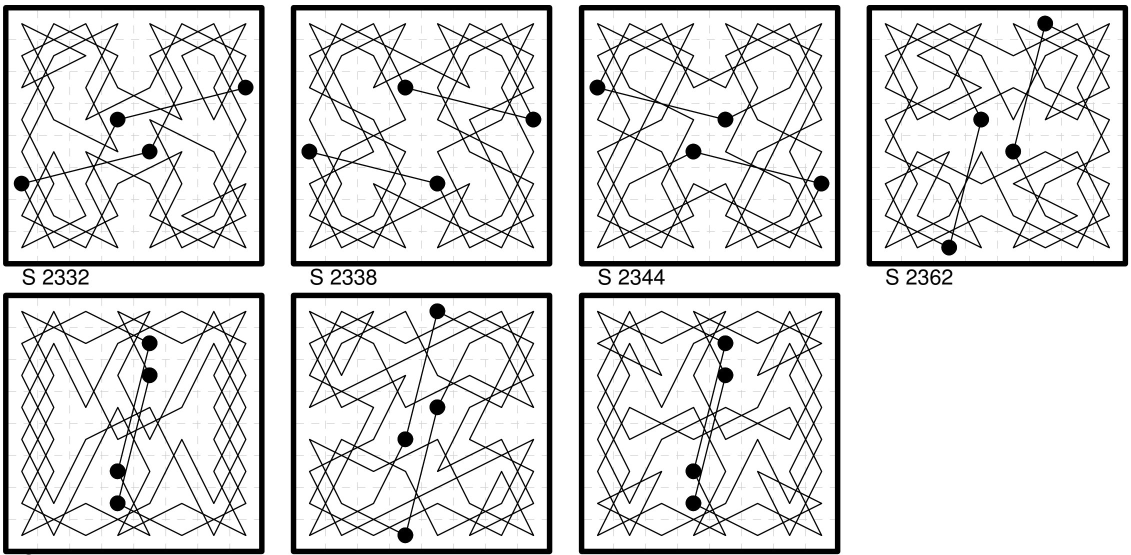


4×8 Biaxially symmetric magic tours. Results from a partial enumeration 17 Oct 2014.



{1,4}{1,2}

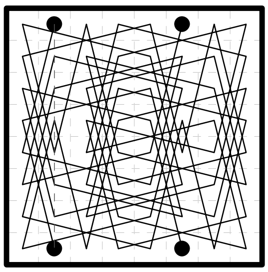
8×8 Magic two-knight rotary tours *Le Siècle* (1) ¶2332 (18 Apr 1884) 224/232 by Reuss. (2) ¶2338 (25 Apr 1884), diags 260±12 by Béligne. (3) ¶2344 (2 May 1884) 260±20 by M. A. F. (4) ¶2362 (23 May 1884) 312/272 by M. A. D. a Rouvres. (5) ¶2482 (10 Oct 1884) 260±4 by Béligne. (6) ¶2494 (24 Oct 1884) 264/224 by Palamede (Ligondes). (7) ¶2506 (7 Nov 1884) 260±32 Anon.



S 2332 S 2338 S 2344 S 2362 S 2482 S 2494 S 2506

{1,4}+{0,7}

7	32	9	18	39	64	41	50
38	61	44	51	6	29	12	19
3	28	13	22	35	60	45	54
34	57	48	55	2	25	16	23
31	8	17	10	63	40	49	42
62	37	52	43	30	5	20	11
27	4	21	14	59	36	53	46
58	33	56	47	26	1	24	15



8×8 Magic Two-Giraffe Tour (Jelliss 2013): two copies of a 32-cell giraffe tour joined 32-33.

{3,4}+{0,5} Fiveleaper

6×6 Board. Closed paths (not tours) by a fiveleaper on this board were considered in *Chess Amateur* Apr 1925 and Mar 1926 where paths with two axes of symmetry were given by H. A. Adamson, F. Douglas, R. J. French and J. Sunyer, and in *L'Echiquier* Dec 1930 and Jan 1931 where five paths with one axis of symmetry by W. E. Lester were given.

6×9 Board. In *Chessics* (#24 1985 p.93) T. W. Marlow gave a closed tour with a horizontal axis of symmetry. This is the smallest rectangular area, without holes, that a 5-leaper can tour.

54	21	14	31	18	11	28	15	52
23	50	7	46	33	4	49	8	45
36	43	26	39	2	35	42	25	38
19	12	29	16	53	20	13	30	17
32	5	48	9	22	51	6	47	10
1	34	41	24	37	44	27	40	3

4	23	46	27	42	11	2
25	40	29	36	19	44	9
16	13	34	7	38	17	14
21	32	1		5	22	31
48	43	10	3	24	47	28
37	18	45	26	41	12	35
6	39	30	15	20	33	8

4	23	36	43	26	17	2
21	34	45	12	41	28	19
32	47	14	7	10	39	30
25	16	1		5	24	37
42	27	18	3	22	35	44
11	40	29	20	33	46	13
6	9	38	31	48	15	8

7×7 Board. T. H. Wilcocks in *Chessics* (#24 1985 p.93) gave an open tour of a centreless 7×7 board (the other example given here is my own).

8×8 Board. Kraitchik (1927 p.74) noted that the fiveleaper has four moves at every cell of the board and gave an asymmetric closed tour (using 36 lateral + 28 skew moves). However he did not point out that the moves not used, two at every cell, in this case also form a tour! I show this complementary tour alongside the original (it uses 12 lateral + 52 skew moves). It seems likely that the tour was constructed to have this property. In the case of a random tour the complementary moves would more likely form a pseudotour of two or more circuits. There are 48 lateral moves and 80 skew moves on the board to be shared between the pair of tours.

40	33	16	59	62	13	34	47
45	26	51	6	29	44	23	52
64	11	56	19	38	3	8	57
31	14	35	48	41	32	15	36
28	61	22	53	46	27	60	21
39	4	17	58	63	12	5	18
42	25	50	7	30	43	24	49
1	10	55	20	37	2	9	54

Kraitchik 1927

6	63	58	15	24	5	8	59
41	38	29	26	61	48	39	36
18	51	10	43	34	19	50	53
23	12	45	2	55	32	21	16
14	47	4	7	64	57	30	25
27	62	49	40	37	28	9	60
42	33	20	17	52	11	44	35
1	56	31	22	13	46	3	54

Kraitchik complementary

19	54	35	60	5	30	11	46
44	3	22	17	48	43	62	21
57	32	9	14	51	56	33	8
6	29	12	39	26	53	36	59
49	42	63	20	45	2	23	16
18	55	34	61	4	31	10	47
27	52	37	58	7	28	13	38
40	1	24	15	50	41	64	25

Vinje 1939

40	63	34	13	20	47	62	33
11	18	37	54	27	4	49	42
58	23	6	51	44	57	24	7
21	46	61	32	9	16	35	60
28	3	48	41	64	29	14	53
39	56	25	12	19	38	55	26
10	17	36	59	22	5	50	43
1	30	15	52	45	2	31	8

Huber-Stockar 1942

O. E. Vinje *FCR* Nov 1939 gave an axisymmetric example (20 lateral + 44 skew). The symmetry is shown by the fact that when numbered from b1 to g1 (the ends of one of the two symmetric cross-axis rook moves) all pairs of numbers on either side of the vertical axis add to 65, which means it is semi-magic. However the original was numbered 0 to 63 beginning at a1 and the symmetry was not noted. If numbered 1 to 64 from a1 as in the other examples 25 pairs of numbers add to 51 and 7 pairs to 115 and the tour is not semi-magic in this numbering.

S. H. Hall had proposed the fiveleaper tour problem in *FCR* Dec 1938 ¶3463 and E. Huber-Stockar sent a solution published later in *FCR* (vol 5 Aug 1942 p.5 and Oct 1942 p.14 ¶5273/4). This is a closed tour with rotary symmetry (24 lateral + 40 skew). This symmetry is shown by diametrally opposite numbers differing by 32. The complementary moves form pseudotours.

Around 1973, when I first became interested in tours, not knowing the above results I reinvented the fiveleaper calling it the Vaulter - symbol V - and composed two examples. The first (37 lateral + 27 skew) may be an attempt to maximise the number of lateral moves (the theoretical maximum is 39, formed of 12 on the middle ranks and files plus 27 along three edges of nine squares). The second, which appeared in *Chessics* #5 1978, has rotary symmetry and equal numbers of rook and skew moves. The third shows biaxial symmetry (12 lateral + 52 skew). Pairs of numbers in the files add to 65 making it semi-magic. Pairs of numbers in the ranks add to 33 and 97, making the six middle ranks also magic, but not the top and bottom ranks.

10	21	40	29	52	9	20	63
61	50	13	6	43	32	49	12
4	57	24	37	16	3	56	27
45	34	19	64	59	46	35	18
42	31	54	11	62	41	30	53
7	22	39	28	51	8	23	38
60	47	14	5	44	33	48	15
1	58	25	36	17	2	55	26

Jelliss 1973

60	31	34	17	8	57	30	33
19	36	53	14	47	22	37	52
10	45	26	41	50	11	44	27
7	56	3	64	61	6	55	16
48	23	38	29	32	35	24	39
59	12	43	18	9	58	13	42
20	5	54	15	46	21	4	51
1	62	25	40	49	2	63	28

Jelliss 1978

3	48	29	22	11	4	49	30
64	27	20	41	56	13	6	33
25	18	51	58	39	46	15	8
10	53	60	31	2	37	44	23
55	12	5	34	63	28	21	42
40	47	14	7	26	19	50	57
1	38	45	24	9	52	59	32
62	17	36	43	54	61	16	35

Jelliss 2018

Complementary Fiveleaper Tours.

Being unaware of the Kraitchik result I raised the question of whether a complementary pair of tours was possible in *Variant Chess* (vol.1 #6 Apr-Jun 1991 p.75). The question was answered by Tom Marlow in a letter to me of 17 Nov 1991, but due to an oversight his result was not published until ten years later, in the last issue of the *Games and Puzzles Journal* (vol.2 #18 Mar 2001 p.347). He solves the problem with a single tour (24 lateral + 40 skew). The complementary tour being the same tour rotated a half-turn.

He argued as follows: “The 5-leaper has exactly four moves available on every square of the 8×8 board. In all there are 128 leaps, each being possible from either end. The two closed tours below make use between them of all these leaps. The method of construction was to build a tour starting at a1 and at each leap to mark as unavailable the corresponding leap after 180 degree rotation; e.g. the opening a1-a6 barred h8-h3 and h3-h8. When the tour was complete the same route, rotated 180 degrees, could be travelled using the barred leaps. That tour was then renumbered to start at a1.”

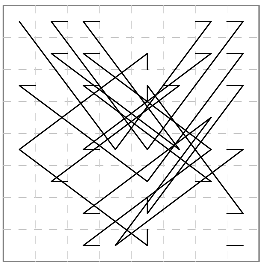
20	47	62	55	6	21	46	63
31	42	57	50	11	34	29	44
2	59	16	9	52	3	36	17
13	22	39	26	19	14	23	38
54	5	28	45	64	61	56	7
51	48	35	30	43	58	49	10
32	41	24	37	12	33	40	25
1	60	15	8	53	4	27	18

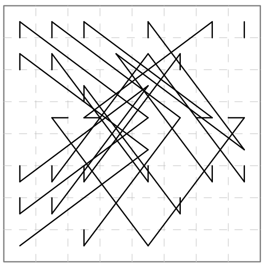
46	37	60	11	56	49	4	63
39	24	31	52	27	40	23	32
54	15	6	21	34	29	16	13
57	8	3	64	19	36	59	10
26	41	50	45	38	25	42	51
47	28	61	12	55	48	5	62
20	35	30	53	14	7	22	33
1	18	43	58	9	2	17	44

This work was followed up by John D. Beasley in articles in *Variant Chess*: ‘Complementary five-leaper tours’ (#62 Oct 2009 p.131) and ‘Complementary five-leaper (and other) tours with rotational symmetry’ (#64 Aug 2010 p.232-233). He calls Tom's tour ‘rotationally antisymmetric’. Instead he looks for tours unaltered by a half-turn (i.e. rotationally symmetric), but in which a quarter turn gives the complementary tour.

By applying a computer programme John found that there are **125217** rotationally symmetric fiveleaper tours, of which **373** are also laterally symmetric (in other words they have biaxial symmetry). He found **224** tours of this new complementary type, showing one example. A print-out of all 224 tours is available on the JSB website. Here is his example.

1	4	7	50	45	64	3	6
48	9	30	53	20	47	10	29
27	56	23	60	17	26	57	22
44	63	40	37	34	43	14	51
19	46	11	2	5	8	31	12
54	25	58	49	28	55	24	59
61	42	15	52	21	62	41	16
38	35	32	13	18	39	36	33

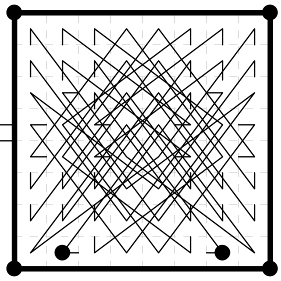


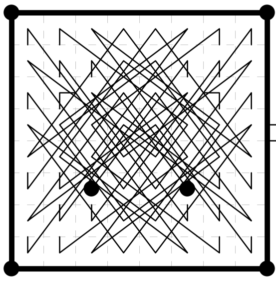


Magic Fiveleaper Tours

T. W. Marlow constructed 58 magic fiveleaper tours (42 closed and 16 open) in September 1990. They are presented in colourful form on the KTN web pages. Tours #5 (closed) and #47 (open) in Marlow's list are diagonally magic. These were published in *The Problemist* March 1991. The closed tour remains magic when cyclically renumbered g6 to b6. These tours are shown also in diagram form, with just the end sections of the lateral moves marked, which illustrates why the numerical presentation of fiveleaper tours is preferable.

5	56	51	44	21	14	9	60
28	41	16	7	58	49	24	37
35	32	47	26	39	18	33	30
54	13	10	63	2	55	52	11
43	20	23	4	61	42	45	22
6	57	50	29	36	15	8	59
27	40	17	34	31	48	25	38
62	1	46	53	12	19	64	3





14	59	36	55	10	29	6	51
49	18	63	12	53	2	47	16
24	33	4	45	20	61	32	41
39	28	7	22	43	58	37	26
54	9	30	15	50	35	56	11
13	60	1	48	17	64	5	52
44	19	62	25	40	3	46	21
23	34	57	38	27	8	31	42

The other results, with two examples, were reported in *Variant Chess* (#6 Apr-Jun 1991 p.75). All the other tours are shown in the following diagrams though in a different orientation from their presentation on the KTN web-pages. They are oriented here with the axis of symmetry vertical and the start cell in the a1 quadrant, as in the Vinje tour above.

The method of construction is to find sequences of 32 five-leaper steps that, when reflected in the vertical axis, cover the remaining 32 squares of the board. Then if the 32nd square is on the second or seventh file its reflection is five squares away and the second half of the tour proceeds in reverse order to the first half. The result is that all ranks sum to 260 because each consists of four pairs adding to 65. It then remains to find cases where the files also total 260. The closed tours begin in the second file, so that the end cell is one five-leaper move from the start.

There are 32 of the closed tours that are symmetric about the horizontal axis as well as the vertical axis (i.e. they are bizaxial). We show them in batches:

Magic Fiveleaper Closed Tours with Biaxial Symmetry

batch of 8 (2 and 7-14, less 12)

3	**32**	61	52	13	4	**33**	62
16	59	50	11	54	15	6	**49**
57	42	9	26	39	56	23	8
44	19	28	63	2	37	46	21
53	14	5	34	31	60	51	12
40	55	24	7	58	41	10	25
17	38	47	22	43	18	27	**48**
30	**1**	36	45	20	29	**64**	35

2

19	**32**	45	24	41	20	**33**	46
16	59	22	35	30	43	6	**49**
61	2	53	28	37	12	63	4
40	55	26	47	18	39	10	25
57	42	7	50	15	58	23	8
36	31	44	5	60	21	34	29
17	38	11	62	3	54	27	**48**
14	**1**	52	9	56	13	**64**	51

7

25	**32**	41	20	45	24	**33**	40
6	**49**	22	35	30	43	**16**	59
61	2	55	28	37	10	63	4
46	53	18	39	26	47	12	19
51	44	15	58	7	50	21	14
36	31	42	5	60	23	34	29
27	**48**	11	62	3	54	**17**	38
8	**1**	56	13	52	9	**64**	57

8

51	**32**	41	28	37	24	**33**	14
16	43	26	35	30	39	22	**49**
45	2	57	12	53	8	63	20
4	59	10	47	18	55	6	61
29	38	23	50	15	42	27	36
52	31	40	21	44	25	34	13
17	54	7	62	3	58	11	**48**
46	**1**	56	5	60	9	**64**	19

9

55	**32**	61	14	51	4	**33**	10
8	59	**16**	53	12	**49**	6	57
43	2	35	26	39	30	63	22
20	47	28	41	24	37	18	45
13	50	5	56	9	60	15	52
54	31	62	7	58	3	34	11
25	38	**17**	44	21	**48**	27	40
42	**1**	36	19	46	29	**64**	23

10

56	43	8	61	4	57	22	9
49	**32**	59	20	45	6	**33**	**16**
14	53	26	47	18	39	12	51
3	28	63	10	55	2	37	62
30	5	34	23	42	31	60	35
19	44	7	50	15	58	21	46
48	**1**	38	13	52	27	**64**	**17**
41	54	25	36	29	40	11	24

11

36	59	30	43	22	35	6	29
17	**32**	41	52	13	24	**33**	**48**
46	39	8	15	50	57	26	19
21	10	63	28	37	2	55	44
12	23	34	5	60	31	42	53
51	58	25	18	47	40	7	14
16	**1**	56	45	20	9	**64**	**49**
61	38	3	54	11	62	27	4

13

20	43	30	39	26	35	22	45
49	**32**	37	24	41	28	**33**	**16**
14	55	4	47	18	61	10	51
57	6	63	12	53	2	59	8
40	27	34	21	44	31	38	25
19	42	29	50	15	36	23	46
48	**1**	60	9	56	5	**64**	**17**
13	54	3	58	7	62	11	52

14

continued

batch of 12 (16-29, less 26)

8	51	56	43	22	9	14	57
59	**32**	5	**16**	**49**	60	**33**	6
30	47	36	63	2	29	18	35
53	10	13	26	39	52	55	12
44	23	20	7	58	45	42	21
3	50	61	34	31	4	15	62
38	**1**	28	**17**	**48**	37	**64**	27
25	46	41	54	11	24	19	40

16

14	43	52	59	6	13	22	51
49	**32**	57	4	61	8	**33**	**16**
30	55	24	63	2	41	10	35
37	26	21	18	47	44	39	28
60	7	12	15	50	53	58	5
3	42	9	34	31	56	23	62
48	**1**	40	29	36	25	**64**	**17**
19	54	45	38	27	20	11	46

17

4	51	56	43	22	9	14	61
59	**32**	7	**16**	**49**	58	**33**	6
30	47	40	63	2	25	18	35
53	10	13	28	37	52	55	12
44	23	20	5	60	45	42	21
3	50	57	34	31	8	15	62
38	**1**	26	**17**	**48**	39	**64**	27
29	46	41	54	11	24	19	36

18

12	55	60	35	30	5	10	53
49	**32**	15	8	57	50	**33**	**16**
20	41	46	63	2	19	24	45
39	4	27	22	43	38	61	26
58	29	6	11	54	59	36	7
13	56	51	34	31	14	9	52
48	**1**	18	25	40	47	**64**	**17**
21	42	37	62	3	28	23	44

19

28	43	38	57	8	27	22	37
35	**32**	**17**	20	45	**48**	**33**	30
14	53	50	63	2	15	12	51
41	26	23	4	61	42	39	24
56	7	10	29	36	55	58	9
19	44	47	34	31	18	21	46
62	**1**	**16**	13	52	**49**	**64**	3
5	54	59	40	25	6	11	60

21

28	45	54	41	24	11	20	37
35	**32**	**49**	26	39	**16**	**33**	30
6	51	18	63	2	47	14	59
57	10	21	4	61	44	55	8
40	23	12	29	36	53	42	25
27	46	15	34	31	50	19	38
62	**1**	**48**	7	58	**17**	**64**	3
5	52	43	56	9	22	13	60

22

28	41	46	53	12	19	24	37
35	**32**	**17**	26	39	**48**	**33**	30
6	57	50	63	2	15	8	59
43	20	23	4	61	42	45	22
54	13	10	29	36	55	52	11
27	40	47	34	31	18	25	38
62	**1**	**16**	7	58	**49**	**64**	3
5	56	51	44	21	14	9	60

23

14	37	52	55	10	13	28	51
35	**32**	57	**16**	**49**	8	**33**	30
18	59	26	63	2	39	6	47
43	24	21	4	61	44	41	22
54	9	12	29	36	53	56	11
15	38	7	34	31	58	27	50
62	**1**	40	**17**	**48**	25	**64**	3
19	60	45	42	23	20	5	46

24

22	37	42	57	8	23	28	43
35	**32**	**49**	20	45	**16**	**33**	30
12	51	18	63	2	47	14	53
39	24	27	4	61	38	41	26
58	9	6	29	36	59	56	7
21	46	15	34	31	50	19	44
62	**1**	**48**	13	52	**17**	**64**	3
11	60	55	40	25	10	5	54

25

21	46	43	56	9	22	19	44
36	**49**	30	7	58	35	**16**	29
27	**32**	63	60	5	2	**33**	38
40	23	18	13	52	47	42	25
57	10	15	20	45	50	55	8
6	**1**	34	37	28	31	**64**	59
61	**48**	3	26	39	62	**17**	4
12	51	54	41	24	11	14	53

27

41	12	23	28	37	42	53	24
14	**49**	30	39	26	35	**16**	51
57	**32**	63	18	47	2	**33**	8
6	61	54	45	20	11	4	59
27	36	43	52	13	22	29	38
40	**1**	34	15	50	31	**64**	25
19	**48**	3	58	7	62	**17**	46
56	21	10	5	60	55	44	9

28

43	28	21	14	51	44	37	22
26	**49**	40	63	2	25	**16**	39
55	**32**	9	18	47	56	**33**	10
30	45	36	59	6	29	20	35
3	52	61	38	27	4	13	62
42	**1**	24	15	50	41	**64**	23
7	**48**	57	34	31	8	**17**	58
54	5	12	19	46	53	60	11

29

batch of 8 (30-38, less 33)

55	8	27	4	61	38	57	10
14	**49**	52	63	2	13	**16**	51
43	**32**	21	18	47	44	**33**	22
30	37	58	41	24	7	28	35
3	63	39	56	9	26	5	62
54	**1**	12	15	50	53	**64**	11
19	**48**	45	34	31	20	**17**	46
42	25	6	29	36	59	40	23

30

55	40	59	36	29	6	25	10
14	**49**	52	63	2	13	**16**	51
43	**32**	21	18	47	44	**33**	22
30	5	26	9	56	39	60	35
3	28	7	24	41	58	37	62
54	**1**	12	15	50	53	**64**	11
19	**48**	45	34	31	20	**17**	46
42	57	38	61	4	27	8	23

31

61	6	11	24	41	54	59	4
14	**49**	52	63	2	13	**16**	51
35	**32**	21	18	47	44	**33**	30
8	55	58	37	28	7	10	57
25	42	39	60	5	26	23	40
62	**1**	12	15	50	53	**64**	3
19	**48**	45	34	31	20	**17**	46
36	27	22	9	56	43	38	29

32

61	2	53	8	57	12	63	4
16	51	10	59	6	55	14	**49**
37	**32**	43	18	47	22	**33**	28
26	41	20	35	30	45	24	39
7	56	13	62	3	52	9	58
60	**1**	54	15	50	11	**64**	5
17	46	23	38	27	42	19	**48**
36	31	44	25	40	21	34	29

34

43	2	21	14	51	44	63	22
24	**49**	40	37	28	25	**16**	41
59	**32**	7	18	47	58	**33**	6
4	45	62	55	10	3	20	61
29	52	35	42	23	30	13	36
38	**1**	26	15	50	39	**64**	27
9	**48**	57	60	5	8	**17**	56
54	31	12	19	46	53	34	11

35

39	2	21	14	51	44	63	26
24	**49**	42	37	28	23	**16**	41
59	**32**	11	18	47	54	**33**	6
4	45	62	57	8	3	20	61
29	52	35	40	25	30	13	36
38	**1**	22	15	50	43	**64**	27
9	**48**	55	60	5	10	**17**	56
58	31	12	19	46	53	34	7

36

38	57	28	63	2	37	8	27
51	**16**	13	10	55	52	**49**	14
22	41	44	47	18	21	24	43
3	**32**	61	26	39	4	**33**	62
30	**1**	36	7	58	29	**64**	35
11	56	53	50	15	12	9	54
46	**17**	20	23	42	45	**48**	19
59	40	5	34	31	60	25	6

37

52	43	14	63	2	51	22	13
7	**16**	57	36	29	8	**49**	58
60	55	24	27	38	41	10	5
3	**32**	47	12	53	18	**33**	62
30	**1**	50	21	44	15	**64**	35
37	42	9	6	59	56	23	28
26	**17**	40	61	4	25	**48**	39
45	54	19	34	31	46	11	20

38

continued

batch of 4 (39-42)

42	53	24	63	2	41	12	23
17	26	47	4	61	18	39	**48**
28	45	14	59	6	51	20	37
35	**32**	57	22	43	8	**33**	30
62	**1**	40	11	54	25	**64**	3
5	52	19	38	27	46	13	60
16	7	50	29	36	15	58	**49**
55	44	9	34	31	56	21	10

39

6	57	60	63	2	5	8	59
51	**16**	13	10	55	52	**49**	14
22	41	44	47	18	21	24	43
35	**32**	29	26	39	36	**33**	30
62	**1**	4	7	58	61	**64**	3
11	56	53	50	15	12	9	54
46	**17**	20	23	42	45	**48**	19
27	40	37	34	31	28	25	38

40

26	43	34	51	14	31	22	39
37	**16**	29	24	41	36	**49**	28
8	55	62	47	18	3	10	57
45	**32**	21	6	59	44	**33**	20
52	**1**	12	27	38	53	**64**	13
25	42	35	50	15	30	23	40
60	**17**	4	9	56	61	**48**	5
7	54	63	46	19	2	11	58

41

30	43	34	51	14	31	22	35
37	**16**	27	24	41	38	**49**	28
8	55	58	47	18	7	10	57
45	**32**	21	4	61	44	**33**	20
52	**1**	12	29	36	53	**64**	13
25	42	39	50	15	26	23	40
60	**17**	6	9	56	59	**48**	5
3	54	63	46	19	2	11	62

42

#40 on middle rank has arithmetic progressions CD3.

Magic Fiveleaper Tours continued

There remain 9 closed and 15 open tours. The eight closed tours that follow are cyclic. Tour #20, the ninth diagram below, is a closed tour but unlike the other eight is not cyclic. #6 and #33 have arithmetic progression with CD 3 in 32-33 line.

27	**48**	39	8	57	26	**17**	38
36	3	30	43	22	35	62	29
5	**32**	15	20	45	50	**33**	60
58	25	18	53	12	47	40	7
23	56	63	28	37	2	9	42
44	**49**	34	61	4	31	**16**	21
13	46	51	6	59	14	19	52
54	**1**	10	41	24	55	**64**	11

1

3	**48**	11	36	23	54	**17**	62
58	9	6	15	50	59	56	7
45	**32**	21	24	41	44	**33**	20
30	53	26	63	2	39	12	35
51	28	55	18	47	10	37	14
4	**49**	60	57	8	5	**16**	61
23	40	43	34	31	22	25	42
46	**1**	38	13	52	27	**64**	19

3

29	54	37	**16**	**49**	28	11	36
4	47	60	21	44	5	18	61
41	26	23	34	31	42	39	24
56	7	10	63	2	55	58	9
45	50	19	12	53	46	15	20
30	43	38	**17**	**48**	27	22	35
3	**32**	59	40	25	6	**33**	62
52	**1**	14	57	8	51	**64**	13

4

19	**48**	45	6	59	20	**17**	46
54	9	12	15	50	53	56	11
35	**32**	29	26	39	36	**33**	30
42	21	24	63	2	41	44	23
7	60	57	18	47	8	5	58
14	**49**	52	55	10	13	**16**	51
27	40	37	34	31	28	25	38
62	**1**	4	43	22	61	**64**	3

6

36	59	30	51	14	35	6	29
11	**32**	53	**48**	**17**	12	**33**	54
56	39	8	23	42	57	26	9
15	20	63	28	37	2	45	50
18	13	34	5	60	31	52	47
41	58	25	10	55	40	7	24
22	**1**	44	**49**	**16**	21	**64**	43
61	38	3	46	19	62	27	4

12

40	59	30	51	14	35	6	25
11	**32**	53	**48**	**17**	12	**33**	54
56	37	4	23	42	61	28	9
15	20	63	26	39	2	45	50
18	13	34	7	58	31	52	47
41	60	29	10	55	36	5	24
22	**1**	44	**49**	**16**	21	**64**	43
57	38	3	46	19	62	27	8

15

38	**17**	28	51	14	37	**48**	27
11	56	53	34	31	12	9	54
22	41	44	63	2	21	24	43
15	60	**49**	26	39	**16**	5	50
58	13	8	47	18	57	52	7
35	**32**	29	10	55	36	**33**	30
62	**1**	4	23	42	61	**64**	3
19	40	45	6	59	20	25	46

26

27	40	37	34	31	28	25	38
6	55	60	63	2	5	10	59
53	**16**	43	8	57	22	**49**	12
18	45	24	51	14	41	20	47
35	**32**	29	26	39	36	**33**	30
62	**1**	4	11	54	61	**64**	3
7	56	21	**48**	**17**	44	9	58
52	15	42	19	46	23	50	13

33

24	59	40	13	52	25	6	41
61	**32**	15	54	11	50	**33**	4
30	21	56	63	2	9	44	35
19	**48**	7	28	37	58	**17**	46
12	51	26	5	60	39	14	53
23	10	43	34	31	22	55	42
62	**1**	**16**	45	20	**49**	**64**	3
29	38	57	18	47	8	27	36

20

18	61	**48**	31	34	**17**	4	47
55	12	9	6	59	56	53	10
42	21	24	27	38	41	44	23
35	**16**	29	46	19	36	**49**	30
14	**33**	52	3	62	13	**32**	51
7	60	57	54	11	8	5	58
26	37	40	43	22	25	28	39
63	20	**1**	50	15	**64**	45	2

43

6	**33**	56	45	20	9	**32**	59
61	22	3	18	47	62	43	4
50	25	**16**	29	36	**49**	40	15
27	10	31	52	13	34	55	38
46	63	42	5	60	23	2	19
7	**48**	57	44	21	8	**17**	58
12	35	54	39	26	11	30	53
51	24	**1**	28	37	**64**	41	14

44

40	27	**48**	31	34	**17**	38	25
21	46	43	36	29	22	19	44
8	53	58	61	4	7	12	57
51	**16**	63	10	55	2	**49**	14
30	**33**	18	39	26	47	**32**	35
41	28	23	20	45	42	37	24
60	3	6	13	52	59	62	5
9	54	**1**	50	15	**64**	11	56

45

20	**33**	44	25	40	21	**32**	45
35	**16**	27	56	9	38	**49**	30
14	63	54	7	58	11	2	51
61	22	5	46	19	60	43	4
24	39	**48**	31	34	**17**	26	41
57	10	**1**	50	15	**64**	55	8
36	59	28	3	62	37	6	29
13	18	53	42	23	12	47	52

46

8	51	56	39	26	9	14	57
49	18	63	6	59	2	47	**16**
30	**33**	4	45	20	61	**32**	35
53	10	13	22	43	52	55	12
40	27	24	15	50	41	38	25
7	60	**1**	**48**	**17**	**64**	5	58
44	19	62	31	34	3	46	21
29	42	37	54	11	28	23	36

48

43	24	21	62	3	44	41	22
30	**33**	36	39	26	29	**32**	35
11	56	53	50	15	12	9	54
18	45	**48**	7	58	**17**	20	47
63	4	**1**	42	23	**64**	61	2
38	25	28	31	34	37	40	27
51	**16**	13	10	55	52	**49**	14
6	57	60	19	46	5	8	59

49

11	60	53	30	35	12	5	54
58	**33**	8	3	62	57	**32**	7
39	20	25	50	15	40	45	26
18	13	**48**	43	22	**17**	52	47
63	36	**1**	6	59	**64**	29	2
10	61	56	31	34	9	4	55
23	**16**	41	46	19	24	**49**	42
38	21	28	51	14	37	44	27

50

The last seven above are open tours (for #47 see diagonally magic tours above).

The final batch of 8 magic fiveleaper open tours (51-58):

3	28	63	58	7	2	37	62
30	**33**	56	5	60	9	**32**	35
45	54	39	50	15	26	11	20
22	41	**48**	13	52	**17**	24	43
59	8	**1**	36	29	**64**	57	6
4	27	10	31	34	55	38	61
51	**16**	25	44	21	40	**49**	14
46	53	18	23	42	47	12	19

51

39	24	63	20	45	2	41	26
30	**33**	36	43	22	29	**32**	35
11	58	53	50	15	12	7	54
60	3	**48**	9	56	**17**	62	5
21	46	**1**	40	25	**64**	19	44
38	23	28	31	34	37	42	27
51	**16**	13	6	59	52	**49**	14
10	57	18	61	4	47	8	55

52

53	10	63	6	59	2	55	12
18	**49**	4	57	8	61	**16**	47
43	24	21	14	51	44	41	22
30	**33**	36	39	26	29	**32**	35
7	60	**1**	54	11	**64**	5	58
52	9	62	**17**	**48**	3	56	13
19	50	45	42	23	20	15	46
38	25	28	31	34	37	40	27

53

9	18	57	22	43	8	47	56
54	**33**	12	45	20	53	**32**	11
35	62	29	50	15	36	3	30
60	7	**48**	39	26	**17**	58	5
21	42	**1**	10	55	**64**	23	44
14	19	52	31	34	13	46	51
27	**16**	37	4	61	28	**49**	38
40	63	24	59	6	41	2	25

54

13	18	57	22	43	8	47	52
54	**33**	10	45	20	55	**32**	11
35	62	25	50	15	40	3	30
60	7	**48**	37	28	**17**	58	5
21	42	**1**	12	53	**64**	23	44
14	19	56	31	34	9	46	51
27	**16**	39	4	61	26	**49**	38
36	63	24	59	6	41	2	29

55

36	55	28	41	24	37	10	29
61	20	39	8	57	26	45	4
2	**33**	14	59	6	51	**32**	63
23	**16**	43	30	35	22	**49**	42
18	25	46	11	54	19	40	47
7	56	27	62	3	38	9	58
60	21	50	**1**	**64**	15	44	5
53	34	13	**48**	**17**	52	31	12

56

29	54	35	60	5	30	11	36
2	**33**	62	23	42	3	**32**	63
57	26	9	**48**	**17**	56	39	8
6	45	50	37	28	15	20	59
43	4	31	12	53	34	61	22
24	55	40	**1**	**64**	25	10	41
47	**16**	19	58	7	46	**49**	18
52	27	14	21	44	51	38	13

57

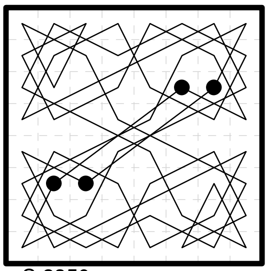
25	54	35	60	5	30	11	40
2	**33**	62	23	42	3	**32**	63
57	28	13	**48**	**17**	52	37	8
6	45	50	39	26	15	20	59
43	4	31	10	55	34	61	22
24	53	36	**1**	**64**	29	12	41
47	**16**	19	58	7	46	**49**	18
56	27	14	21	44	51	38	9

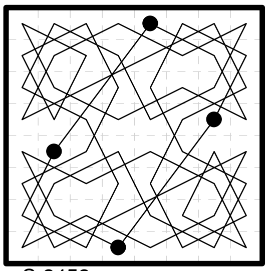
58

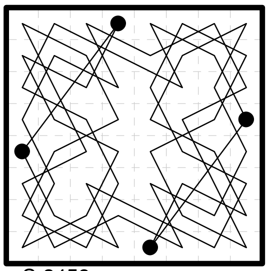
For more on fiveleapers, such as pseudotours, see the website.

{3,4}{1,2}

8×8 Board. Magic two-knight tours *Le Siècle* (1) ¶2350 (9 May 1884) rotary, diagonals 260±4 by Palamede (Ligondes). (2) ¶2452 (5 Sep 1884) rotary, diagonals 296 & 288 by Béligne. (3) ¶2458 (12 Sep 1884) rotary, diagonals 248 & 292 by Palamede (Ligondes).

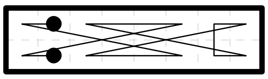

S 2350


S 2452

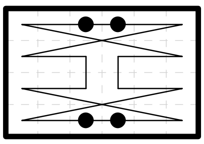

S 2458

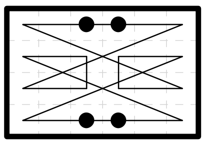
{1,5}+{0,1}

2×8 On this board there is one magic tour by {0,1}+{1,5}. **4×6** Biaxial (Jelliss 10 Aug 1991).



2	1	11	12	13	14	8	7
15	16	6	5	4	3	9	10





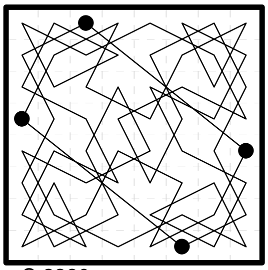
{2,5}+{0,1}

4×6 Board. Biaxial magic 4×6 (Jelliss 10 Aug 1991) diagram above right.

These were the last of 29 tours by two-pattern movers found on this board.

{4,5}+{1,2}

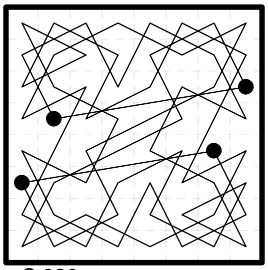
8×8 Board. Magic two-knight tour *Le Siècle* (1) ¶2380 (13 Jun 1884) rotary, diagonals 280 & 296 by M. A. F. (Feisthamel).



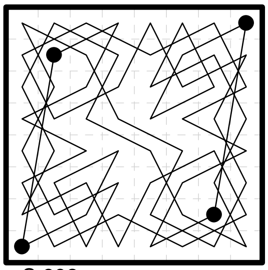
S 2380

{1,6}+{1,2}

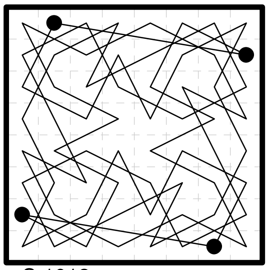
8×8 Magic two-knight tours *Le Siècle* (1) ¶826 (27 Jun 1879) rotary, diagonals 260±4. (2) ¶892 (12 Sep 1879) rotary, diagonals 296 & 188. (3) ¶1012 (30 Jan 1880) rotary, diagonals 260±20.



S 826



S 892



S 1012

{1,7}+{5,5} Root 50 Leaper

12×12 Kraitchik (1927) gave a closed √50-leaper tour of the cells of one colour on a 12×12 board, the smallest square board on which a tour is possible. It is equivalent to a fiveleaper tour on a serrated board formed by 45 degree rotation of the cells of one colour.

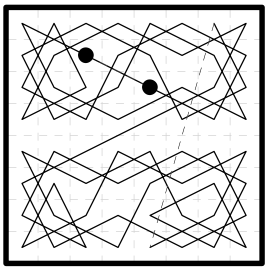
	8		52		64		22		12		10
23		69		39		19		43		35	
	42		36		6		24		50		40
47		49		61		33		57		37	
	72		66		16		46		30		4
63		31		13		9		53		65	
	20		44		2		70		26		14
7		25		51		41		21		11	
	68		56		38		18		60		34
17		59		29		5		67		55	
	48		54		62		32		58		28
1		71		27		15		45		3	

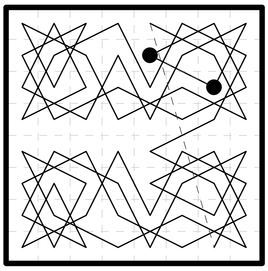
{2,7}+{1,2}

H. J. R. Murray (1917) in his article on knight tours on the half-board in the *British Chess Magazine* mentions six tours formed by joining two semi-magic 4×8 tours so that they add to 260 in the files and to 132 in the upper ranks and to 388 in the lower ranks (examples had previously been given by Brede and Wenzelides).

He writes: “The two which I diagram could on a cylindrical board in which the upper and lower edges were contiguous be renumbered to give a magic tour”. The linking knight move across the cylinder join is in effect a {2,7} leap.

63	8	53	12	57	4	49	14
54	11	64	5	52	13	58	3
7	62	9	56	1	60	15	50
10	55	6	61	16	51	2	59
27	38	17	44	29	36	23	46
18	41	28	37	24	45	32	35
39	26	43	20	33	30	47	22
42	19	40	25	48	21	34	31



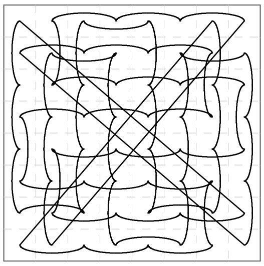


55	8	57	12	49	14	63	2
58	11	54	5	64	3	50	15
7	56	9	60	13	52	1	62
10	59	6	53	4	61	16	51
23	38	27	44	17	46	31	34
26	41	24	37	32	35	18	47
39	22	43	28	45	20	33	30
42	25	40	21	36	29	48	19

{6,7}+{0,2}

8×8 Board. (Jelliss 19 Dec 1990). This consists of simple boustrophedonal tours of the four dabbaba domains, joined together by crossboard links.

1	52	8	51	9	50	16	49
32	45	31	44	30	37	29	36
2	53	7	54	10	55	15	56
25	46	26	43	27	38	28	35
3	60	6	59	11	58	14	57
24	47	23	42	22	39	21	34
4	61	5	62	12	63	13	64
17	48	18	41	19	40	20	33

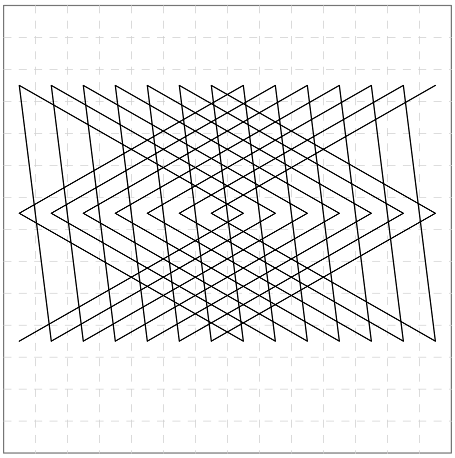


The {6,7} = √85 mover is a freeleaper on a sufficiently large board, but not on the 8×8 board, so this combination can be regarded as an ‘amphibian’ on this board.

{1,8}{4,7} Root 65 Leaper

14×14 Board. In *Chessics* (#24 1985 p.93) I gave a closed centrosymmetric tour of a 14×14 board. Only the first half of the tour is shown in numerical form here. The rest, 99 to 196, rotates the first part 180 degrees (diametrally opposite numbers differ by 98). Part of the route (1 to 42) is shown to illustrate the system of construction; the route 99 to 140 is similar.

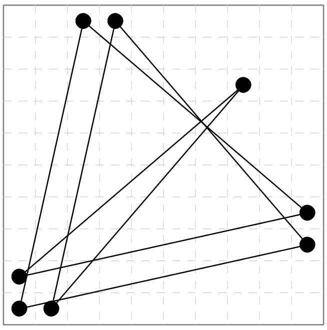
75		79		83		87		91		95		71	
58		62		70		68		44		50		54	
3	6	9	12	15	18	21	24	27	30	33	36	39	42
	90		94		98		74		78		82		86
	47		49		53		57		61		65		67
23	26	29	32	35	38	41	2	5	8	11	14	17	20
	76		80		84		88		92		96		72
	59		63		69		45		43		51		55
1	4	7	10	13	16	19	22	25	28	31	34	37	40
89		93		97		73		77		81		85	
46		48		52		56		60		64		66	



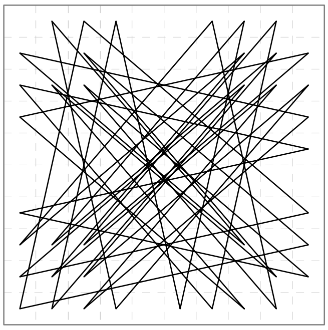
This is a combination of two freeleapers.

{2,9}{6,7} Root 85 Leaper

10×10 Board. In *PFCS* (Jun 1932 prob 404 a/b) W. E. Lester gave a shortest closed path of 8 moves, and E. J. van den Berg gave a longest closed path on 10×10 of 40 moves.



	20	7	12			29	14	37	
22		4					27	16	39
35	2	31					10	25	18
40									23
									34
33									8
28	15	38					21		5
9	26	17						3	32
6	11	24	19		13	36	1	30	

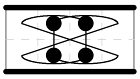


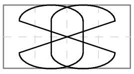
Three-Move Beasts

{1,3}+{0,1}{0,2}

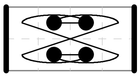
2×4 Board. Two semi-magic biaxial closed tours by WDC (Wazir + Dabbaba + Camel).

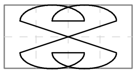
2	5	1	6
7	4	8	3





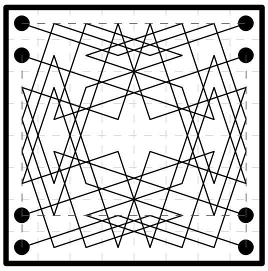
2	8	1	7
6	4	5	3





{1,3}+{0,6}{0,7}

8×8 Board. Magic four-camel tour from *Le Siècle* (13/14/21 May 1887) by 'Adsum a Saint-P' (Bouvier) symmetric about the vertical axis. Diagonal sums complementary (adding to 520). This is the earliest mention of a {1,3} mover that I know of.

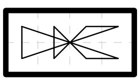


1	62	13	58	7	52	3	64
49	21	53	2	63	12	44	16
9	39	11	50	15	54	26	56
51	14	59	8	57	6	51	4
20	35	22	41	24	43	30	45
40	10	38	31	34	27	55	25
32	60	28	47	18	37	5	33
48	19	36	23	42	29	46	17

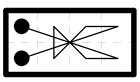
{1,3}+{1,1}{0,1}

2×4 Board. This WFC mover (camel + king) can make a closed 2×4 tour with axial symmetry that is a magic in ranks and files when numbered in two ways (these are permutes of the magic king tour) and also semi-magic in the ranks when numbered from other cells.

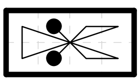
It also produces two semi-magic biaxial closed tours, similar to the camel+rook examples.



1	4	6	7
8	5	3	2



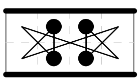
4	1	7	6
5	8	2	3

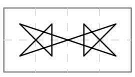


6	3	1	8
7	2	4	5

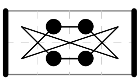
3	6	8	1
2	7	5	4

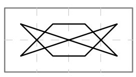
2	8	4	6
7	1	5	3





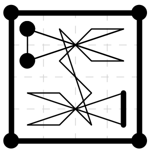
2	5	4	7
6	1	8	3





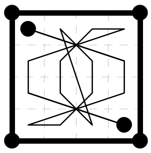
4×4 Board. Diamagic tours (#24/#216 and #112 in the Frénicle list). The first is reentrant, giving a centrosymmetric pattern when closed, the second is open and centrosymmetric.

1	4	14	15
16	13	3	2
7	6	12	9
10	11	5	8



16	13	3	2
1	4	14	15
10	11	5	8
7	6	12	9

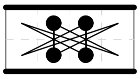
1	8	12	13
14	11	7	2
15	10	6	3
4	5	9	16

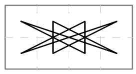


{1,3}{1,2}+{0,1}

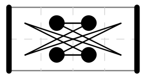
2×4 Board. This three-way WNC moving piece has two semi-magic biaxial closed tours.

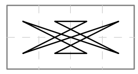
2	4	8	6
7	5	1	3





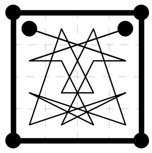
2	4	5	7
6	8	1	3



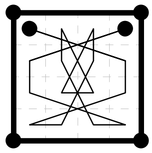


4×4 Board. Diamagic WNC tours (#81 and #138 in the Frénicle list). Both axisymmetric.

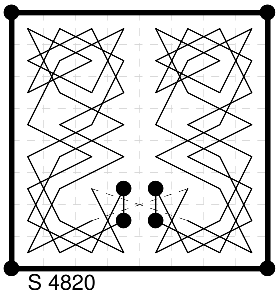
1	7	10	16
15	14	3	2
12	9	8	5
6	4	13	11



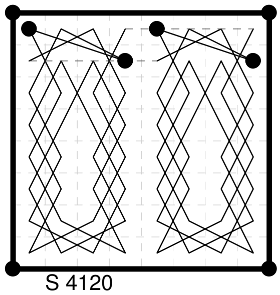
1	10	7	16
15	11	6	2
14	8	9	3
4	5	12	13



8×8 Board. Diamagic four-knight tour *Le Siècle* ¶4820 (15 Jan 1892) by Adsum (Bouvier).



S 4820



S 4120

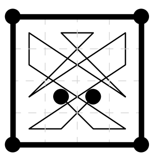
1	63	15	49	32	34	18	48
16	56	8	64	17	41	25	33
62	2	50	14	35	31	47	19
55	9	57	7	42	24	40	26
3	61	13	51	30	36	20	46
10	54	6	58	23	43	27	39
60	4	52	12	37	29	45	21
53	11	59	5	44	22	38	28

{1,3}{1,2}+{0,4}

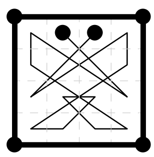
(Above) Diamagic four-knight tour from *Le Siècle* ¶4120 (24 Jan 1890) by Adsum (Bouvier).

{2,3}+{1,1}{0,1}

4×4 Board. Diamagic axisymmetric tours from the Frénicle list. Cyclic renumbering.



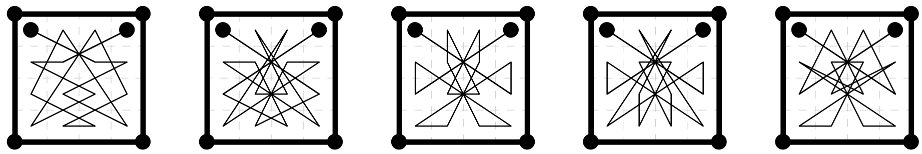
428 WFZ



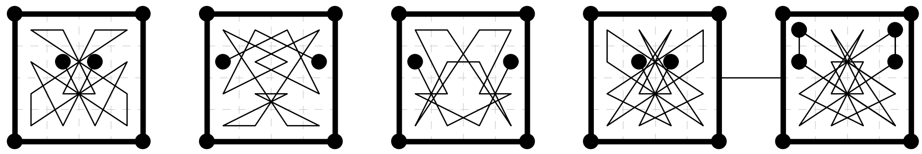
594 WFZ

{2,3}{1,2}+{0,1}

4×4 Board..Fifteen diamagic tours fom the Frénicle list with three move types WNZ.

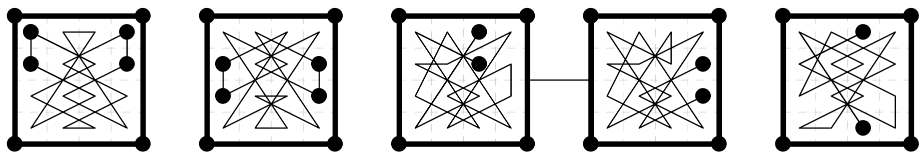


52 WNZ   131 WNZ   134 WNZ   166 WNZ   186 WNZ



631 WNZ   764 WNZ   793 WNZ   627 WNZ   205/373 WNZ

Nine are axisymmetric, but the ninth has a second asymmetric magic numbering.

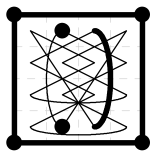


180/551 WNZ   538/552 WNZ   271/303 WNZ   486/510 WNZ   272/644 WNZ

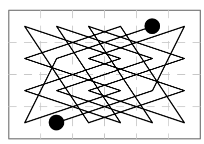
The others are asymmetric, two being axisymmetric when closed. Frénicle numbers are shown.

{2,3}{1,2}+{0,3}

4×4 board. Diamagic tour with three move types TNZ, axisymmetric when closed.



349/701 NTZ

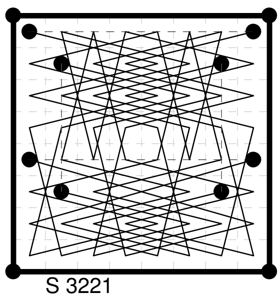


{2,3}{1,3}{1,2}

4×6 board. Bison+Knight with {3,3} closure, by E. Huber Stockar (1935). Diagram above.

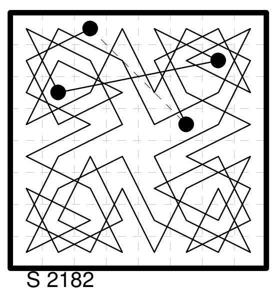
{1,4}+{0,4}{0,7}

8×8 Diamagic four-giraffe tour from *Le Siècle* ¶3221 (5/12 Mar 1887) by ‘X a Belfort’ (Reuss) symmetric about the vertical axis.



S 3221

1	25	9	23	42	56	40	64
43	49	39	63	2	26	16	22
3	27	15	21	44	50	38	62
45	51	37	61	4	28	14	20
32	8	24	10	55	41	57	33
54	48	58	34	31	7	17	11
30	6	18	12	53	47	59	35
52	46	60	36	29	5	19	13



S 2182

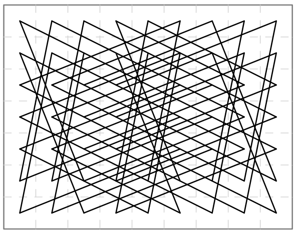
{1,5}{1,2}+{3,3}

8×8 Board. Magic two-knight tour from *Le Siecle* ¶2182 (26 Oct 1883) by Palaméde (Ligondès). The symmetry of this is described as ‘pivotante quadrangulaire’ diagonals 368 & 144.

{2,5}{1,5}{2,4}

7×9 Board. E. Huber-Stockar (*FCR* Apr 1944 p.85 and Jun 1944 p.94, prob 5935). Symmetric open or closed tour with moves in four given directions. The centre cell has the middle number 32, and diametrally opposite pairs add to 64. The {2,4} closure move 1-63 passes over the centre cell, the moves through the centre 31-32-33 overlapping in the same line.

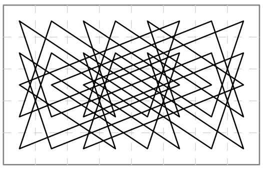
51	23	58	9	44	16	30	2	37
33	26	61	54	47	19	12	5	40
15	29	1	36	50	22	57	8	43
18	11	4	39	32	25	60	53	46
21	56	7	42	14	28	63	35	49
24	59	52	45	17	10	3	38	31
27	62	34	48	20	55	6	41	13



{2,5}{2,3}{1,3}

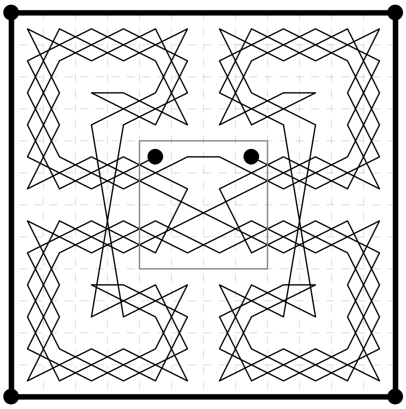
5×8 Board. E. Huber-Stockar (*Fairy Chess Review*, vol.5, Feb 1943 p.28 and Apr 1943 p.37, prob 5447). A symmetric closed tour moves in four given directions. Bergholtian type since b2-g4 and c4-f2 cross at the centre point. Renumbered from b2 to g4 so that diametrally opposite pairs all add to 41 (in the source the tour was numbered a5 to d3, giving two different diametral sums 5 and 45).

2	37	12	7	22	17	32	27
10	15	20	25	30	35	40	5
18	33	28	3	38	13	8	23
36	1	6	11	16	21	26	31
14	9	24	19	34	29	4	39



{1,6}{1,2}+{0,1}

12×12 diagonally magic tour (Jelliss 2018) based on 4×4 Emperor diagonally magic tour by variant of Joachim Brügge method (braids forming G shape instead of S).

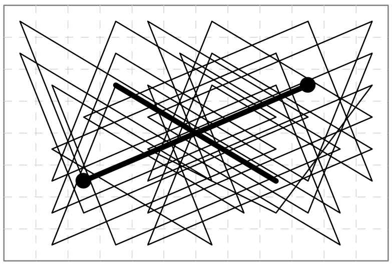


40	77	24	31	38	79	66	7	14	21	68	5
23	32	39	78	25	30	15	20	67	6	13	22
76	41	28	27	80	37	8	65	18	17	4	69
33	22	81	36	29	26	19	16	9	64	23	12
42	75	20	35	44	73	72	1	10	25	70	3
21	34	43	74	19	36	09	26	71	2	11	24
88	29	38	53	90	27	18	55	92	07	16	57
39	52	89	28	37	54	91	08	17	56	93	06
30	87	46	45	34	83	62	11	00	99	58	15
51	40	35	82	47	44	01	98	63	10	05	94
86	31	42	49	84	33	12	61	96	03	14	59
41	50	85	32	43	48	97	02	13	60	95	04

{3,7}{3,5}{2,5}

8×12 Board. E. Huber-Stockar (*FCR* Apr 1945 p.132 and Jun 1945 p.142, prob 6380). Moves in four given directions. The diagram shows the moves 1-48 of the first half of the tour, and the middle {3,5} move 48-49 in bold which passes through the central point (as does the closure {3,7} move 96-1). The other half is diametrally related to the first half. Diametrally opposite nmbers add to 97.

2	58	82	42	34	90	18	74	66	26	50	10
37	93	85	45	69	29	21	77	5	61	53	13
72	32	88	48	8	64	24	80	40	96	56	16
75	67	27	51	11	3	59	83	43	35	91	19
78	6	62	54	14	38	94	86	46	70	30	22
81	41	1	57	17	73	33	89	49	9	65	25
84	44	36	92	20	76	68	28	52	12	4	60
87	47	71	31	23	79	7	63	55	15	39	95



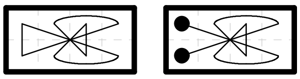
===

Four-Move Beasts

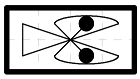
===

{1,3}+{1,1{0,1}{0,2}

2×4 Board. This camel + restricted queen has two magic tours 2×4, that are alternative numberings of the same closed tour.



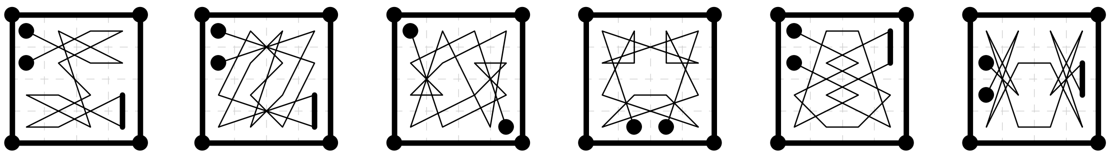
1	6	4	7
8	3	5	2



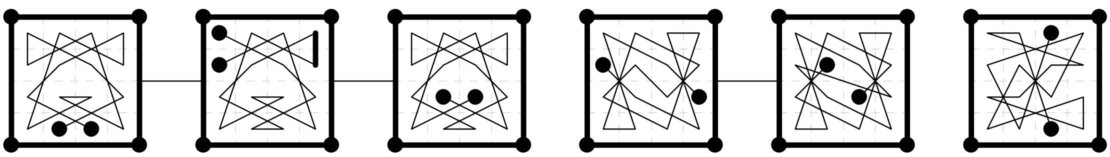
4	7	1	6
5	2	8	3

{1,3}{1,2}+{1,1}{0,1}

4×4 Diamagic WFNC tours. The Frénicle numbers are shown.



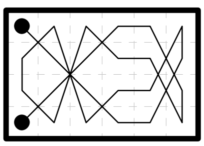
30 WFNC 60 WFNC 126 WFNC 381 WFNC 197 WFNC 414/416 WFNC



636 WFNC 147 WFNC 296 WFNC 478 WFNC 632 WFNC 442/588 WFNC

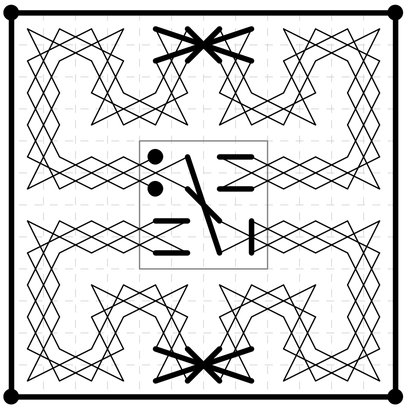
4×6 Magic tour (Jelliss) derived from a king tour.Filesum 50. Ranksum 75.

1	11	15	21	20	7
12	2	22	16	17	6
13	23	3	9	8	19
24	14	10	4	5	18



This with 1-2 and 3-4 ranks transposed has 5 move types. Other permutes have 5 or 6 types.

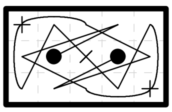
12 ×12 (Jelliss 2018) Brügge method tour using a P or R pattern braid based on Frenicle #24.



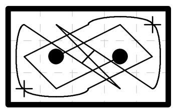
5	32	39	14	9	28	18	35	24	13	22	31
40	13	6	31	36	17	27	10	21	32	23	14
33	4	15	38	29	8	34	19	12	25	30	23
12	41	30	7	16	37	11	26	33	20	15	22
3	34	43	10	1	36	26	27	20	17	24	29
42	11	2	35	44	09	19	18	25	28	21	16
57	52	89	92	55	54	08	73	38	71	06	75
88	93	56	53	90	91	37	72	07	74	39	70
51	58	97	84	47	62	80	01	66	43	76	05
94	87	48	61	98	83	65	44	79	02	69	40
59	50	85	96	63	46	00	81	42	67	04	77
86	95	60	49	82	99	45	64	03	78	41	68

{1,3}{1,2}+{1,1}{0,2}and **{2,3}{1,2}+{1,1}**{0,2}

3×5 board. In *Chessics* #26 (1986) I reported finding 39 basic magic numberings for the 3×5 board. These occur in 19 pairs that are reversals of each other, and one which is its own reversal. This special case allows for four symmetric magic tours. Two of these, shown here, use four move types, DFNC or DFNZ. The magic constants are 24 and 40.



5	7	4	14	10
13	15	8	1	3
6	2	12	9	11

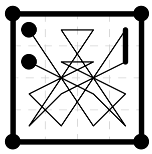


6	9	12	2	11
13	1	8	15	3
5	14	4	7	10

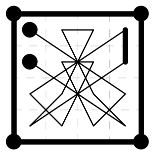
This work was anticipated by Charles Planck who mentions the figure of 39 solutions, and gives one magic tour using six move types in Andrews (1917, Fig.454).

{2,3}{1,2}+{1,1}{0,1}

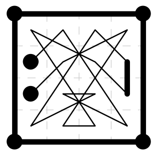
4×4 Board. Diamagic tours with four move types WFNZ, in different directions.



195 WFNZ



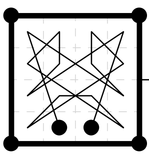
196 WFNZ



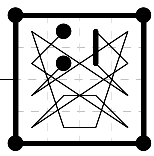
517 WFNZ

{2,3}{1,3}+{1,1}{0,1}

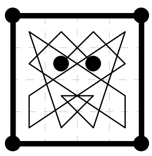
4×4 Board. Bison + King WGCZ. Diamagic tours 4×4.



370 WFCZ



525/531 WFCZ

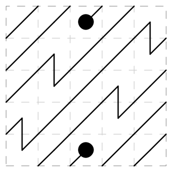


629 WFCZ

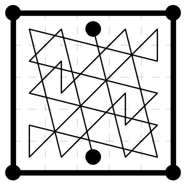
{1,4}+{1,1}{0,1}{0,4}

5×5 On this board regarded as a torus there are 24 distinct magic tours of the step-sidestep type. Here ‘distinct’ means that we do not count rotations and reflections of the magic square as different. The diagrams shown here are oriented according to the Frénicle rule and with the middle number, 13, in the centre cell. The magic sum is 5×13 = 65. There are two with three types of move, nine with four types. The other thirteen use 5 or 6 move types.

Step {1,1} and sidestep {0,1} on the torus plus {1,4} on the normal board [plus {0,4} if the closure move is counted]. This tour is Fig. 2 in Andrews.

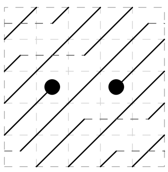


9	2	25	18	11
3	21	19	12	10
22	20	13	6	4
16	14	7	5	23
15	8	1	24	17

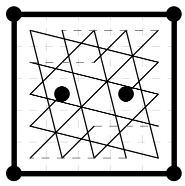


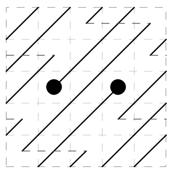
{1,4}+{1,1}{0,2}{0,3}

5×5 Step {1,1} and sidestep {0,2} on the torus plus {1,4} and {0,3} on the normal board. This tour is Fig.41 in Andrews (also in Carus p.113). The second has the same component moves.

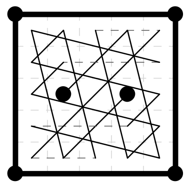


3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23





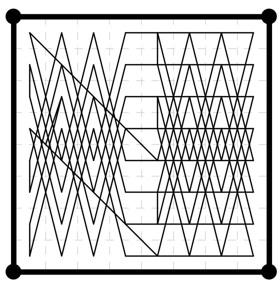
4	17	10	23	11
18	6	24	12	5
7	25	13	1	19
21	14	2	20	8
15	3	16	9	22



{1,4}+{4,4}**{0,1}**{0,4)

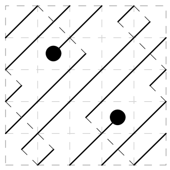
8×8 The following remarkable magic square is due to **H. J. Kesson** (**‘Ursus’**) ‘Caissan Magic Squares’ *The Queen*, The Lady's Newspaper, London, Aug, Sep, Oct 1881. It uses Giraffe moves combined with lateral and diagonal moves. Besides adding to the magic constant in ranks and files (rook moves) it is also magic in the all 16 diagonals, and also magic in the knight and camel lines. As noted in the Magic Squares section (⌘ 1) it has the same numbers in the ranks as the ‘Intermediate’ magic square of the semi-natural type. This tour is also in Andrews (1917, Fig.262)

1	58	3	60	8	63	6	61
16	55	14	53	9	50	11	52
17	42	19	44	24	47	22	45
32	39	30	37	25	34	27	36
57	2	59	4	64	7	62	5
56	15	54	13	49	10	51	12
41	18	43	20	48	23	46	21
40	31	38	29	33	26	35	28

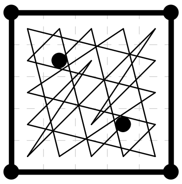


{1,4}{2,3}+{1,1}{2,2}

5×5 Board. Step {1,1} and sidestep {2,2} on the torus becoming {1,1}+{1,4}+{2,3} on the normal board [plus {2,2} when the closure move is counted].



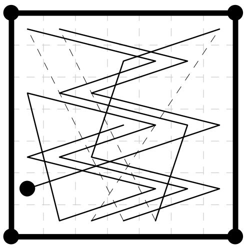
7	3	24	20	11
4	25	16	12	8
21	17	13	9	5
18	14	10	1	22
15	6	2	23	19



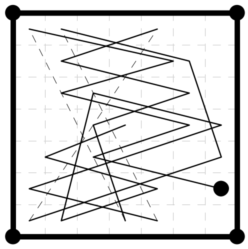
{1,4}{4,6}**{1,3}**{3,6}

7×7 Board. Four step-sidestep tours using camel moves for both steps. A camel move across a side or corner of the 7x7 torus is equivalent to giraffe move, double zebra or triple kight move. Only the half-tour is shown. The other half is the same rotated 180 degrees.

11	6	43	38	33	28	16
37	32	27	15	10	5	49
21	9	4	48	36	31	26
47	42	30	25	20	8	3
24	19	14	2	46	41	29
1	45	40	35	23	18	13
34	22	17	12	7	44	39

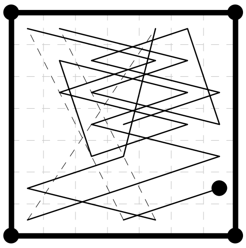


6	16	33	43	11	28	38
49	10	27	37	5	15	32
36	4	21	31	48	9	26
30	47	8	25	42	3	20
24	41	2	19	29	46	14
18	35	45	13	23	40	1
12	22	39	7	17	34	44

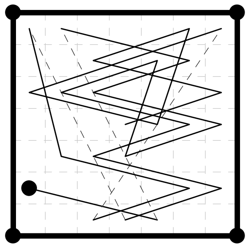


Features of interest are the triangles such as those formed of knight, camel and giraffe moves.

3	18	33	48	14	22	37
49	8	23	38	4	19	34
39	5	20	35	43	9	24
29	44	10	25	40	6	21
26	41	7	15	30	45	11
16	31	46	12	27	42	1
13	28	36	2	17	32	47

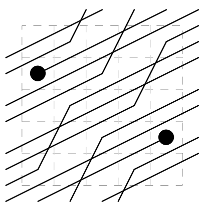


14	3	48	37	33	22	18
38	34	23	19	8	4	49
20	9	5	43	39	35	24
44	40	29	25	21	10	6
26	15	11	7	45	41	30
1	46	42	31	27	16	12
32	28	17	13	2	47	36

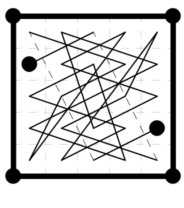


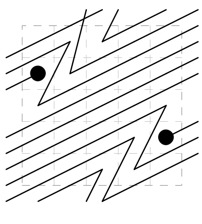
{3,4}{1,3}{1,2}{2,4}

5×5. Step and sidestep {1,2} on the torus. [Figs.20 & 19 in Andrews.] After entering the first 5 numbers, say in the (1, 2) direction, there is a choice of directions for the knight sidestep. This cannot be forwards (1, 2) or backwards (−1, −2) since these lead to cells already used, also they cannot be the other 'vertical' moves (−1, 2), (1, −2) since the first of these does not alter the file and the second does not alter the rank on which the next sequence of (1, 2) moves begins, so if the step is vertical the sidestep must be horizontal. This is a general rule, applicable to other leapers and larger boards.

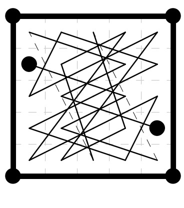


3	11	24	7	20
25	8	16	4	12
17	5	13	21	9
14	22	10	18	1
6	19	2	15	23





4	15	21	7	18
25	6	17	3	14
16	2	13	24	10
12	23	9	20	1
8	19	5	11	22

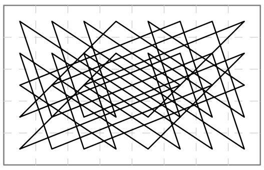


In the 5×5 case the two horizontal sidesteps at right angles to the step, that is (2, −1) and (−2, 1) are also blocked, since they lead to cells already used; however this is not a general rule.

{2,5}{2,3}{1,3}+{3,3}

5×8 Board. E. Huber-Stockar (*FCR* April-June 1943 problem 5500). Symmetric closed 5×8 tour by this mover, in four given directions. The original numbering is retained. The symmetry is Bergholtian and the diametrally opposite numbers add to 41.

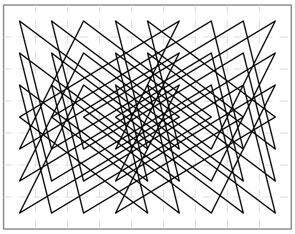
19	24	9	14	39	4	29	34
11	36	1	6	31	16	21	26
3	28	33	18	23	8	13	38
15	20	25	10	35	40	5	30
7	12	37	2	27	32	17	22



{3,5}{3,4}{1,4}{2,4}

7×9 Board. E. Huber-Stockar (*FCR* Aug 1943 p.53 and Oct 1943 p.63 prob 5659). Closed symmetric tour 7×9 with four types of move in four directions.

33	47	61	12	26	40	54	5	19
14	28	21	56	7	63	35	49	42
58	9	44	37	51	23	16	30	2
39	53	4	18	32	46	60	11	25
62	34	48	41	13	27	20	55	6
22	15	29	1	57	8	43	36	50
45	59	10	24	38	52	3	17	31



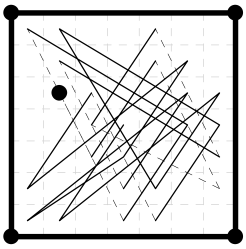
1	15	29	43	57	8	22	36	50
45	59	52	24	38	31	3	17	10
26	40	12	5	19	54	47	61	33
7	21	35	49	63	14	28	42	56
30	2	16	9	44	58	51	23	37
53	46	60	32	25	39	11	4	18
13	27	41	55	6	20	34	48	62

In the original numbering (shown on the right) the centre cell is 63 and diametrally opposite pairs add to 63. I prefer the numbering on the left where the middle number 32 is in the centre and the 1-63 move passes over the centre. In this diametrally opposite numbers add to 64.

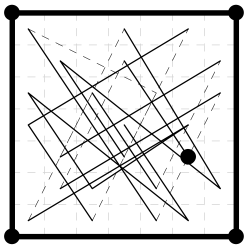
{4,5}{3,5}{2,3}{2,4}

7×7 Board. Zebra. Only the half-tour is shown. The other half is the same rotated 180 degrees.

4	21	31	48	9	26	36
43	11	28	38	6	16	33
40	1	18	35	45	13	23
30	47	8	25	42	3	20
27	37	5	15	32	49	10
17	34	44	12	22	39	7
14	24	41	2	19	29	46

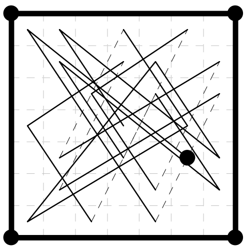


7	26	45	15	41	11	30
36	13	32	2	28	47	17
23	49	19	38	8	14	4
10	29	6	25	44	21	40
46	16	42	12	31	1	27
33	3	22	48	18	37	14
20	39	9	35	5	24	43

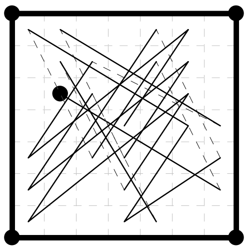


There are some very sharp bends! Pairs of the {2,4} moves overlap in the first three.

2	24	46	19	41	14	29
39	12	34	7	22	44	17
27	49	15	37	10	32	5
8	30	3	25	47	20	42
45	18	40	13	35	1	23
33	6	28	43	16	38	11
21	36	9	31	4	26	48



5	20	35	43	9	24	39
48	14	22	37	3	18	33
42	1	16	31	46	12	27
29	44	10	25	40	6	21
23	38	4	19	34	49	8
17	32	47	13	28	36	2
11	26	41	7	15	30	45

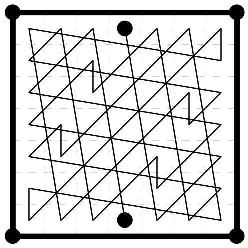


Magic Sum 175.

{1,6}+{1,1}{0,1}{0,6}

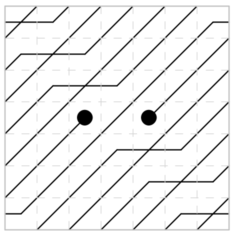
7×7 Step {1,1} sidestep {0,1} on torus, the {1,1} becoming {1,6} on normal board where the move crosses the edges. plus {0,6} closure. By the Loubère plan (see ⌘ 1). [Andrews Fig.3].

20	11	2	49	40	31	22
12	3	43	41	32	23	21
4	44	42	33	24	15	13
45	36	34	25	16	14	5
37	35	26	17	8	6	46
29	27	18	9	7	47	38
28	19	10	1	48	39	30

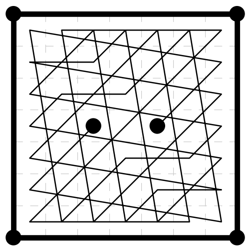


{1,6}+{1,1}{0,2}{0,5}

7×7 This magic square is formed by the method decribed by Manuel Moschopoulos (1315), [Andrews Fig.4]. This is a particular case of the step-sidestep method of construction (see ⌘ 1).



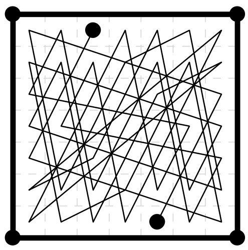
4	29	12	37	20	45	28
35	11	36	19	44	27	3
10	42	18	43	26	2	34
41	17	49	25	1	33	9
16	48	24	7	32	8	40
47	23	6	31	14	39	15
22	5	30	13	38	21	46



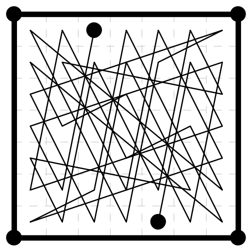
{5,6}{2,6}**{1,5}{1,2}**

7×7. Knight {1,2} on torus plus {1,5}+{2,6}+{5,6} on normal board. {2,6} is a camelrider move.

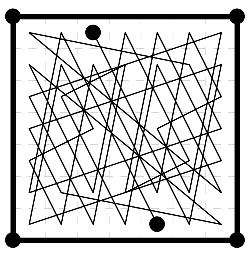
4	23	49	19	38	8	34
41	11	30	7	26	45	15
22	48	18	37	14	33	3
10	29	6	25	44	21	40
47	17	36	13	32	2	28
35	5	24	43	20	39	9
16	42	12	31	1	27	46



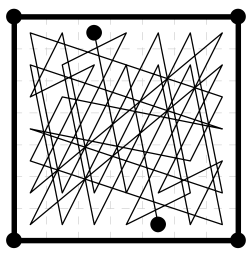
10	5	49	37	32	27	15
41	29	24	19	14	2	46
16	11	6	43	38	33	28
47	42	30	25	20	8	3
22	17	12	7	44	39	34
4	48	36	31	26	21	9
35	23	18	13	1	45	40



8	4	49	38	34	23	19
41	30	26	15	11	7	45
18	14	3	48	37	33	22
44	40	29	25	21	10	6
28	17	13	2	47	36	32
5	43	39	35	24	20	9
31	27	16	12	1	46	42



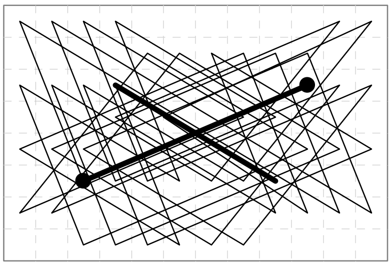
5	27	49	15	37	10	32
41	14	29	2	24	46	19
28	43	16	38	11	33	6
8	30	3	25	47	20	42
44	17	39	12	34	7	22
31	4	26	48	21	36	9
18	40	13	35	1	23	45



{3,7}{4,5}{3,5}{2,5}

8×12 Board. This is our only example of a Quadruple Beast. Tour on 8×12 with moves in four directions. E. Huber-Stockar (*FCR* vol.5 Dec 1944 p,117 and Feb 1945 p.126, prob 6229).

2	10	18	26	82	90	50	58	66	74	34	42
85	93	53	61	21	29	37	45	5	13	69	77
24	32	40	48	56	64	72	80	88	96	8	16
59	67	75	35	43	3	11	19	27	83	91	51
46	6	14	70	78	86	94	54	62	22	30	38
81	89	1	9	17	25	33	41	49	57	65	73
20	28	84	92	52	60	68	76	36	44	4	12
55	63	23	31	39	47	7	15	71	79	87	95



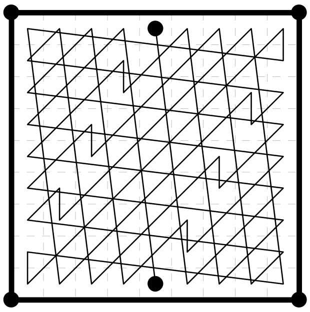
Half tour shown and cross-centre links 48-49 and 96-1.The rank sums are 612, 576, 540, 600, 564, 624, 588, 552 which form an arithmetic progression (cd 12). The rank constant is 582. The file sums are 404, 388, 468, 404, 388, 372, 404, 388, 372, 308, 388 372 (file constant 388). The numbers 308, 372, 388, 404, 468 have differences 64, 16, 16, 64.

There is a large tour 11×23 of four moves {1,4} {3,5} {3,8} {2,9} in Huber Stockar's original paper (1935), but too large to reproduce adequately here.

{1,8}+{1,1}{0,1}{0,8}

9×9 Diamagic tour is by the de la Loubère method. Andrews Fig,97. Magic constant 369.

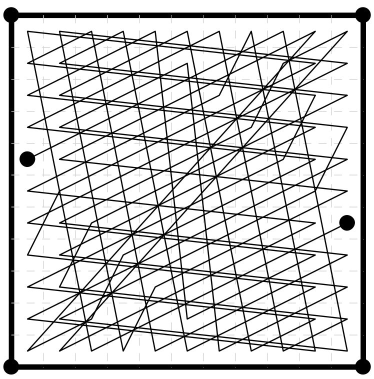
35	24	13	2	81	70	59	48	37
25	14	3	73	71	60	49	38	36
15	4	74	72	61	50	39	28	26
5	75	64	62	51	40	29	27	16
76	65	63	52	41	30	19	17	6
66	55	53	42	31	20	18	7	77
56	54	43	32	21	10	8	78	67
46	44	33	22	11	9	79	68	57
45	34	23	12	1	80	69	58	47



{9,10}{2,10}{1,9}{1,2}

11×11 board. Four magic knight tours by the step-sidestep torus method are again possible.

6	46	97	16	56	07	26	77	17	36	87
98	17	57	08	27	67	18	37	88	7	47
58	09	28	68	19	38	78	8	48	99	18
29	69	20	39	79	9	49	89	19	59	10
21	40	80	10	50	90	20	60	00	30	70
81	11	51	91	21	61	01	31	71	11	41
52	92	22	62	02	32	72	12	42	82	1
12	63	03	33	73	13	43	83	2	53	93
04	23	74	14	44	84	3	54	94	13	64
75	15	34	85	4	55	95	14	65	05	24
35	86	5	45	96	15	66	06	25	76	16



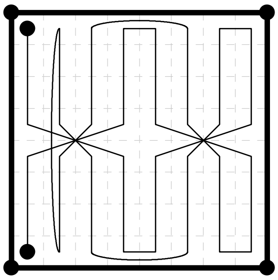
As shown by one example here. Knight {1,2} on torus plus {1,9}{2,10}{9,10} on normal board. The {2,10} is a {1,5}-rider move.

Five-Move Beasts

{1,3}+{1,1}{0,1}{0,3}{0,7}

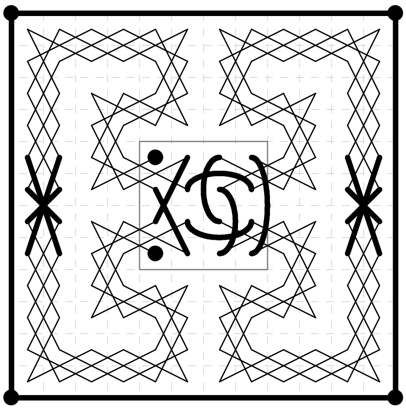
8×8 Board. A diamagic camel + four queen-move types:by L. S. Frierson in Andrews Fig.270.

1	32	40	57	56	41	17	16
2	31	39	58	55	42	18	15
3	30	38	59	54	43	19	14
4	29	37	60	53	44	20	13
61	36	28	5	12	21	45	52
62	35	27	6	11	22	46	51
63	34	26	7	10	23	47	50
64	33	25	8	9	24	48	49



{1,3}{1,2}+{1,1}{0,2}{0,3}

12 ×12 (Jelliss 2018) Brügge method tour using P or R pattern braid, using Frenicle #6.

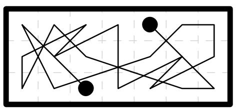


7	33	24	14	5	31	86	68	49	03	84	66
25	15	6	32	23	13	50	04	85	67	48	02
34	8	11	21	30	4	69	87	06	52	65	83
16	26	29	3	12	22	05	51	70	88	01	47
9	35	20	10	1	27	90	72	53	07	82	64
27	17	2	28	19	09	54	08	89	71	46	00
18	28	43	17	26	36	91	37	56	74	99	45
36	10	25	35	44	18	55	73	92	38	63	81
29	19	16	42	33	23	40	94	75	57	44	98
11	37	34	24	15	41	76	58	39	93	80	62
20	30	39	13	22	32	95	41	60	78	97	43
38	12	21	31	40	14	59	77	96	42	61	79

{1,3}{1,2}+{1,1}{2,2}**{0,1}**

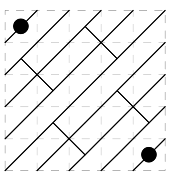
3×7 Board. Magic rectangle tour using five move types WFNAC.

20	5	8	10	1	17	16
7	9	4	11	18	13	15
6	19	21	12	14	3	2

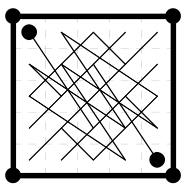


{2,3}+{1,1}{2,2}{3,3}{4,4}

5×5 Board. This is formed by {2,2} moves on the torus with {1,1} sidestep, which become {2,3}+{3,3} on the normal board. {4,4} closure. [Andrews Fig.16 and 23.] Diagonal moves overlap.

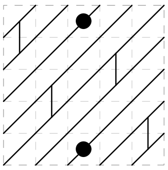


1	18	10	22	14
20	7	24	11	3
9	21	13	5	17
23	15	2	19	6
12	4	16	8	25

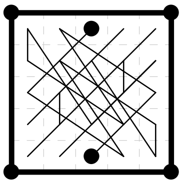


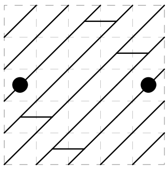
{2,3}+{2,2}{3,3}**{0,1}**{0,4}

5×5. Tours formed by the step-sidestep method using {2,2} with {0,1} sidestep, becoming {2,3} {3,3} on the normal board. {0,4} for closure. The second is by H. A. Sayles in Andrews Fig.408.

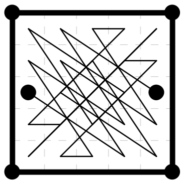


6	3	25	17	14
5	22	19	11	8
24	16	13	10	2
18	15	7	4	21
12	9	1	23	20



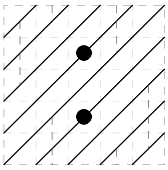


8	2	21	20	14
4	23	17	11	10
25	19	13	7	1
16	15	9	3	22
12	6	5	24	18

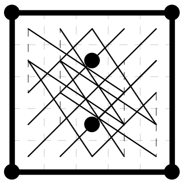


{2,3}+{2,2}{3,3}{0,2}{0,3}

5×5. The {0,2}-mover is a dabbaba, and the {2,2} an alfil, so together they give an **alibaba** tour!

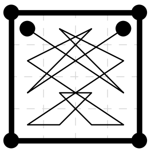


5	16	7	23	14
18	9	25	11	2
6	22	13	4	20
24	15	1	17	8
12	3	19	10	21

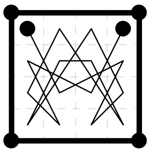


{2,3}{1,2}+{1,1}{0,1}{0,3}

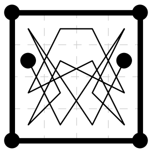
4×4 Board. Diamagic tours with five move types WFNZ and closure {0,3}.



14 WFNZ



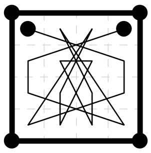
154 WFNZ



521 WFNZ

{2,3}{1,3}{1,2}+{0,1}{0,3}

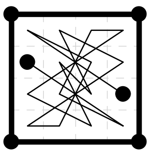
4×4 Board. Bison + Knight + Wazir and closure {0,3}. WNCZ. Diamagic tour 4×4.



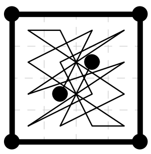
167 WNCZ

{2,3}{1,3}{1,2}+{1,1}{0,1}

4×4 Board. **Wizard**. Diamagic tours with five move types WFNCZ. These are all the possible moves of a Wizard on this board using all its powers.



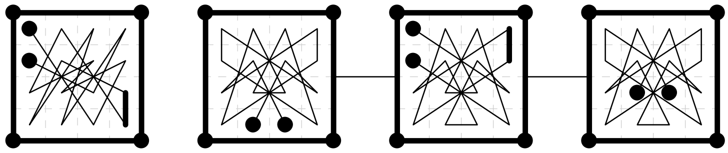
299 WFNZ



637 WNCZ

In the first the closure is the Camel move of the Bison, in the second it is the Fers move.

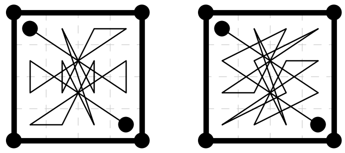
For full definitionof the Wizard see the end pages.



84 WFNCZ 630 WFNCZ 163 WFNCZ 475 WFNCZ

{2,3}{1,3}{1,2}+{3,3}{0,1}

4×4 Board. Diamagic tours with four move types WNCZ and closure {3,3}.

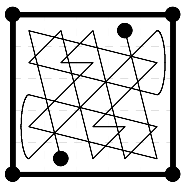


113 WNCZ 122 WNCZ

{1,4}{2,4}+{1,1}{0,1}{0,2}

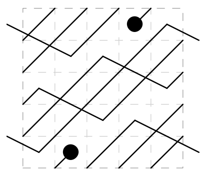
5×5 This tour is by L. S. Frierson in Andrews Fig.440. This can be regarded as a {1,1} torus path with two alternative sidesteps {0,1} and {0,2}. Closure {2,4} double knight move.

2	8	19	25	11
9	20	21	12	3
16	22	13	4	10
23	14	5	6	17
15	1	7	18	24

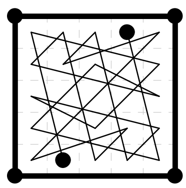


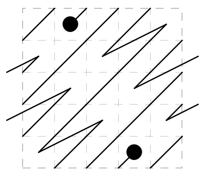
{1,4}{1,3}{1,2}{2,4}+{1,1}

5×5 Step {1,1} and sidestep {1,2} on torus plus {1,3}{1,4} on normal board and {2,4} closure. [Andrews Figs.18 and 17] . The {1,4}{1,3} is now apparently known as a **Zebu**.

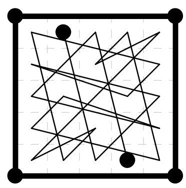


2	18	9	25	11
19	10	21	12	3
6	22	13	4	20
23	14	5	16	7
15	1	17	8	24





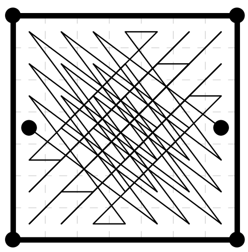
8	1	24	17	15
5	23	16	14	7
22	20	13	6	4
19	12	10	3	21
11	9	2	25	18



{3,4}+{3,3}{4,4}{0,1}{0,6}

7×7. Step {3,3} sidestep {0,1} on torus plus {3,4}{4,4} on normal board (antelope+queen).

18	10	2	43	42	34	26
12	4	45	37	29	28	20
6	47	39	31	23	15	14
49	41	33	25	17	9	1
36	35	27	19	11	3	44
30	22	21	13	5	46	38
24	16	8	7	48	40	32

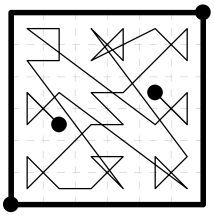


By H. A. Sayles in an article on Lozenge Magic Squares reproduced in Andrews (1917, Fig.405).

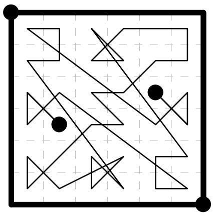
{3,4}{1,3}{1,2}+{1,1}{0,1}

6×6 Board. A magic square of the type shown here, formed of nine 2×2 king paths connected in the pattern of a 3×3 magic square, appears in Falkener (1892, p.294). His version is magic in the ranks and files but adds to 111 ± 3 in the diagonals. A diamagic version is given in Andrews (1917, p.163). However the number of different moves used in these are 7 and 8 respectively. In the examples shown here I have reduced these numbers to 4 or 5 (including the camel closure move in this case) where one diagonal adds to 112 instead of 111.

5	6	28	26	24	22
8	7	25	27	21	23
35	33	19	20	1	3
34	36	17	18	4	2
14	16	10	11	31	29
15	13	12	9	30	32



5	6	28	25	24	23
8	7	26	27	21	22
35	33	19	20	1	3
34	36	17	18	4	2
14	16	10	12	30	29
15	13	11	9	31	32



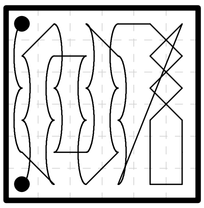
See the six-move section for examples with both diagonals magic.

C. Planck (in Andrews 1917 p.266) says "It is a demonstrable fact that squares of orders 4p + 2, (i.e., 6, 10, 14 etc.) cannot be made perfectly magic in columns and rows and at the same time either associated or pandiagonal when constructed with consecutive numbers." He gives two 6×6 squares in which two of the files differ from the magic constant +/-1 (Andrews Figs 463 and 464).

{2,5}+{1,1}{0,1}{0,2}{0,5}

6×6 Board. Magic tour after Falkener (1892 p.292). Permuted, reoriented. Diags 100 and 133.

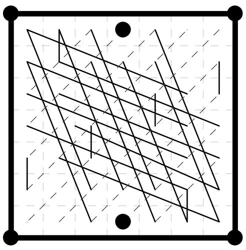
1	33	28	12	13	24
34	6	7	27	23	14
2	32	29	11	15	22
35	5	8	26	21	16
3	31	30	10	20	17
36	4	9	25	19	18

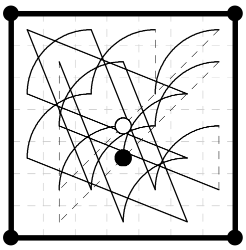


{2,5}+{2,2}{5,5}{0,1}{0,6}

7×7 Board. Step {2,2} Sidestep {0,1} becoming {2,5} and {5,5) on normal board. Closure move is a fifth type {0,6}. Diagonal moves indicated by dashed lines. Some are switchbacks.

19	8	4	49	38	34	23
11	7	45	41	30	26	15
3	48	37	33	22	18	14
44	40	29	25	21	10	6
36	32	28	17	13	2	47
35	24	20	9	5	43	39
27	16	12	1	46	42	31





3	32	12	41	21	43	23
35	8	37	17	46	26	6
11	40	20	49	22	2	31
36	16	45	25	5	34	14
19	48	28	1	30	10	19
44	24	4	33	13	42	15
27	7	29	9	38	18	47

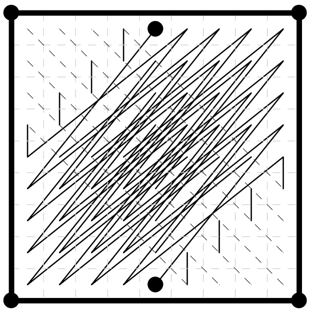
{2,5}+{2,2}{5,5}{0,2}{0,5}

7×7 Board. Step {2,2} sidestep {0,2} plus {0,5) {2,5} {5,5} (5 types). The diagram (above right) shows the first half of the tour. The other half is the same rotated a half-turn. I have a note that these two tours may be due to T. H. Willcocks, but no source.

{4,5}+{4,4}{5,5}**{0,1}**{0,8}

9×9 Board. H. A. Sayles in Andrews (1917 Fig.410). This is a {4,4}-step {0,1}-sidestep diamagic tour on the torus. On the normal board the cross-edge {4,4} moves become {4,5} and {5,5}. The dashed lines indicate where the diagonal moves go, but they are difficult to distinguish since they overlap (e.g. the moves 37-38-39-40-41-42-43-44-45 are all along the same diagonal line.)

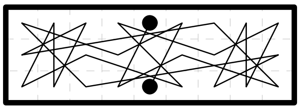
42	34	26	18	1	74	66	58	50
52	44	36	19	11	3	76	68	60
62	54	37	29	21	13	5	78	70
72	55	47	39	31	23	15	7	80
73	65	57	49	41	33	25	17	9
2	75	67	59	51	43	35	27	10
12	4	77	69	61	53	45	28	20
22	14	6	79	71	63	46	38	30
32	24	16	8	81	64	56	48	40



{1,6}{1,3}{1,2}+{1,1}{0,2}

3×9 Board. This tour by M. Trenkler (1999), is based on three 3×3 magic squares.

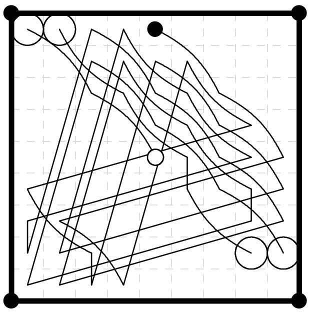
4	18	20	13	27	2	22	9	11
21	5	16	3	14	25	12	23	7
17	19	6	26	1	15	8	10	24



{2,7}+{2,2}{7,7}**{0,1}**{0,8}

9×9 board. Andrews (1917 Fig.25). This is a {2,2}-step {0,1}-sidestep diamagic tour on the torus. On the normal board the {2,2} cross-edge moves become {2,7} and {7,7}. Only half the tour is shown here. The {2,2} moves are curved. The {7,7} moves are omitted, they join the circled cells.

39	34	20	15	1	77	72	58	53
49	44	30	25	11	6	73	68	63
59	54	40	35	21	16	2	78	64
69	55	50	45	31	26	12	7	74
79	65	60	46	41	36	22	17	3
8	75	70	56	51	37	32	27	13
18	4	80	66	61	47	42	28	23
19	14	9	76	71	57	52	38	33
29	24	10	5	81	67	62	48	43

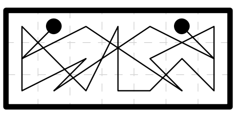


Six-Move Beasts

{2,3}{1,2}+{1,1}{0,1}{0,2}{0,4}

3×7 Board. Asymmetric magic tour

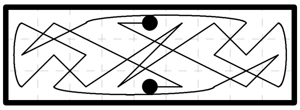
15	1	3	12	19	21	6
2	14	17	11	5	8	20
16	18	13	10	9	4	7



{2,3}{1,2}+{1,1}{0,1}{0,2}{0,5}

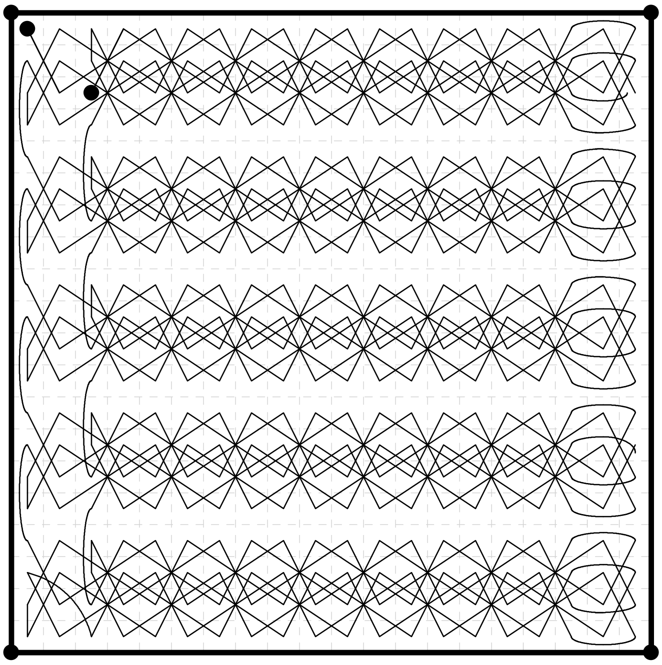
3×9 Board. This can be permuted to give a {1,4} version (see next page below).

9	24	22	2	1	13	16	21	18
23	11	8	25	14	3	20	17	5
10	7	12	15	27	26	6	4	19



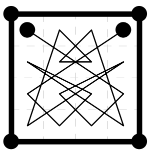
{2,3}{1,2}+{2,2}**{0,1}**{0,2}{0,3}

20×20 board. This diagonally magic ornamented square by Charles Planck is shown (in numerical form) as Fig.700 in *Magic Squares and Cubes* by W. S. Andrews (1917). The {2,2} move is used only for the middle move 200 to 201 (a3-c1) and the closure move.

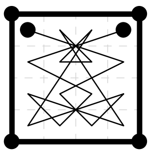


{2,3}{1,3}{1,2}+{1,1}{0,1}{0,3}

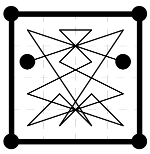
4×4 Board. Diamagic Bison + Centaur tours with five move types WFNCZ, plus closure move {0,3}. Can be regarded as open Wizard tours.



72 WFNCZ

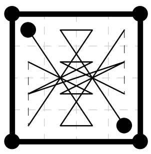


136 WFNCZ



341 WFNCZ

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16



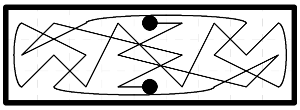
{2,3}{1,3}{1,2}+{3,3}**{0,1}**{0,2}

4×4 Board. This magic square (above right) #176 in the Frénicle list, can be described as 'semi-natural' since half the numbers occur in natural order and half in reverse order (see ⌘ 1).

{1,4}{1,2}+{1,1}{0,1}{0,2}{0,5}

3×9 Board. This is a permute of the tour in the {2,3} section (see previous page).

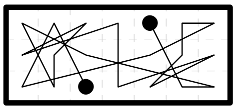
9	24	22	13	1	2	16	21	18
23	11	8	3	14	25	20	17	5
10	7	12	26	27	15	6	4	19



{1,4}{1,3}{1,2}+{1,1}{2,2}**{0,1}**

3×7 Board. Asymmetric magic tour.

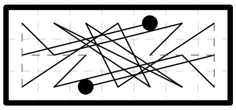
5	20	8	10	1	16	17
9	7	4	11	18	15	13
19	6	21	12	14	2	3



{1,4}{2,3}{1,2}+{1,1}{2,2}{0,2}

3×7 Board. Symmetric magic tour. Unlike other examples this is without wazir moves.

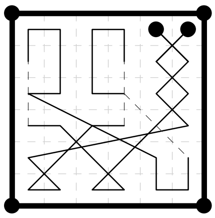
16	12	9	14	1	18	7
2	17	3	11	19	5	20
15	4	21	8	13	10	6



{2,4}{1,5}+{1,1}{2,2}**{0,1}**{0,2}

6×6 Board. Andrews Fig.321. Diagonally magic. Moves: (6 types)

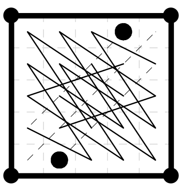
25	24	13	12	1	36
26	23	14	11	35	2
21	22	15	16	3	34
27	28	9	10	33	4
5	8	29	32	20	17
7	6	31	30	19	18



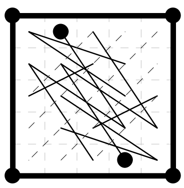
{2,4}{2.3}{1,3}{1,2}+{2,2}{3,3}

5×5 Board. These step sidestep tours have the alfil {2,2} move as the step and the knight as sidestep. These add the moves {1,3}{2,3}{3,3} on the normal board and the closure move is {2,4}. The {2,2} and {3,3} moves shown by dashed lines overlap.

3	17	6	25	14
19	8	22	11	5
10	24	13	2	16
21	15	4	18	7
12	1	20	9	23



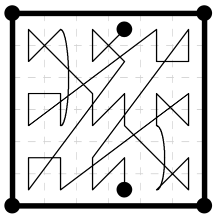
10	1	22	18	14
3	24	20	11	7
21	17	13	9	5
19	15	6	2	23
12	8	4	25	16



{3,4}+{1,1}{0,1}{0,2}{0,3}{0,5}

6×6 Board. Queen+Antelope diagonally magic tour of the Falkener type. Previous attempts used nine different moves. My four solutions (two more below and one in the Augmented Knight section) accomplish the task in only six move types. Non diamagic examples in five moves are also possible.

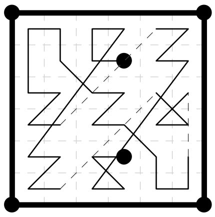
15	13	34	36	8	5
14	16	35	33	7	6
10	11	17	19	26	28
9	12	18	20	25	27
31	30	4	2	21	23
32	29	3	1	24	22



{3,4}+{1,1}{3,3}**{0,1}**{0,2}{0,3}

6×6 Board. Another Queen+Antelope type solution.

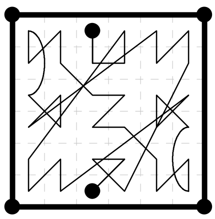
14	15	34	33	8	7
13	16	35	36	6	5
12	11	17	18	28	25
10	9	19	20	26	27
32	31	2	1	21	24
30	29	4	3	22	23



{3,4}{2,4}**+{1,1}{0,1}**{0,2}{0,5}

6×6 Board. Queen+Lancer+Antelope.

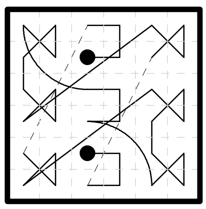
13	15	36	33	8	6
14	16	35	34	7	5
12	10	17	18	26	28
9	11	19	20	27	25
32	30	3	2	21	23
31	29	1	4	22	24



{3,4}{2,4}**+{1,1}**{2,2}**{0,1}**{0,3}

6×6 Board. Another Queen+Lancer+Antelope 6×6 magic tour. Diagonals 108 & 114 (sum 222).

16	14	33	34	8	6
13	15	36	35	5	7
12	10	17	18	28	26
9	11	20	19	25	27
32	30	1	2	24	22
29	31	4	3	21	23



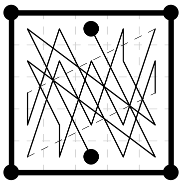
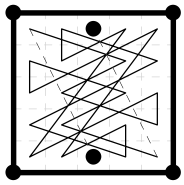
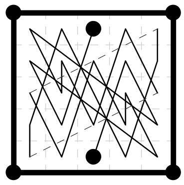
{3,4}{1,3}{1,2}{2,4}**+{0,1}**{0,4}

5×5 Board. Here are some more 5×5 tours of step-sidestep type on the torus using three additional moves on the normal board. These are Knight+Wazir (Emperor) tours on the torus. Five move types plus {0,4} closure The third is in Andrews (1917, p.4, Fig.5 inverted).

3	14	25	6	17
22	8	19	5	11
16	2	13	24	10
15	21	7	18	4
9	20	1	15	24

2	11	25	9	18
21	10	19	3	12
20	4	13	22	6
14	23	7	16	5
8	17	1	15	24

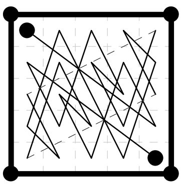
4	12	25	8	16
23	6	19	2	15
17	5	13	21	9
11	24	7	20	3
10	18	1	14	22

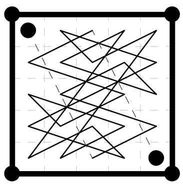
{3,4}{1,3}{1,2}{2,4}**+{1,1}**{4,4}

5×5 Board. These two are Knight+Fers (Prince) tours on the torus. These are in Andrews (1917, p.11 Figs 15 and 14, though differently oriented). They have a sixth move as closure {4,4}.

1	14	22	10	18
24	7	20	3	11
17	5	13	21	9
15	23	6	19	2
8	16	4	12	25



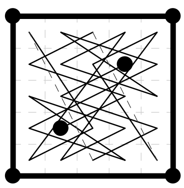
1	15	24	8	17
23	7	16	5	14
20	4	13	22	6
12	21	10	19	3
9	18	2	11	25



{3,4}{2,3}{1,3}{1,2}{2,4}**+**{2,2}

5×5 Board. Knght step, Alfil sidestep torus tour. Strangely the two linking {2,2}-moves on the torus become {2,3} moves on the normal board apart from the closure move.

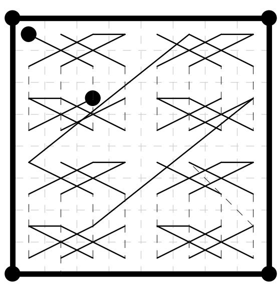
5	11	22	8	19
23	9	20	1	12
16	2	13	24	10
14	25	6	17	3
7	18	4	15	21



{4,5}{1,2}+{2,2}**{0,1}**{0,2}{0,3}

8×8 Board. Diamagic multileaper tour with 6 move types by L. S. Frierson in Andrews Fig.267.

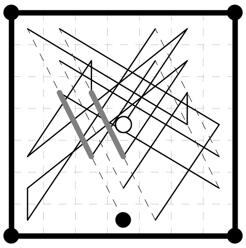
64	57	4	5	56	49	12	13
3	6	63	58	11	14	55	50
61	60	1	8	53	52	9	16
2	7	62	59	10	15	54	51
48	41	20	21	40	33	28	29
19	22	47	42	27	30	39	34
45	44	17	24	37	36	25	32
18	23	46	43	26	31	38	35



{4,5}{3,5}{2,4}**{2,3}+{0,1}**{0,6}

7×7 Board. Kraitchik 1930 Step {2,3} sidestep {0,1}. Only the first half of the tour is shown. The grey lines indicate where pairs of (2,4} moves, shown dashed, overlap.

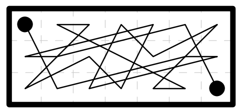
6	18	30	49	12	24	36
47	10	22	41	4	16	35
39	2	21	33	45	8	27
31	43	13	25	37	7	19
23	42	5	17	29	48	11
15	34	46	9	28	40	3
14	26	38	1	20	32	44



{2,6}**{1,4}{1,2}+{1,1}**{2,2}**{0,1}**

3×7 Board. Symmetric magic tour. This uses the same magic files as an asymmetric example by Trenkler (1999) but reduces the number of types of move from 9 to 6 counting the closure move:

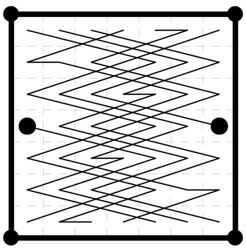
1	12	13	6	17	20	8
18	19	15	11	7	3	4
14	2	5	16	9	10	21

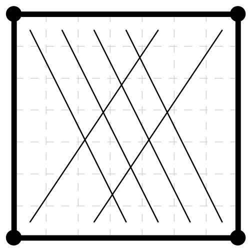


{4,6}{3,6}**{1,4}{1,3}+{0,1}**{0,6}

7×7 Board. C. C. L. Sells [aka Cedric Lytton] 1973 Step {1,3} and sidestep {0,1}. For clarity the {3,6} and {4,6} moves are shown separately. These triple knight and double-zebra moves are in fact equivalent to camel moves across one or two edges of the torus.

12	4	45	37	29	28	20
36	35	27	19	11	3	44
18	10	2	43	42	34	26
49	41	33	25	17	9	1
24	16	8	7	48	40	32
6	47	39	31	23	15	14
30	22	21	13	5	46	38

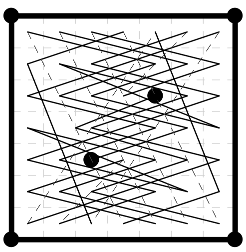




{4,6}{3,6}**{2,5}{1,4}{1,3}**+{2,2}

7×7 Board. T.H.Willcocks {1,3}+{2,2} step-sidestep tour, plus {1,4} {2,5} {3,6} {4,6} (6 types)

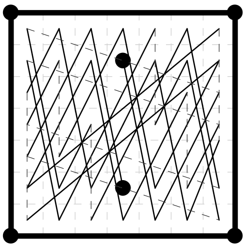
5	18	31	44	8	28	41
43	14	27	40	4	17	30
39	3	16	29	49	13	26
35	48	12	25	38	2	15
24	37	1	21	34	47	11
20	33	46	10	23	36	7
9	22	42	6	19	32	45



{5,6}{2,6}**{1,5}{1,2}+{0,3}**{0,4}

7×7 Board. Lines (p.63) {1,2}+{0,3} step-sidestep tour plus {0,4} {1,5}{2,6} {5,6} (6 types)

5	23	48	17	42	11	29
38	14	32	1	26	44	20
22	47	16	41	10	35	4
13	31	7	25	43	19	37
46	15	40	9	34	3	28
30	6	24	49	18	36	12
21	39	8	33	2	27	45

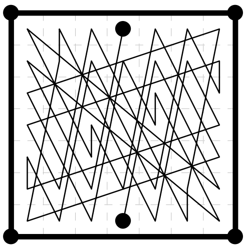


This and the examples below all use the double camel {2,6} move.

{5,6}{2,6}**{1,5}{1,2}+{0,1}**{0,6}

7×7 Board. W. Stead 1974 {1,2} step and {0,1} sidestep tour, the knight move becoming {1,5}, {2,6}, {5,6} on normal board. Five types plus closure move {0,6}.

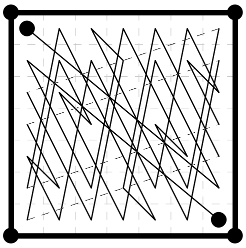
5	15	32	49	10	27	37
46	14	24	41	2	19	29
38	6	16	33	43	11	28
30	47	8	25	42	3	20
22	39	7	17	34	44	12
21	31	48	09	26	36	4
13	23	40	1	18	35	45



{5,6}{2,6}**{1,5}{1,2}+{1,1}**{6,6}

7×7 Board. Jelliss 1974.{1,2}+{1,1} plus {1,5} {2,6} {5,6} (5 types plus closure move {6,6})

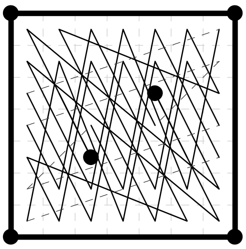
1	18	35	45	13	23	40
48	9	26	36	4	21	31
39	7	17	34	44	12	22
30	47	8	25	42	3	20
28	38	6	16	33	43	11
19	29	46	14	24	41	2
10	27	37	5	15	32	49



{5,6}{2,6}**{2,5}{1,5}{1,2}+**{2,2}

7×7 Board. Lines (p.45) {1,2}+{2,2} step-sidestep tour plus {1,5} {2,5} {2,6} {5,6} (6 types)

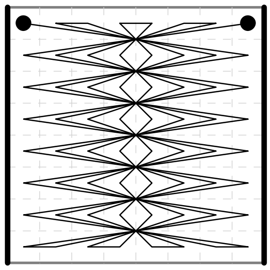
3	21	32	43	12	23	41
46	8	26	37	6	17	35
40	2	20	31	49	11	22
34	45	14	25	36	5	16
28	39	1	19	30	48	10
15	33	44	13	24	42	4
9	27	38	7	18	29	47



{1,7}{1,5}{1,3}+{1,1}{0,1}{0,7}

8×8 Board. A quasi-magic multileaper square by James Parton (Andrews Fig.97) using five different move types plus {0,7} closure. Ranks magic (4×65), files 260±4 (4×64 and 4×66).

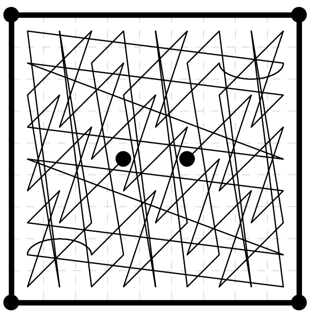
1	16	17	32	33	48	49	64
63	50	47	34	31	18	15	2
3	14	19	30	35	46	51	62
61	52	45	36	29	20	13	4
5	12	21	28	37	44	53	60
59	54	43	38	27	22	11	6
7	10	23	26	39	42	55	58
57	56	41	40	25	24	9	8



{3,8}{1,8}{1,6}+{1,1}{0,2}{0,3}

9×9 board. This diamagic square on the 9×9 board by L. S. Frierson is Fig.254 in Andrews (1917). On the torus board it is a three-direction tour formed of two diagonal steps {1,1} followed by a camel leap {0,3} these being repeated 9 times followed by a side-step {0,2}. On the bounded board three longer moves are needed {1,6} + {1,8} + {3,8} where the moves cross the boundary.

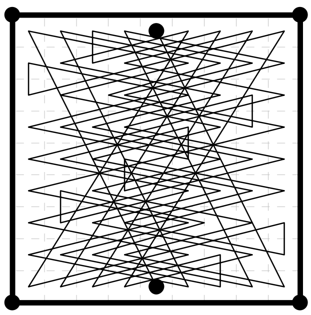
29	16	33	20	61	24	65	52	69
72	32	19	36	23	64	27	68	28
31	75	35	22	39	25	67	3	71
74	34	78	38	25	42	2	70	6
9	77	37	81	41	1	45	5	73
76	12	80	40	57	44	4	48	8
11	79	15	56	43	60	47	7	51
54	14	55	18	59	46	63	50	10
13	30	17	58	21	62	49	66	53



{5,8}{4,8}**{1,5}{1,4}+{0,1}**{0,8}

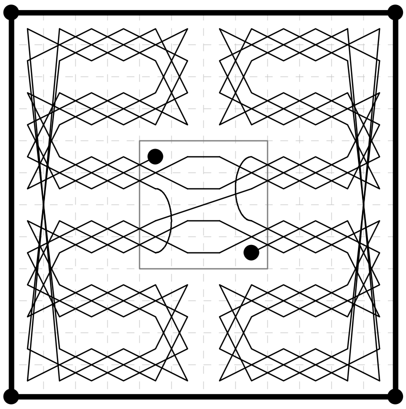
9×9 board. This is a diamagic step-sidestep tour on 9×9 torus using Giraffe step moves and Wazir sidestep. It is Fig.24 in Andrews (1917). On the bounded board the Giraffe moves that cross a parallel edge, like 1-2, become {4,8} quadruple knight moves, while those like 2-3 that cross a perpendicular edge become {1,5} moves, and those like 12-13 that cross two edges become {5,8}. The wazir move to close the tour becomes a sixth move type {0,8}.

2	24	37	59	81	13	35	48	70
73	14	36	49	71	3	25	38	60
72	4	26	39	61	74	15	28	50
62	75	16	29	51	64	5	27	40
52	65	6	19	41	63	76	17	30
42	55	77	18	31	53	66	7	20
32	54	67	8	21	43	56	78	10
22	44	57	79	11	33	46	68	9
12	34	47	69	1	23	45	58	80



{1,11}{1,9}{1,3}{1,2}+{0,1}{0,2}

12×12 Board. The original Joachim Brügge tour by adding a braid to a 4×4 magic square.



36	27	44	83	38	25	12	15	98	65	10	17
45	82	37	26	43	84	97	66	11	16	99	64
22	41	86	41	24	39	14	13	68	95	12	15
87	40	23	40	85	42	67	96	13	14	69	94
42	21	38	89	44	19	18	09	92	71	16	11
39	88	43	20	37	90	91	72	17	10	93	70
75	52	35	28	73	54	55	08	25	2	57	06
34	29	74	53	36	27	26	1	56	07	24	3
51	76	31	32	49	78	03	60	5	22	05	58
30	33	50	77	32	31	6	21	04	59	4	23
81	46	29	34	79	48	61	02	19	8	63	00
28	35	80	47	30	33	20	7	62	01	18	9

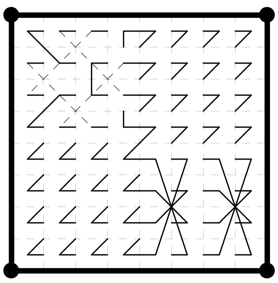
This is derived from the Durer 1514 magic square (Frenicle #175) in the 4×4 centre.

Seven-Move Beasts

{1,3}+{1,1}{2,2}{4,4}**{0,1}**{0,3}{0,4}

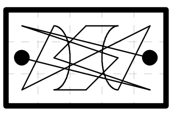
8×8 Board. Diamagic Queen+Camel tour by L. S. Frierson in Andrews Fig.265. Only the ends of the {0,4} and {4,4} are shown. Each successive 4 numbers occupy the corners of a square 5×5.

1	25	56	48	2	26	55	47
40	64	17	9	39	63	18	10
57	33	16	24	58	34	15	23
32	8	41	49	31	7	42	50
3	27	54	46	4	28	53	45
38	62	19	11	37	61	20	12
59	35	14	22	60	36	13	21
30	6	43	51	29	5	44	52

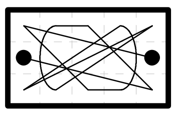


{1,4}{1,3}{1,2}+{1,1}{0,1}{0,2}{0,4}

3×5 Board. Symmetric magic tour of rectangle using seven move types including the {0,4} closure move (four is possible see earlier).



2	6	12	11	9
15	13	8	3	1
7	5	4	10	14



2	11	12	6	9
15	3	8	13	1
7	10	4	5	15

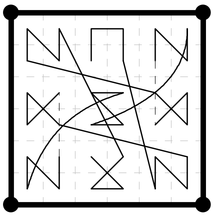
{1,4}{2,3}{1,2}+{1,1}{0,1}{0,2}{0,4}

3×5 Board. Symmetric magic tour of rectangle (above right) using Zebra instead of Camel.

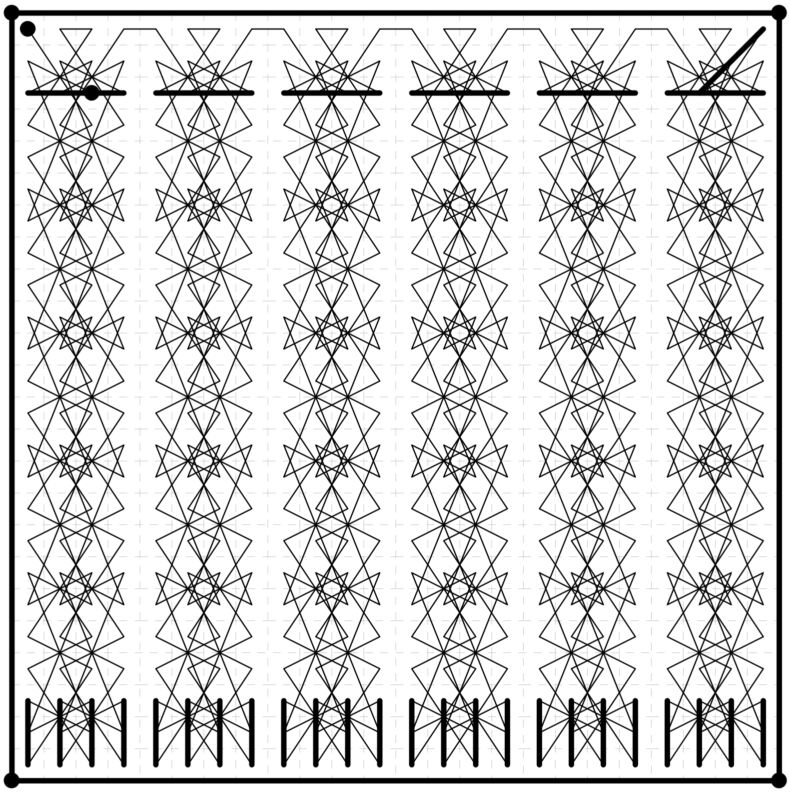
{2,4}**{1,4}+{1,1}**{3,3}**{0,1}**{0,2}{0,3}

6×6 Board. A Falkener style diamagic tour using 7 types of move including closure.

7	5	35	34	14	16
8	6	36	33	13	15
27	25	19	20	9	11
26	28	17	18	12	10
22	24	1	4	31	29
21	23	3	2	32	30



{2,5}{2,3}{1,2}+{2,2}+{0,1}{0,2}{0,3}

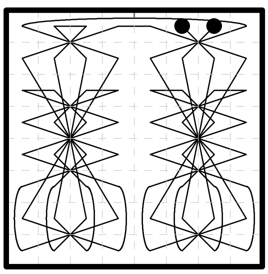


24×24 Board. Diamagic square by Charles Planck (Fig.701 in numerical form in Andrews 1917). The {0,2} and {2,2}used for the middle move 288 to 289 and closure, are shown in bold.

{3,5}{1,5}{1,3}+{1,1}{0,1}{0.2}{0.7}

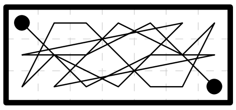
8×8 Board. A magic tour by Benjamin Franklin (Andrews Fig.183, 191, 291). Diagonals 228 and 292 (sum 520). All 2×2 sub-squares add to 130.

17	32	33	48	49	64	1	16
47	34	31	18	15	2	63	50
24	25	40	41	56	57	8	9
42	39	26	23	10	7	58	55
22	27	38	43	54	59	6	11
44	37	28	21	12	5	60	53
19	30	35	46	51	62	3	14
45	36	29	20	13	4	61	52



{2,6}**{1,5}{1,2}+{1,1}**{2,2}**{0,1}**{0,2}

3×7 Board. A symmetric tour.

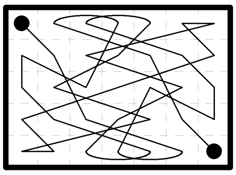


1	13	12	6	20	17	8
18	15	19	11	3	7	4
14	5	2	16	10	9	21

{4,6}**{1,4}{1,3}{1,2}+{1,1}{0,1}**{0,2}

5×7 Board. This symmetric magic tour using six move types (seven including the closure move {4,6}) was composed by me on 12 Jun 2016 (our Queen's official 90th birthday) and is the best so far found on this board. The magic constants are 90 and 126 (i.e. 5×18 and 7×18).

1	32	10	31	11	20	21
29	2	22	12	33	9	19
28	13	30	18	6	23	8
17	27	3	24	14	34	7
15	16	25	5	26	4	35

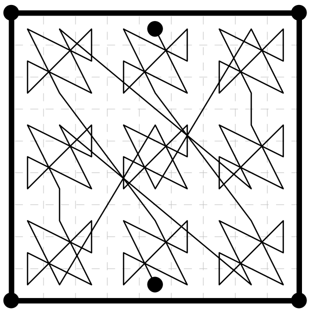


This was derived in part from the arithmic king tour on this board, but also used the method of complementary differences by Charles Planck, as described in Andrews (1917, p.257-262). The example by Planck shown there uses ten move types.

{5,6}{3,5}{1,2}{2,4}**+{1,1}{0,1}**{0,8}

9×9 board. This diamagic square is Fig.96 in Andrews (1917) roted by a half-turn, and is formed of nine 3×3 magic squares linked as if the nine 3×3 components are cells of another 3×3 magic square. The three moves of the small square are thus augmented by three longer moves. A seventh move {0,8} being needed to make the tour closed.

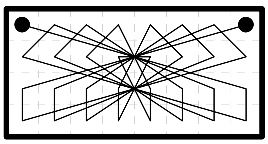
11	18	13	74	81	76	29	36	31
16	14	12	79	77	75	34	32	30
15	10	17	78	73	80	33	28	35
56	63	58	38	45	40	20	27	22
61	59	57	43	41	39	25	23	21
60	55	62	42	37	44	24	12	69
47	54	49	2	9	4	65	72	67
52	50	48	7	5	3	70	68	66
51	46	53	6	1	8	69	64	71



{2,7}{2,5}{2,3}{1,2}+{1,1}{0,1}{0,7}

4×8 Board. Magic rectangle by L. S. Frierson [Andrews Fig.271 reflected]. Adds to 66 & 132.

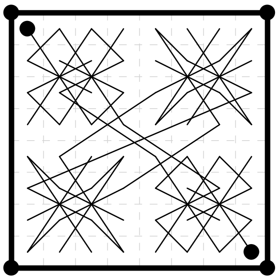
1	5	9	13	20	24	28	32
4	8	12	16	17	21	25	29
31	27	23	19	14	10	6	2
30	26	22	18	15	11	7	3

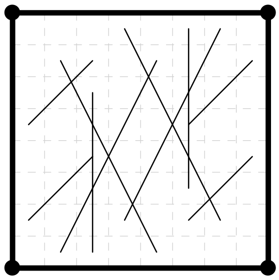


{3,7}{2,3}{1,2}{3,6}+{1,1}{2,2}{0,5}

8×8 Board. Diamagic tour by L. S. Frierson [Andrews Fig.449]. The {0,5} {2,2} {3,6} links that pass over intermediate cells are shown separately.

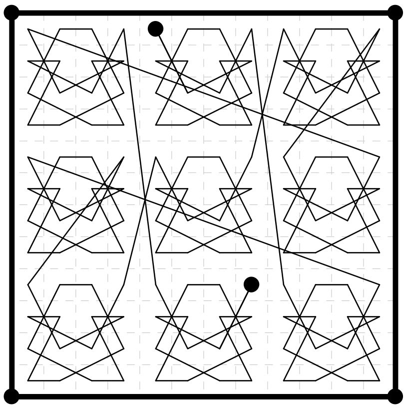
1	14	55	60	35	48	21	26
15	4	57	54	45	34	27	24
58	53	16	3	28	23	46	33
56	59	2	13	22	25	36	47
18	29	40	43	52	63	6	9
32	19	42	37	62	49	12	7
41	38	31	20	11	8	61	50
39	44	17	30	5	10	51	64





{4,11}{3,8}{1,8}{3,4}{1,4}{1,2}+{0,1}

12×12 board. The magic constant for the 12×12 board is 870. Using the diamagic 4×4 Emperor tour joined to copies of itself in a 3×3 magic array (after Andrews Fig 98). Moves {0.1} {1,2} (two moves within the 4×4) plus four different joins) and {3,8} closure move so 7 types.

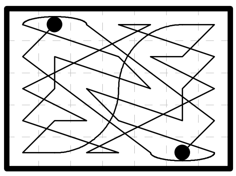


Eight-Move Beasts

{3,4}{1,3}{1,2}+{1,1}{2,2}{4,4}{0,1}{0,2}

5×7 Board. Symmetric magic tour (Jelliss 13 June 2016) one of four that are permutes of one another. The magic constants are 90 and 126 (i.e. 5×18 and 7×18).

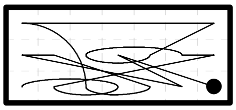
32	1	33	10	11	19	20
2	29	12	31	22	21	9
13	28	6	18	30	8	23
27	15	14	5	24	7	34
16	17	25	26	3	35	4



{1,5}{1,3}{1,2}+{2,2}**{0,1}**{0,2}{0,3}{0,6}

3×7 Board. This tour has the arithmetic sequence 8, 9, 10, 11, 12, 13, 14 in the first rank. This is from Pickover (p.146). The Q+N tour that is a permute of this was found independently.

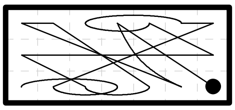
8	9	10	11	12	13	14
4	5	16	2	15	17	18
21	19	7	20	6	3	1



{2,5}{2,3}{1,2}+{2,2}**{0,1}**{0,2}{0,3}{0,6}

3×7 Board. This tour has the arithmetic sequence 8, 9, 10, 11, 12, 13, 14 in the middle rank. It is related to the tour from Pickover above which has the sequence in the first rank.

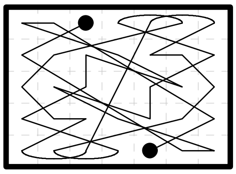
4	5	16	2	15	17	18
8	9	10	11	12	13	14
21	19	7	20	6	3	1



{3,5}{1,4}{1,3}{1,2}{2,4}**+{1,1}{0,1}**{0,2}

5×7 Board. Symmetric magic tour (Jelliss 13 June 2016) one of four permutes (see above).

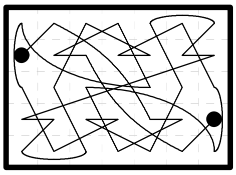
32	33	1	10	19	11	20
2	12	29	31	21	22	9
13	6	28	18	8	30	23
27	14	15	5	7	24	34
16	25	17	26	35	3	4



{1,3}{2,6}**{1,2}**{2,4}**+{1,1}**{3,3}**{0,1}**{0,2}

5×7 Board. Symmetric magic (Jelliss 13 June 2016) one of four permutes (see above).

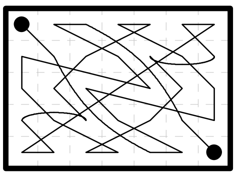
29	2	12	31	22	9	21
1	32	33	10	11	20	19
28	13	6	18	30	23	8
17	16	25	26	3	4	35
15	27	14	5	24	34	7



{1,4}{2,3}{4,6}**{1,2}+{1,1}**{3,3}**{0,1}**{0,2}

5×7 Board. Symmetric magic tour (Jelliss 13 June 2016) one of four that permutes (see above),

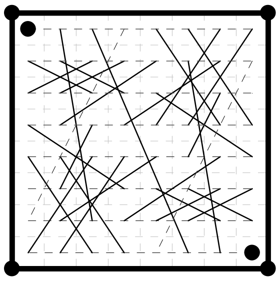
1	32	33	10	11	20	19
29	2	12	31	22	9	21
28	13	6	18	30	23	8
15	27	14	5	24	34	7
17	16	25	26	3	4	35



{3,7}{1,6}{2,3}{1,2}{3,6}**+**{7,7}{0,2}{0,4}

8×8 Board. Panmagic associated square by L. S. Frierson [Andrews Fig.468]. All the odd moves are horizontal moves of {0,4} type, which overlap and are indicated here by a single dotted line. The closure move is {7,7}. If the files are permuted so that the {0,4} moves are closed up to become {0,1} moves (the files being taken in the sequence 1,5,2,6,3,7,4,8) as in the right-hand numerical diagram the tour remains diamagic and associated (no longer panmagic) but uses 9 move types.

1	44	32	53	2	43	31	54
58	19	39	14	57	20	40	13
38	15	59	18	37	16	60	17
29	56	4	41	30	55	3	42
23	62	10	35	24	61	9	36
48	5	49	28	47	6	50	27
52	25	45	8	51	26	46	7
11	34	22	63	12	33	21	64



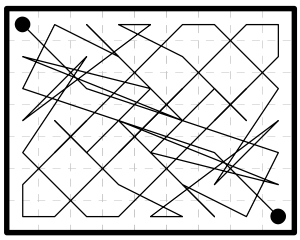
1	2	44	43	32	31	53	54
58	57	19	20	39	40	14	13
38	37	15	16	59	60	18	17
29	30	56	55	4	3	41	42
23	24	62	61	10	9	35	36
48	47	5	6	49	50	28	27
52	51	25	26	45	46	8	7
11	12	34	33	22	21	63	64

More-Move Beasts

{2,5}{3,4}{1,4}{2,3}{2,6}**{1,2}+{11}**{2,2}{3,3}**{0,1}**

7×9 Board. The best I have found on the 7×9 board uses ten types of move and five types of beast (*Jeepyjay Diary* 18 Dec 2014) derived from a King tour. Some of the diagonal moves overlap.

1	59	57	14	15	42	43	29	28
55	2	18	58	41	13	30	44	27
60	17	3	40	54	33	11	25	45
16	52	38	8	32	56	26	12	48
19	39	53	31	10	24	61	47	4
37	20	34	51	23	6	46	62	9
36	35	21	22	49	50	7	5	63



The magic sums are 254 (7×32) and 288 (9×32)

There are many tours to be found in works on magic squares that use even more types of move, but larger numbers are only justified if they are needed to show features that cannot be more clearly shown by using fewer.

Wizard

A Magic Wizard Tour?

The **wizard** is a multimover able to make any leap {r,s} where r and s have no common factor. In contrast the **witch** makes only moves {r,s} where r and s have a common factor. This means that a witch move passes over the centre of at least one intermediate cell. The wizard's moves were first illustrated on the cover of *Chessics* #24 (Winter 1985), the special issue on Theory of Leapers.

My problem begins from the 10×10 magic square formed of two orthogonal latin squares, found by Ernest Tilden Parker 1960. It is shown in a modified form as the frontispiece in the later editions of Rouse Ball's *Mathematical Recreations*. The original version in the 1960 paper shown here is almost symmetric, number ab being opposite ba, except in the corner 3×3 area. Diagonals not magic.

00	67	58	49	91	83	75	12	24	36
76	11	07	68	59	92	84	23	35	40
85	70	22	17	08	69	93	34	46	51
94	86	71	33	27	18	09	45	50	62
19	95	80	72	44	37	28	56	61	03
38	29	96	81	73	55	47	60	02	14
57	48	39	90	82	74	66	01	13	25
21	32	43	54	65	06	10	77	88	99
42	53	64	05	16	20	31	89	97	78
63	04	15	26	30	41	52	98	79	87

74	48	13	25	39	57	01	90	82	66
83	67	24	36	58	00	12	49	91	75
92	11	35	40	07	76	23	68	59	84
$\boxed{69}$	$\boxed{70}$	46	51	22	85	34	17	08	93
18	86	50	62	71	94	45	33	27	$\boxed{09}$
06	32	88	99	43	21	77	54	65	$\boxed{10}$
20	53	97	78	64	42	89	05	16	31
41	04	79	87	15	63	98	26	30	52
37	95	61	03	80	19	56	72	44	28
55	29	02	14	96	38	60	81	73	47

The problem is to permute the ranks, or the files, or the initial digits, or the final digits, or any combination of these so as to produce a Wizard Tour. The best result I have found to date by permuting the ranks and files is shown on the right. It uses only four witch moves (or three if the closure move 99-00 is not counted). Can the number be reduced to zero? There is always one wazir move in a rank and one in a file, 69-70 and 9-10 in the case shown.

Puzzle Solution

Bison Cryptotour Puzzle Camel + Zebra = Bison ANSWER

“If in chess you may chance to set eyes on
Some camels with black and white stripes on
Or zebras with humps making camellar jumps
You've seen bison, or bisons, go by son!”