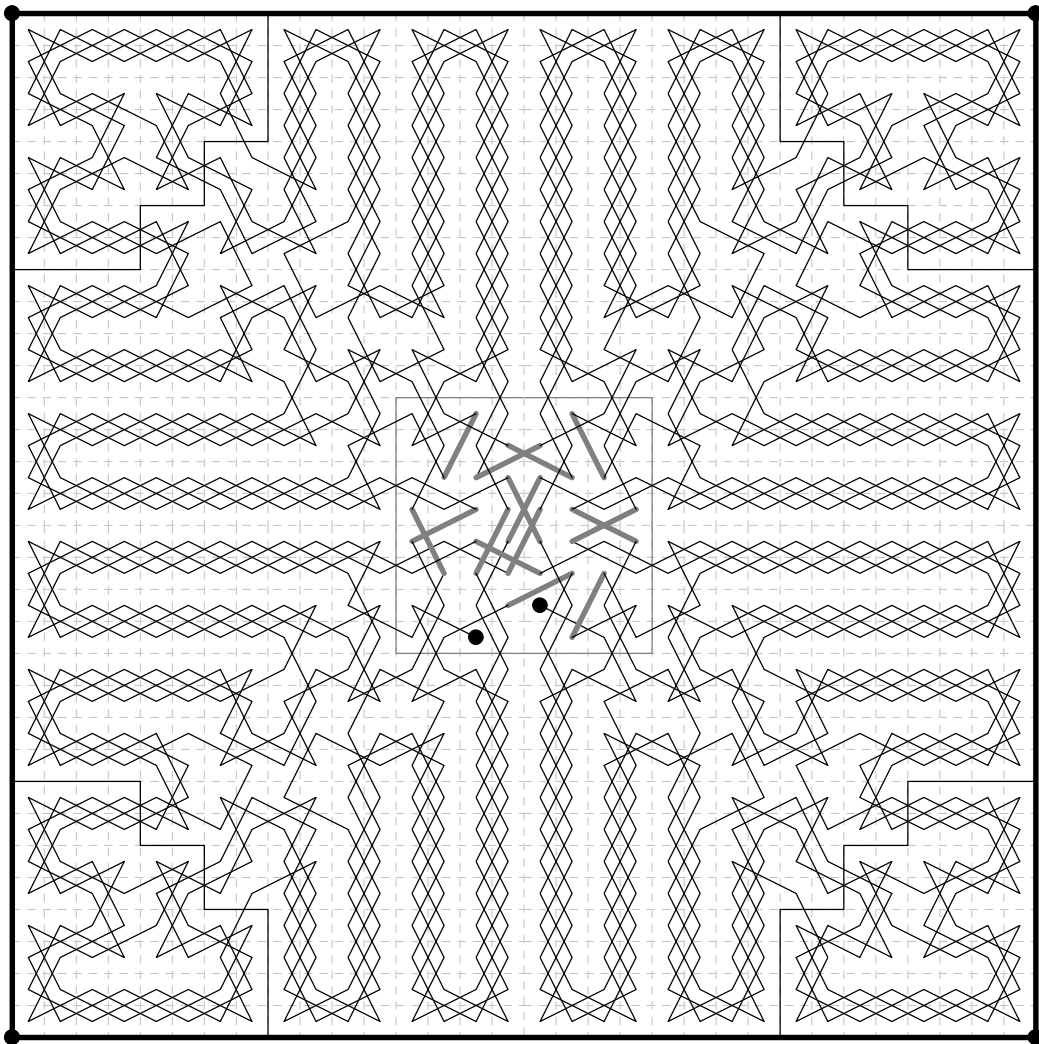


KNIGHT'S TOUR NOTES

⌘ 9

Magic Knight Tours

by G. P. Jelliss



2019

Title Page Illustration

The first diagonally magic 32×32 knight tour. It was constructed by H. E. de Vasa (1956). When numbered from 1 to 1024 beginning at either of the dots, the numbers in the 32 ranks, the 32 files and the 2 main diagonals, add up to the magic constant 16400.

Contents

Construction Methods

- 3. Method of Quartes. Regular Quartes.
- 6. Irregular Quartes. Classification of Regular Magic Tours
- 7. Contraparallel Chains, The X&N Notation,
- 9. Murray's Testings Procedure
- 10. The Method of Braids

History of 8×8 Magic Knight Tours

- 11. First Magic Knight Tours (1848-1876)
Beverley, Wenzelides, Mysore, Jaenisch
- 17. The Age of Magic Tours (1876-1886)
Feisthamel Bouvier Caldwell Exner Reuss Francony Beligne Ligondes
- 29. Taking Stock (1886-1986)
Parmentier, Falkener, Grossetaite, Ligondes, Lehmann, Murray
- 34. Completing the Task (1986-2003)
Jelliss, Marlow, Roberts, Mackay, Meyrignac & Stertenbrink

Catalogue of 8×8 Magic Knight Tours

- 38.. Historical
- 41. Geometric Catalogue
- 48. Arithmetic Catalogue

Magic Knight Tours on Larger Boards

- 63 12×12
- 68 16×16
- 74 20×20
- 75 24×24
- 77 32×32
- 80 48×48

© George Peter.Jelliss 2019

<http://www.mayhematics.com/>

Knight's Tour Notes, Volume 9, Magic Knight Tours.

If cited in other works please give due acknowledgment of the source as for a normal book.

Construction Methods

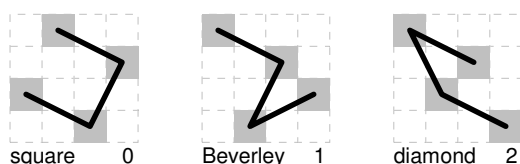
Method of Quartes

The ‘method of quartes’ combining ‘squares and diamonds’ with ‘Beverley quartes’ is essentially due to Wenzelides (1850). My treatment here owes a lot to the account by H. J. R. Murray in his unpublished manuscripts of 1942 chapters X–XII and 1951 chapters I and III–VI. The terminology is simplified from Murray's account. He ‘indexes’ the cells using the letters ACBD, but I prefer to keep to Roget's LEAP scheme (see ¶ 6). He refers to square and diamond quartes as ‘single-indexed’ and Beverley quartes as ‘double-indexed’. Because of the advent of computers many details of his procedures are now only of historical interest. In particular he adopts a number of notational conventions that now seem unwieldy. Where Murray's text is followed closely it is between double “quote marks”. Other passages and [bracketed insertions] are by the present author. In place of Murray's numbering of the tours the system (00b, 12d, 27a, etc) introduced in my 1986 *Chessics* catalogue of the tours is used.

“Any knight's tour on the chessboard may be regarded as composed of 16 four-cell chains, and every number of move can be expressed in the form $4 \cdot x + n$, where x may have any integral value from 0 to 15, and n is one or other of the four numbers 1, 2, 3 and 4. In other words x gives the number of chains which the knight has already completed, and n gives the position of the knight on the current chain.” These four-cell chains, adapting a terminology of Wenzelides (1850), we call **quartes**. In other words the sections of a magic tour numbered 1-4, 5-8, 9-16 and so on are quartes. This usage can be extended to any leaper tours. See also under Magic Tours in ¶ 1.

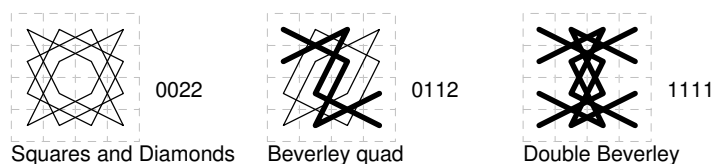
Regular Quartes

A board $4 \cdot h \times 4 \cdot k$ can be divided into $h \cdot k$ areas 4×4 which I call **quads**. On the 8×8 board the quads are the quarters of the board, also termed quadrants. Quartes that occupy a 4×4 area and use one cell in each rank and file of the quad are termed **regular** quartes. The four cells used are either at the corners of a **square** or a **diamond** or form a sort of square root sign (these are known as **Beverley** quartes since they first appeared in the 1848 magic tour by William Beverley). The number of cells in each diagonal of the quad is 0 for a square, 1 for a Beverley quarte and 2 for a diamond.



In the tour diagrams in our text Beverley and irregular quartes are drawn in bold. This makes it easy to see which of three types a tour falls into. (A) **Rhombic tours** which are formed purely on the ‘squares and diamonds’ plan, (B) **Beverley tours** formed of squares, diamonds and Beverley quartes, (C) **Irregular** using at least one irregular quarte.

In a **regular** tour, classes (A) and (B), all the quartes are regular and each quadrant of the 8×8 board therefore contains four quartes. Three types of regular quad are possible; they can be denoted 0022 (two squares and two diamonds), 0112 (square, two Beverley quartes, and diamond) or 1111 (four Beverley quartes).



KNIGHT'S TOUR NOTES

It can be shown that there are 36 geometrically distinct ways of deleting one move from each of the four circuits in the 0022 squares and diamonds quad; in addition there are 8 ways of deleting the moves from the two circuits in the 0112 quad; so there are 45 regular quads in all.

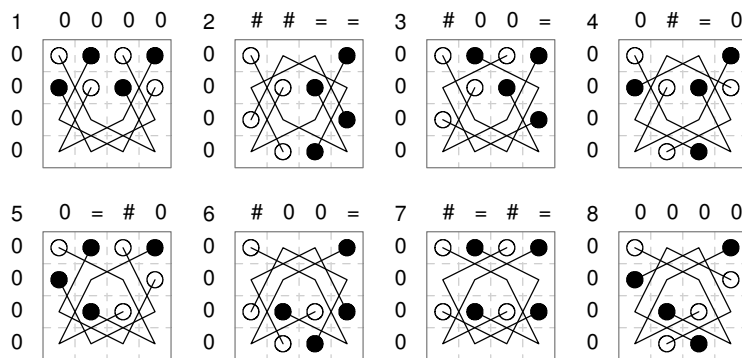
Diagrams of all 45 quads follow.

In these diagrams the signs above each file and beside each rank indicate the excess or defect (variance) of its n-sum from 10. Zero variance is of course shown by 0 (the line will be found to contain the same number of black and white dots). The signs + and - indicate variances of ± 2 (the line will be found to contain one more white than black or one more black than white dots respectively). The signs # and = indicate variances of ± 4 (the line contains an excess of two white or two black dots). This interpretation assumes that the first number, $4 \cdot x + 1$, in each quartet is placed on the black dot and the last, $4 \cdot x + 4$, on the white dot. If the numbering is reversed then the negative and positive signs must be interchanged (- for + and = for #).

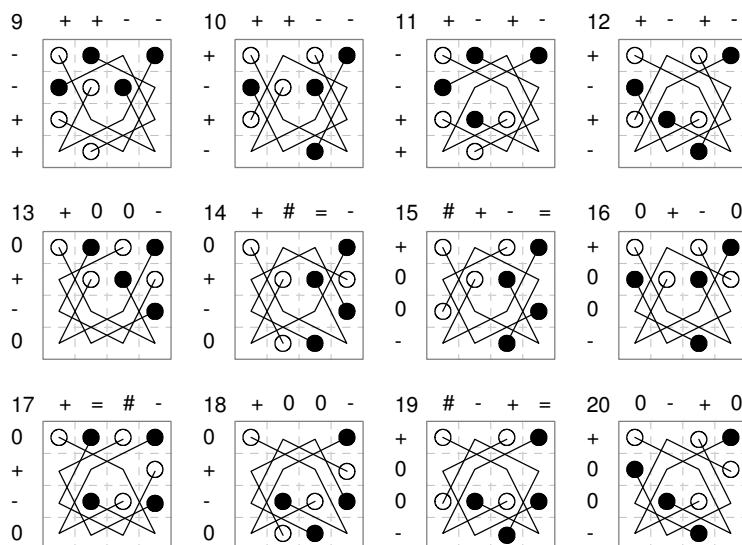
The 45 quads consist of 36 asymmetric, 8 symmetric with one axis and 1 (the double Beverley) symmetric with two axes, so that the total of oriented patterns is $8 \times 36 + 4 \times 8 + 2 \times 1 = 322$. (Murray gives the total as 289, omitting 33 cases with the Beverley quartet in N instead of Z orientation.)

There are four quads in which all the ranks and files have zero variance, namely 1, 8, 44, 45. There are 12 further quads in which either all ranks or all files have zero variance, namely 2, 3, 4, 5, 6, 7, 25, 26, 35, 36, 37, 38. If the 45 quad is used on the 8×8 board or in the corner of any larger board then the two loose ends in the corner of the board must be the ends of the tour.

Numbers 1 to 8 of the squares and diamonds type are symmetric.

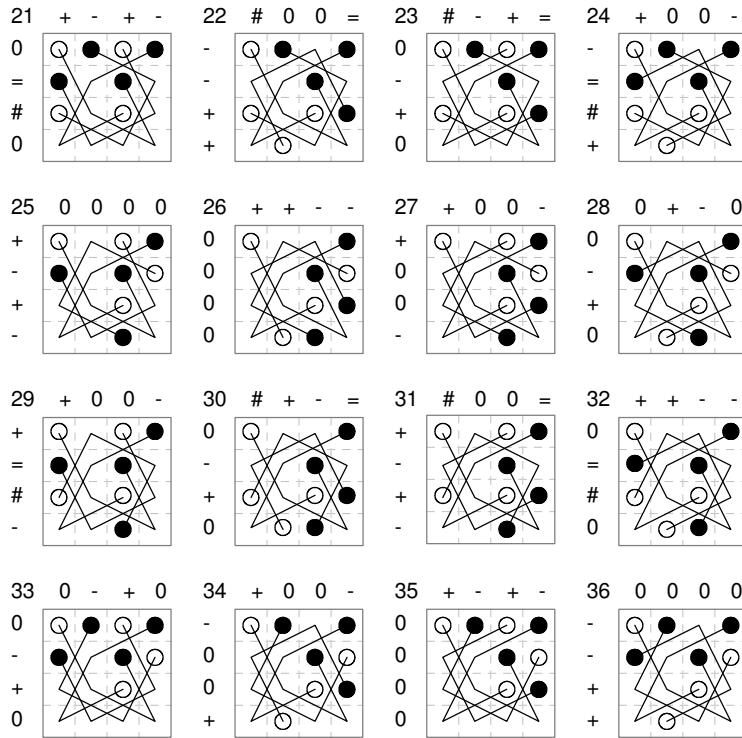


Numbers 9 to 20 of the squares and diamonds type are formed by combining a symmetric pair of diamonds with a pair of squares (which are always symmetric) resulting in an asymmetric quad, since the axes of symmetry do not coincide.

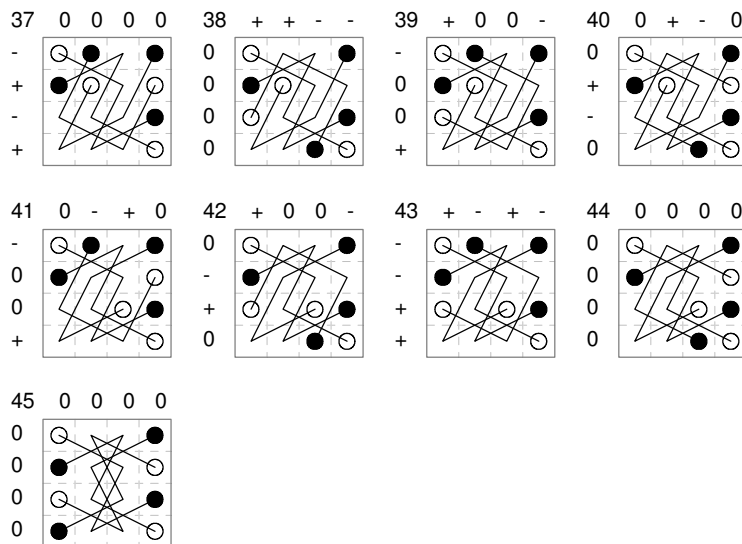


KNIGHT'S TOUR NOTES

Numbers 21 to 36 of the squares and diamonds type all contain the asymmetric pair of diamonds and are therefore asymmetric.



Finally, numbers 37 to 45 are the quads with pairs of Beverley quartets. Number 45 has two pairs of Beverley quartets.

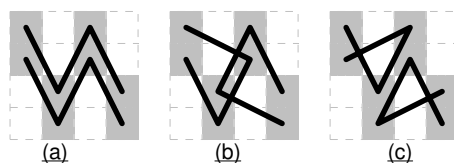


KNIGHT'S TOUR NOTES

Irregular Quartes

Quartes that are 'irregular' differ from the regular quartes either in occupying a quad but not using one cell in each rank and file, or in spreading over two quads. Thinking of the lines separating the quads as bar lines in music I like to call the latter 'syncopated'.

The eight cells used by a pair of Beverley quartes may be combined differently to give another pair of irregular quartes in the following three ways. Only the first of these is mentioned by Murray, since it is the only type occurring in 8x8 magic tours; but the others may be usable on larger boards.



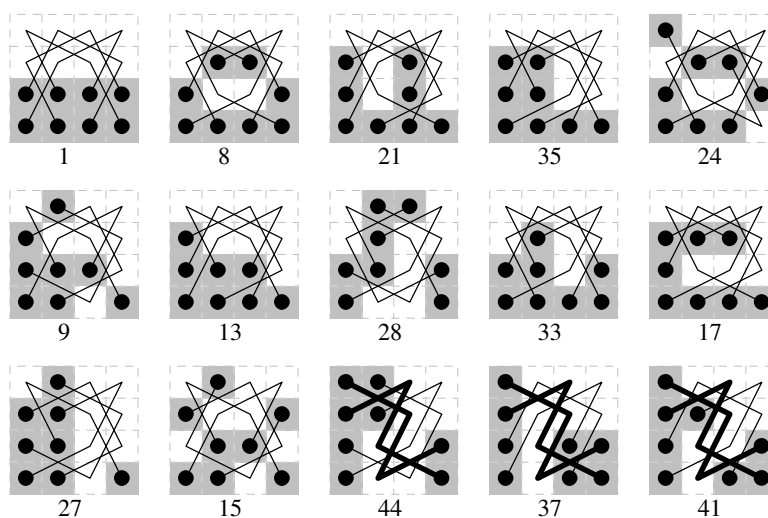
Murray wrote: "The use of irregular quartes [of type (a)] was first shown by C.Bouvier in 1876 (05a) and two other 8x8 tours with these quartes were discovered later by C.E.Reuss (00j) and (27f)." [In fact Murray attributes (27f) to Wihnyk 1885, but it was found earlier by Reuss.]

"The conditions under which this substitution may be made without destroying the magic feature are: (1) The tour must include two pairs of Beverley quartes in the same half of the board and symmetrically placed with respect to the shorter median of that half-board [the half-blocks used being parallel to that median]. (2) If in the tour one pair of Beverley quartes are the quartes numbered m and n in the tour, and m' and n' are the other pair, m being symmetrical to m' and n to n' , then we must have $m + m' = n + n' = 15$. The replacement of the Beverley quartes by the irregular quartes as in (05a) then leaves the resulting tour magic."

The overall effect of the change is an interchange of four pairs of numbers. The tours of this type (00j, 05a, 27f) are related in this way to the tours (00d, 05e, 27d) respectively. Strangely (05a) with irregular quartes was discovered before (05e) with Beverley quartes.

Classification of Regular Magic Tours

In fact the double Beverley quad does not occur in any 8x8 magic tour, though it can occur on larger boards. Out of the 45 quads it is surprising that only 15 different quads occur in regular magic tours on the 8x8 board, namely 1, 8, 9, 13, 15, 17, 21, 24, 27, 28, 33, 35, 37, 41, 44, and only five of these are in the classes with zero variance in all ranks or files (1, 8, 35, 37, 44).



The shaded cells and black dots in these diagrams mark the ends of the quartes, i.e. where the numbers of the forms $4 \cdot x$ and $4 \cdot x + 1$ occur.

Contraparallel Chains

Murray's idea of Contraparallel Chains (defined in general terms in the *Theory of Magic* section in \aleph 1) expresses a general principle, but has not in itself led to many results on the 8x8 board, though it can be seen to underly the more specialised methods of quartes and braids. He wrote: "Most magic tours can be analysed into an aggregate of contraparallel chains of lengths 4, 8, 12, 16, 20 or 24 cells and can be so oriented that the constant sums of each pair of chains lie on the columns of the board. The only exceptions are magic tours (12d) and (12h), by Jolivald."

To these we must now add the tours (00n, 00o, 01g, 01i, 14f, 27t). I have found that it is possible to analyse some tours into contraparallel chains in more than one way, and some can be analysed with corresponding cells in ranks or files. Where this occurs it is presumably best to choose the longest chains, or those closest together.

Table of contraparallel chains in 100 magic tours. The first number counts the pairs of contraparallel chains, the second (in parenthesis) counts the number of quartes in the longest chain.

- 2(6): 01e; 03a
- 2(4): 05f; 16a; 23q; 27a, b, g, h, j, k, l, m, n, o, p, q, r, s
- 3(6): 00b, c, j; 05a
- 3(5): 01d; 07a
- 3(4): 00f, g; 23m, n
- 4(5): 01c; 34f
- 4(4): 03b; 14e
- 4(3): 00a; 01b; 03f; 14a, b; 23e, f, g, h, i, j, k, l; 27i; 34a, b, c, d
- 4(2): 00k, l; 05g; 12i, j, k, l
- 5(4): 12e, g, p; 23a, c
- 5(2): 23o, p; 25b
- 6(3): 00h; 03g
- 6(2): 01f; 12c, f; 14d; 23d; 25a; 27c, d, e, f
- 7(2): 00i; 01a, h; 05d; 12a, b; 34e, g
- 8(1): 00d, e, m; 03c, d, e; 05b, c, e; 12m, n, o; 14c; 23b

The above data is based on a more detailed table by Murray (1951) to which analyses of four of the Marlow tours (except 01g), and the more recent tours by Roberts (14e) and Mackay et al (07a) have been added.

The Marlow tour (03b) can be analysed by ranks into the contraparallel chains 1–12/56–45, 13–16/64–61, 17–20/36–33, 21–24/60–57, 25–32/44–37 (longest chain 12 cells). However it can also be analysed by files into 1–8/60–53, 9–12/36–33, 13–28/52–37, 29–32/64–61 (longest chain 16 cells), so it is classified under 4(4) above.

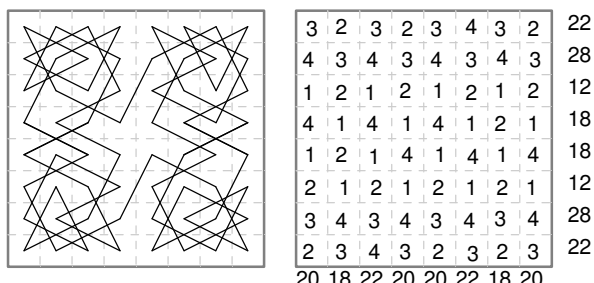
The X and N Notation

This and the following note from the H. J. R. Murray (1951) manuscript are included mainly for their historical interest. Murray's text is between double quotation marks.

"The sum of the eight numbers on any row or column will be of the form $4 \cdot (x_1 + x_2 + \dots + x_8) + (n_1 + n_2 + \dots + n_8)$ which we shall write $4 \cdot X + N$." The average value of n is $(4 + 1)/2 = 2.5$ so the average value of N will be 8 times this = 20. The average value of all the numbers in the array is $(64 + 1)/2 = 32.5$ so the average row sum, the magic constant, is 8 times this = 260. Thus the average value of X is $(260 - 20)/4 = 60$. In a magic tour we require $4 \cdot X + N = 260$ in each line. Thus a unit change in X necessitates a change of 4 in N , in the opposite direction, to preserve the magic total. It follows that in a magic tour N takes the series of values 12, 16, 20, 24, 28. Correspondingly X takes the values 62, 61, 60, 59, 58. "Arithmetically the limiting values of N are 12 (four 1s + four 2s) and 28 (four 3s + four 4s), but since 4 always links with 1, the actual limits are 16 and 24."

KNIGHT'S TOUR NOTES

This result appears to be true empirically but the explanation “since 4 always links with 1” is inadequate. For example in the above symmetric tour of squares and diamonds type constructed by the present author, the rhombs being taken as the quartes, the N-values add to 28 on the second and seventh ranks and 12 on the third and sixth, but the other ranks add to 22 and 18 which are not the required multiples of four. I think this anomaly is resolved by my theorem (Jelliss 2003) about the balance necessary between entries of the forms $4 \cdot n$, $4 \cdot n + 2$ and $4 \cdot n + 1$, $4 \cdot n + 3$.



THEOREM (Murray): “The condition that a tour composed entirely of regular quartes shall have all rows and columns summing to 260 is that $X = 60$ and $N = 20$ for every row and column.”

Proof: “(1) If the quartes on a given quarter-board are the $x + 1$, $y + 1$, $z + 1$, $w + 1$ quartes of the complete tour, then the numbers on each cell of these quartes begin with $4 \cdot x$, $4 \cdot y$, $4 \cdot z$, $4 \cdot w$ respectively, this means that [since each quarte has one entry in each row and column of the quarter] the first term in the summation of each row and column of the quarter is $4 \cdot (x + y + z + w)$ which we may write $4 \cdot Q$. The same is true of each quarter, though the value of Q may be different in each [label them Q_{11} , Q_{12} , Q_{21} , Q_{22}] but the sum of the four values must be $Q_{11} + Q_{12} + Q_{21} + Q_{22} = 0 + 1 + 2 + 3 + \dots + 15 = 120$.”

“(2) The n -terms of the numbers on the different cells are so connected that a 1 on the top row occurs in conjunction with a 3 on the bottom row and vice versa, and a 2 on the second row occurs in conjunction with a 4 on the third, and vice versa. This means that the N -term in the summation of the two outer rows is $10+a$ or $10-a$ in one and $10-a$ or $10+a$ in the other row. So also with the two inner rows and the outer and inner columns. The row-sums for the whole board are of the form $4 \cdot (Q_{11} + Q_{12}) + 20 \pm v$ [with possibly four different values for v], with similar expressions for the columns. These forms are true of all tours that are composed entirely of regular quartes, whether magic or not.”

“(3) If the tour is magic then all of these expressions must equal 260 [which is $4 \cdot 60 + 20$]. It follows first that $v = 0$ for every row and column, and so $N = 20$, and second that $Q_{11} + Q_{12} = Q_{11} + Q_{21} = Q_{22} + Q_{12} = Q_{22} + Q_{21}$, that is $Q_{11} = Q_{22}$ and $Q_{12} = Q_{21}$ [i.e. the Q -values in opposite quarters are equal], and so $X = 60$. [QED]”

Murray writes: “The simplest procedure for obtaining magic tours of regular quartes is to construct diagrams in which the terminals of the quartes are so arranged that every row and column has $N = 20$. Such an arrangement I call a ‘matrix’. All such matrices can be built up in the following way: Obtain all the [322] regular quadrants and above each column and by the side of each row add the excess or defect of its total from 10. Now examine each diagram to see if it can be used on quadrants of the upper half-board. To be usable, every terminal must link with a terminal in another quarte, either on the quadrant itself or on an adjacent quadrant [unless end-points of the tour]. Combine the usable quadrants in such a way that they can link and the defects and excesses of each row cancel one another. Collect together all these half-board diagrams that have the same run of defects and excesses. By combining these half-tours with the inversions of half-tours of the same class we obtain the matrices. If all the terminals on a matrix can be linked together so as to produce a knight’s tour and if the cells of the tour when numbered in the order in which they are entered give for one row and one column the total 260, that tour is magic.”

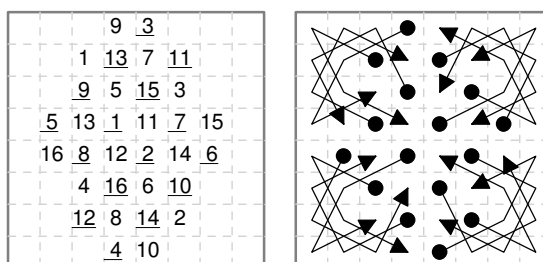
The above account is considerably condensed from Murray's explanation and the task, when carried out manually, is much greater than he admits, but it can now be done (and has been done) by computer methods. The full list of 78 regular magic tours on the 8×8 was completed by the work of T. W. Marlow published in 1988.

KNIGHT'S TOUR NOTES

It is now known that there are 32 matrices possible for tours of regular quartes type, Murray found 30 of these and 2 others must be added to account for two of the Marlow tours, 01g and 23q found in 1987. The matrices are listed in the geometric catalogue of tours that follows.

Murray's Testing Procedure

In constructing the matrices for regular tours Murray numbers the quartes 1 to 16: quartes 1, 5, 9 and 13 being in the a8 quadrant, 3, 7, 11 and 15 in the h8 quadrant, 2, 6, 10 and 14 in the h1 quadrant and 4, 8, 12 and 16 in the a1 quadrant. He numbers only the cells of entry and departure, underlining the cell of entry. This labelling of the quartes 1 to 16 should not be confused with the x-numbering of the quartes 0 to 15, which has yet to be determined.



To determine the magic tours derived from this matrix: “I begin by preparing a table showing all the ways of connecting the ends cells of each quartie with the beginning cells of other quarties. Thus 1 connects to 3, 5, 15 while 3 connects to 1, 2, 6, 13 and so on. With the help of this table and a set of counters numbered 1 to 16 and a sheet of paper divided into columns headed 0, 1 to 15 (to show the position of each quartie in a completed tour) I proceed to work out the permutations of the sixteen counters and so obtain tours which are entered on the sheet of paper.”

Three tours obtained in this way are:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
1	3	6	7	9	11	13	15	2	4	5	8	10	12	14	16	(1)
11	8	10	12	7	9	15	2	6	4	14	1	5	3	13	16	(2)
13	15	10	12	14	16	1	3	2	4	5	7	9	11	6	8	(3)

“We have now to test these formulae of tours to see if both rows and columns give $X = 60$. The row sum is obtained by adding the numbers in the heading which stand above the odd numbers in the formula, and the column sums by adding the numbers on the heading which stand above 1, 4, 5, 8, 9, 12, 13 and 16 in the formula of the tour. If both totals are 60 the tour is magic as it stands. If either total is not 60 the tour is not magic if the tour is open, but if the tour is closed it may be magic with a different starting point, and the effect of changing the starting point must be examined. A convenient way of doing this is to use a strip of paper with the numbers 0, 1, ..., 15, 0, 1, ..., 15 which can be placed over the heading and moved so that any desired column may be 0, i.e. the starting point of the tour. The effect of shifting this strip one column to the right or left is to increase or decrease the sum of any eight terms in the formula by 8. No change of starting point can make the sum 60 unless the original total was $60 \pm 8 \cdot n$. Unless the row and column sums are both an even or both an odd number of 8s from 60, no change of starting point can bring them simultaneously to 60.”

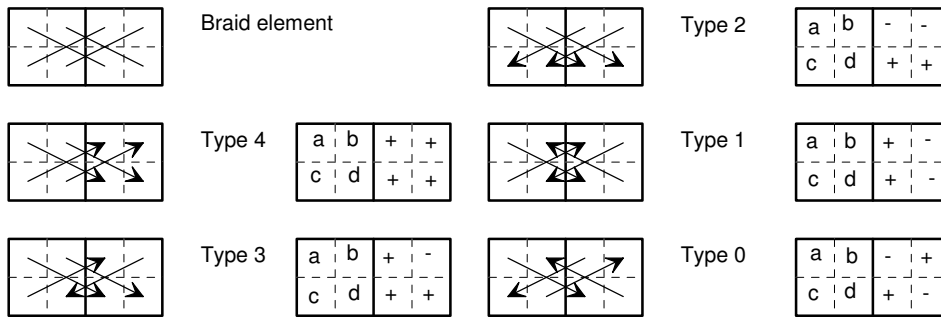
“The row and column sums for the above tours are (1) closed 36 and 68, (2) open 65 and 70, (3) open 60 and 60. It will be seen at once that (3) is magic and (2) not. The closed tour (1) is not magic with the given starting point but both sums are of the form $60 \pm 8 \cdot n$ and in both cases n is odd, so other starting points must be tested. To convert 36 to 60 three shifts are necessary, so I move the strip until 0 covers 5. Now both row and column sums are 60, so the tour is magic provided it starts from quartie 11. Since a closed tour may have several starting points I next try the effect of seven shifts, but this brings the row sum to 76, and as the matrix is in diametral symmetry there is no need to try more than eight shifts.”

The Method of Braids

This seems an appropriate point at which to describe this simple method of extending magic tours to larger boards. It is a composite of methods devised by Wihnyk (based on magic quads) and Lange (based on borders) and by Murray (based on gnomons), all of which are exemplified in the 12×12 and 16×16 and larger tours shown later in this volume.

A **braid** consists of a series of elements each made of four knight's moves connecting cells in one 2×2 block to cells in an adjacent block. Braid elements occur in all but three of the 108 magic knight tours on the 8×8 board (and even the three exceptions 00k, 12k, 12l, all due to Francony 1883, there are 'pseudo-braids' formed of four slant moves). However, braids longer than three elements occur only in seven tours: 14a, 14b, 27a, 27b, 27g, 27h, 27o.

Braids either have all four strands going in the same direction, or one opposed to the other three, or else two going one way and two the other. The two-and-two braid elements can be subclassified into three types according as the two pairs of like-moving strands start in cells in the same rank, file or diagonal of the first block of the 2×4 element. I denote the five types by the codes 4, 3, 2, 1, 0.



If one braid element follows on from another in a straight line then it is of the same type. If it follows on at right angles this remains true for types 4, 3 and 0, but type 2 becomes type 1 and vice versa. Braids of type 0 are especially useful for forming magic tours since the two ranks of an element always add to $a+b+c+d$, while two of the files add to $a+c$ and two add to $b+d$, and these values remain the same if a series of such elements are connected, the +1 and -1 cancelling out both in ranks and files. All five types of braid element occur in the 8×8 magic tours. Those in which braid elements of type 0 occur, with $a+b+c+d = 130$, are 00b, 00c, 03b, 05b, 05g, 12i, 14a, 14b, 23a, 27a, 27b, 27c, 27d, 27e, 27f, 27g, 27h, 27k, 34c.

Starting from an 8×8 magic knight tour, magic tours on larger boards can often be constructed by simply connecting braids to them to fill the extra blocks of cells. The main requirement is that each braid element be matched by another contraparallel braid element of the same type (so that each $a+c$ is opposite $b+d$ to ensure the right sum).

If we represent each 2×2 block on a $2n \times 2n$ board by a single cell on an $n \times n$ board then a braid of four knight paths on the larger board can be represented on the smaller board by a single wazir path. This is the **braid plan** of the tour. Thus, since wazir paths are easy to construct and visualise, it is possible to construct a whole family of magic knight's tours based on the same 8×8 tour (or parts of such a tour) as **kernel**.

Joachim Brüggé has also pointed out the remarkable fact that any of the 4×4 diagonally magic squares can be extended by knight braids to form 12×12 diagonally magic squares in this way, though links between the end-points of the braids require longer leaps. Examples of this are shown in ¶ 10 on Generalised Knights.

History of Magic Tours

The First Magic Knight Tours 1848-76

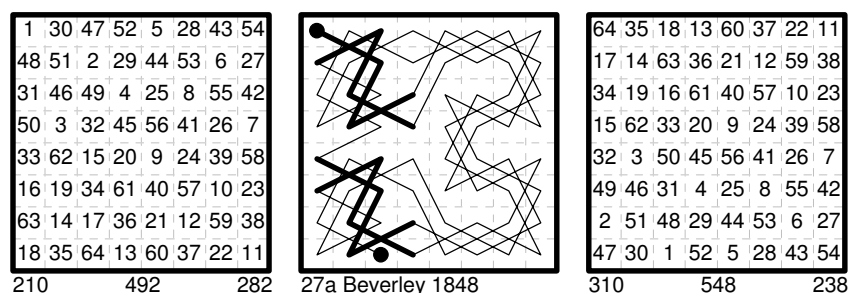
For the history of tours before 1848 see § 6 which includes an account of the Squares and Diamonds method which preceded the discovery of the first magic knight tours.

1848 William Beverley's Tour

The problem of constructing a magic knight's tour was at last solved by William Beverley in 1847. His discovery of this magic tour is his one recorded achievement in the subject. Biographical research detailed in the Bibliography reveals that he was a painter and theatre technician. He sent the tour to his friend the mathematician Henry Perigal Jr on 5 Jun 1847, and Perigal sent it to the editors of the *Philosophical Magazine* on 29 Mar 1848, and it was published in August that year. The precise addresses and dates cited make one wonder if the publishers were aware of the imminent publication of similar results in Germany, and were therefore anxious to ensure the priority of the English work.

This magic tour quickly became widely known. Howard Staunton republished the entire article in the November number of his *Chess Player's Chronicle* (1848 p.344-347), and a similar article signed by Otto von Oppen is: 'William Beverley's Rösselsprung' *Schachzeitung* (vol.4 p.21-24 Jan 1849).

The reverse numbering of any magic tour is magic, and Beverley's tour was presented in both forms, as here, but without a geometrical diagram. In our diagrams the heavy border indicates a magic tour. The numbers below the a1 and h1 corners are the sums of the a1-h8 and h1-a8 diagonals. The number in the middle is the sum of both diagonals.



The surprise about Beverley's solution is that it is not of squares and diamonds type. It introduced a new zigzag path in two quarters which has the same property as the squares and diamonds of having one entry in each rank and file of the quarter. This we now call a 'Beverley quarter', shown in the diagram by heavier lines.

In this tour all the files except those containing the end-numbers 1 and 64 are composed of four complementary pairs of numbers (adding to 65) that are symmetrically placed on either side of the horizontal median, which ensures these files add to the magic constant ($260 = 4 \times 65$). It may also be noted that in the right half-ranks the 'counter-complements' 65 ± 32 (adding to 97 and 33) occur in pairs to ensure the half-file adds to 130). It is noted in the original text that the four quarters are themselves magic, adding to 130, and that the numbers in the 16 blocks 2×2 also add to 130. A footnote on p.247 by S. M. Drach notes that the 4×4 squares centred on the edges are also magic.

H. J. R. Murray's Analysis of Beverley's Tour

In his 1942 ms Murray claims: "Beverley's tour ... was not discovered by chance but was the fruit of a method based on mathematical analysis which was thought out in advance ...". However the analysis he describes, in terms of 'contraparallel chains' is really Murray's own work. I suspect that Beverley found his tour by a more laborious method. Evidence for this is provided by the fact that, when all the tours of quartes type (including the new quartes introduced in Beverley's tour) are oriented and arranged in sequence according to Frénicle's method for magic squares Beverley's tour comes first in the list, and would therefore be the first discovered in a systematic search.

KNIGHT'S TOUR NOTES

The following notes between quotation marks are directly reproduced from the ms by Murray (1942): The numbers (27a), (05f) etc refer to my catalogue of magic knight's tours (1986); the numbers indicate the move {2,7}, {0,5} etc separating the endpoints of the tour.

"Let us begin by examining the first magic tour to be discovered. Beverley himself said nothing about the construction of his tour, but pointed out the special features of the tour: that it was magic, that the quadrants were also magic, and that the sum of the numbers in each 2×2 block was 130."

"If we divide the blocks into halves by vertical lines we obtain another feature which is significant, when we add up the numbers in each half-block. The 32 half-blocks alternately have totals 49 and 81. Beverley's tour is divisible into two pairs of contiguous contraparallel chains: one pair being 1–16 and 48–33 [adding to 49] while the other is 17–32 and 64–49 [adding to 81]."

"Using this construction, every column of a quadrant has sum 130, and every column of the board has sum 260. Beverley must in fact have envisaged that a tour composed in this way must be semi-magic and the possibility that by a suitable arrangement of the pairs of contraparallel chains it might be magic. All this could be seen before putting pen to paper to construct the tour."

"The critical half-blocks are those bearing the terminals of the contraparallel chains, 1 and 48, 49 and 32, 33 and 16, 17 and 64. The natural thing to do is to place 1 on a8 and 48 on a7. Now 49 is fixed: it must be on an '81' half-block and link with a7; it can only be on c6 and this means 32 on c5. Now there is a choice, 33 on a4 or e4: Beverley chose a4, which meant 16 on a3, 17 on c2 and 64 on c1. Thus the layout of the critical cells is possible, one on each row. "

"The next step is to see what other numbers have fixed positions. Obviously 18 must be on a1 and this means 19 on b3, 20 on d4, 63 on a2, 62 on b4 and 61 on d3. Similarly 50, 51 and 52 must be on a5, b7 and d8 respectively and this fixes the positions of 31, 30 and 29. We now have three numbers in four rows of the half-board fixed and, mindful that the columns of the quadrants sum to 130, we look for the fourth number on these rows so that the rows of the quadrants may also sum to 130. To achieve this c4 must be 15 and c3 34, both are possible. In the upper quadrant we find that c7 must be 2 and c8 47. The left-hand half-board is now completed."

"The right-hand half-board is easily completed, having regard to the necessity that each row of the half-board should sum to 130. All the [loose] terminals of the chains on the left-hand half of the board lie on the d-column, and there are two ways in which these terminals may be connected with cells in the half-blocks on the right-hand half of the board to complete the tour. Beverley chose one [27a] and Wenzelides the other [27b], both completions being necessarily symmetrical with respect to the horizontal median. This makes it possible to substitute other symmetrical arrangements, provided the half-block system is abandoned. Four such arrangements exist [27c, 27d, 27e, 27f]."

"It is also possible to construct other tours with the same contraparallel chains as the Beverley tour, but no longer adjacent. These are [05f, 16a, 27q]."

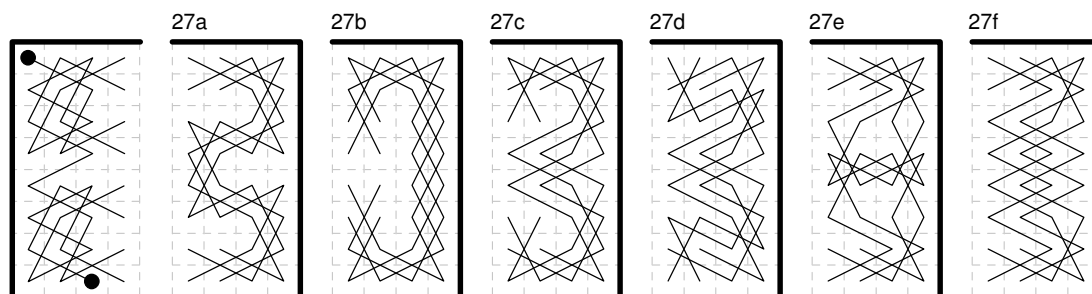
"As we have seen, Beverley exercised a choice between possible positions for the terminals of his contraparallel chains, one when he put '1' on a8, another when he put '33' on a4. There are in fact five different layouts which can be used, two with '1' on a8 and three with '1' on e8. The tours composed of two pairs of contiguous contraparallel chains are: [27a, b, g, h, j, k, l, m, n, o, p, q, r, s]. All of these layouts can be completed in the same way that Beverley adopted but with the abandonment of the condition that every half-row must add to 130, since some of the half-rows will be 128 and others 132."

"There is also an easier way of obtaining these layouts, since the same numbers necessarily occur in the upper half of the board. We have only to arrange the chains 1–8 and 48–41 and the chains 25–32 and 56–49 on the upper half-board in such a way that both pairs are contiguous and the rows all sum to 260 to get all possible arrangements [18] for the upper half of the board. By subtracting each number from 65 we obtain all possible arrangements for the lower half of the board, and by combining the two halves so as to obtain complete tours, we obtain all the magic tours [14] of the Beverley class." This method serves as a check on the enumeration.

KNIGHT'S TOUR NOTES

Magic Tours Related to the Beverley Tour

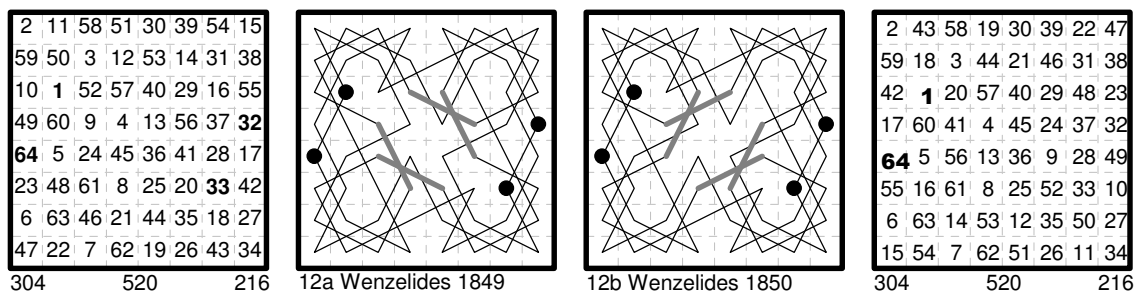
Subsequent composers have shown that the symmetrical half of Beverley's tour (27a) can be replaced by other symmetrical formations without affecting the magic property. Namely: Wenzelides 1850 (27b), Jaenisch 1862 (27c and 27d), Reuss 1880 (27e and 27f). In all these tours the half-board containing the end-points is of the same structure, both geometrically and numerically, and the symmetric half is always formed of symmetrically placed pairs of complements. The sum of the two diagonals is 520 ± 28 (i.e. 492 or 548) in every case, though individual diagonal sums vary a little.



1849-50 Three Magic Tours by Carl Wenzelides

The next three magic tours to be published were by Carl Wenzelides (1770-1852), and appeared in the magazine *Schachzeitung* under the editorship of Wilhelm Hanstein (1811-1850). A further three tours by Wenzelides were published in *Schachzeitung* nine years later in 1858 under the editorship of Otto von Oppen (1783-1860), the delay being due to the deaths of both Hanstein and Wenzelides in the intervening years. There may have been at least one other tour that was lost. The articles by Wenzelides also include much work on construction of tours with geometrical symmetry. For full publication details and biographical notes, see the Bibliography.

The first magic tour by Carl Wenzelides was published in *Schachzeitung* (vol.4 Feb-Mar 1849 p.94-97) as 'Der Rösselsprung in höchster Kunstvollendung' {The Knight's Tour in its Utmost Perfection} and is accompanied by a poetic encomium signed Hn. [Hanstein]. The tour, which is given in both geometrical and arithmetical forms, is the first closed magic knight tour to be discovered (12a in my catalogue), and the first of squares and diamonds type, and also the first symmetric magic tour. The tour also has the extra property that the sum of the two diagonals is 520 (twice the magic constant). It was constructed by Wenzelides on 19-20 Feb 1849. In the geometric diagram the four 5-move paths like a7 to g8 are shown bold, and in the arithmetical form the numbers 1 and 64 are printed bold.

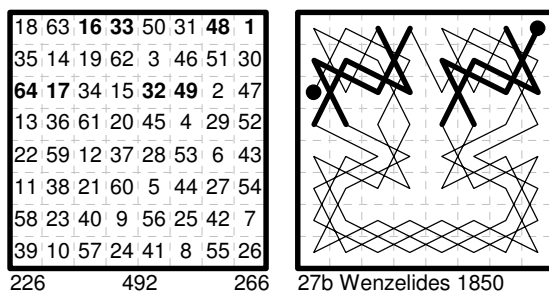


The next year Wenzelides published a similar tour (12b) which differs from 12a only by a 90 degree rotation of the four central links. Because of symmetry these tours can be numbered from any of the four cells marked here by dots, though numbering from the other points does not give any new arithmetical form, merely a rotation. There are eight pairs of complements (adding to 65) placed symmetrically in the ranks and also eight pairs in the files. The same pairs, read in the files and ranks, are also the 'counter complements' (adding to 65 ± 32 , that is 97 or 33).

Much of the article is an explanation of the 'quartes' method under which name he includes squares, diamonds, Beverley and irregular four-move paths within a 4x4 area. Figs 73-84 show these quartes and combinations thereof.

KNIGHT'S TOUR NOTES

This article also includes tour (27b) Fig.107 p.247, the first variant of the Beverley tour, though oriented to have 1 at top right.

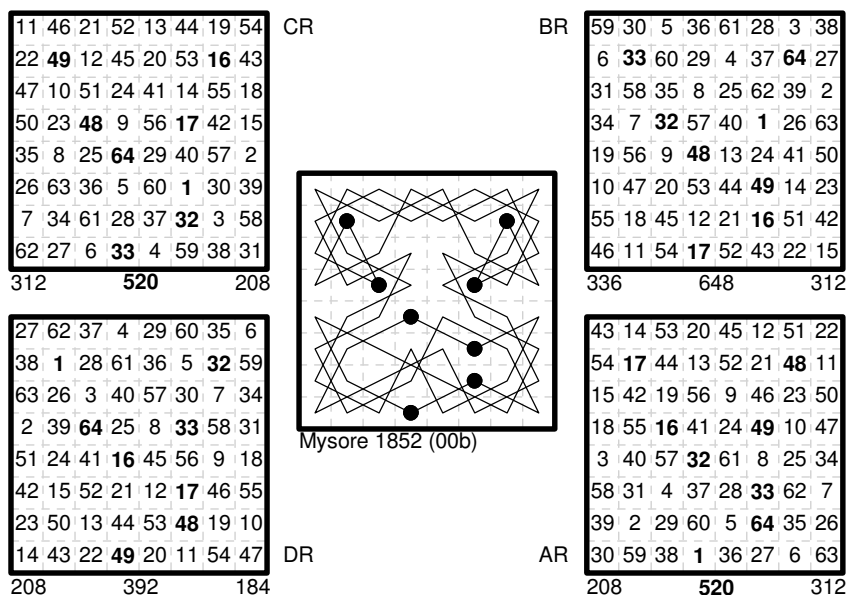


The Lost Work of A. F. Svanberg.

Murray wrote: “In the same year (1849) the *Schachzeitung* announced that A. F. Svanberg, Professor of Mathematics in the Stockholm University, had also discovered four magic tours ‘as a result of mathematical reasoning’, but these were never published and are not to be found among Svanberg’s papers now preserved in Stockholm. All that we know of them is that they were found later than Wenzelides’s first tour and that one was ‘concordant’ with Wenzelides’s first tour, whatever that may mean.” I have not traced where this note appears in *Schachzeitung*.

1852 The Rajah of Mysore’s Tour

Meanwhile in India Krishnaraja Wodeyar III, Maharajah of Mysore (1794-1868) was the third composer to construct a magic knight's tour on the 8x8 board, which he had printed on a silk panel, giving the date of construction as 31 July 1852. However, his composition of this tour did not become known outside of India until 1922, and it was in the meantime discovered independently by Edouard Francony in 1882. His magic tour (00b) is closed and of squares and diamonds type and is fourfold magic in that it can be numbered magically from four different origins by cyclic shift of the numbering. The eight numberings of 00b are numbers 33, 75, 103, 104, 186, 217, 232, 260 in our catalogue of all 280 arithmetical forms (p.47). The silk shows the numerical form 186 inverted as shown here (any one numbering can be shown in eight orientations). On the silk the cells on the diagonals of the quarters are shaded black, unlike our usual chequering, in other words the diamond quartes are dark and the square quartes light.



See the entry for Mysore in the Bibliography (§ 12) for more biographical details. The Rajah also did much other work on tours. In particular see § 7 for 12x12 tours and § 11 for Figured Tours.

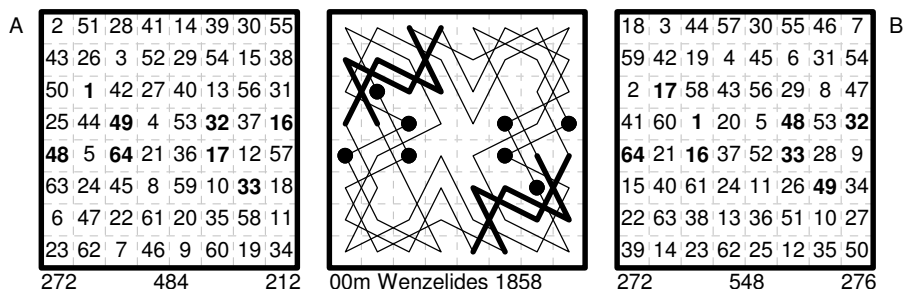
KNIGHT'S TOUR NOTES

1858 Three More Tours by Wenzelides

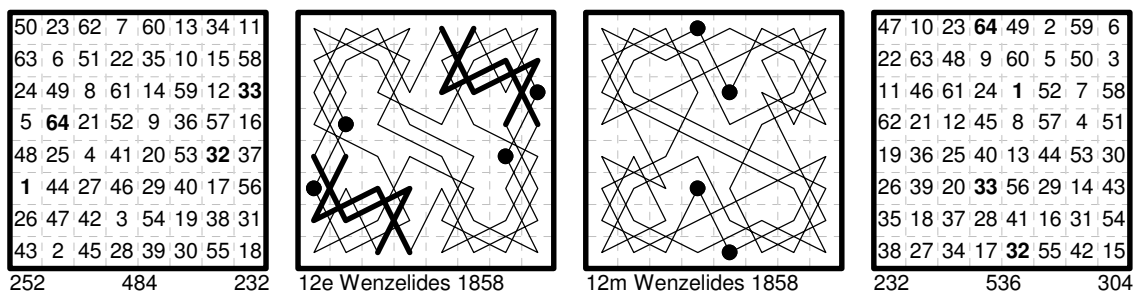
The other three magic tours (00m, 12e, 12m) by Carl Wenzelides were published in *Schachzeitung* (vol.13 May 1858 p.174–175) in an article signed ‘v. Oppen’ the new editor.

Jaenisch (1859) wrote: “Mr C. Wenzelides, an Austrian amateur, was the first to give, in the *Schachzeitung* for 1849, a tour of this kind [with diagonals adding to 520] and one which besides is symmetrical. He has since announced (*Schachzeitung* 1850, p.238) the discovery of six more symmetrical tours, the sum of whose figures in each rank and file amount to precisely the same. Death having unfortunately prevented him from giving them to the public, they would have been entirely lost if the Privy Councillor O. von Oppen had not succeeded in finding three of them, among some old letters addressed to the Editors of the *Schachzeitung*. And connoisseurs certainly owe a debt of gratitude to Mr Von Oppen for the publication of these beautiful solutions in the number of that magazine for May 1858.”

These tours are on a separate page of four figures in *Schachzeitung* lettered A to D and are shown graphically. The first is (00m) in our geometric catalogue (41, 147, 152, 222 in the arithmetical list) and uses two pairs of Beverley quartes. Among the 16 symmetric magic tours now known this is still the only cyclic example. The numbering derived from the horizontal links (00mB) differs from that derived from the vertical links (00mA) by a cyclic shift of 16 cells. Since the tour is symmetric a shift of 32 cells in A merely rotates it, and a cyclic shift of 48 cells in A gives a rotated form of B.



The second is (12e) in the catalogue also with two pairs of Beverley quartes.
The third is (12m) of squares and diamonds type.

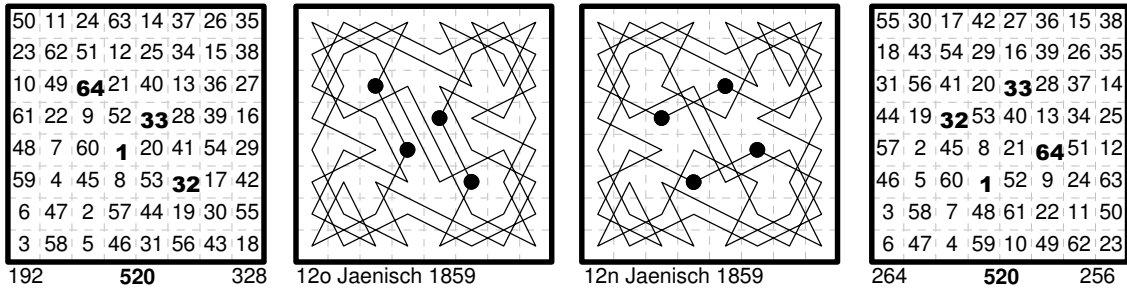


The diagram D (not included here) is a semi-magic tour formed of two semi-magic half-board tours one above the other. Besides the 5 symmetric magic tours found by Wenzelides a further 11 are now known, all in the (12) class.

1859 Two Tours by Carl Jaenisch

The fourth composer of magic knight tours **Carl Friedrich von Jaenisch** (1813 - 1872) from St Petersburg published his first two examples in three issues of *Chess Monthly* (1859) as a preview of his forthcoming treatise (1862). See the Bibliography (§ 12) for more biographical details and page references. These articles contain two new symmetric magic knight tours of rhombic type (12o) and (12n). He follows the convention that the number 1 is to appear in the triangle a1-d1-d4. He terms these tours as ‘thrice reentering’ since the pairs of cells 1-32 and 33-64 are knight moves as well as 64-1, so that the numbers 1, 32, 33, 64 in each case form a knight circuit and are shown in bold. The diagonals together add to 520, and each differs from the magic sum only by ± 4 .

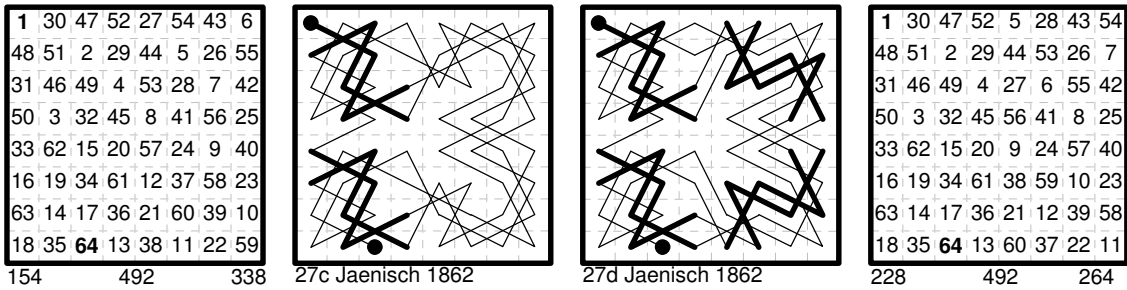
KNIGHT'S TOUR NOTES



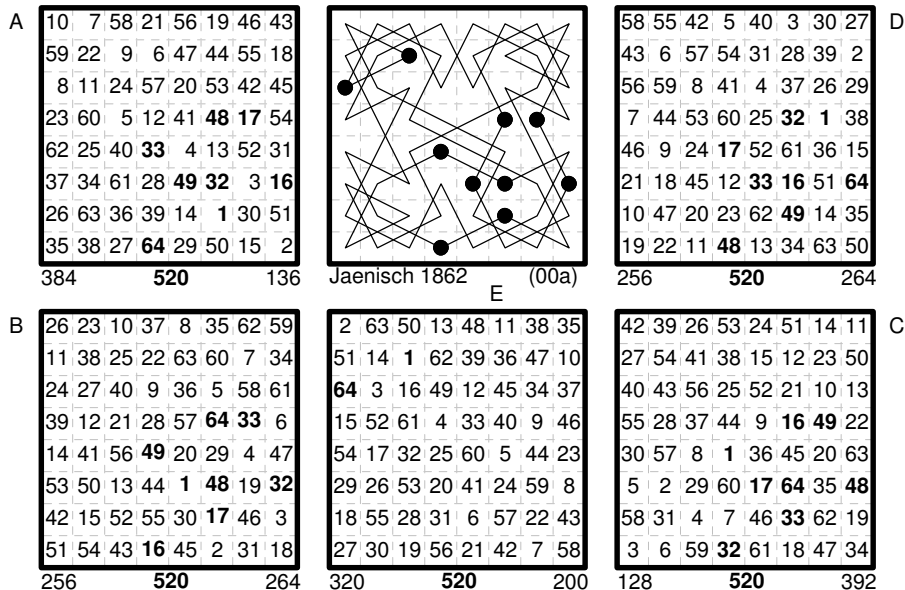
1862 Four More Tours by Jaenisch

In his *Traité des Applications de l'Analyse Mathématique au Jeu des Echecs* (vol.2 1862) Jaenisch gives four further magic tours. These are magic tours 11-14 in the historical sequence.

Tours (27c) Fig.55 and (27d) Fig.56 are of Beverley type. Shown p.187 numbered from c1 as 1. (27d) employs four pairs of Beverley quartets.

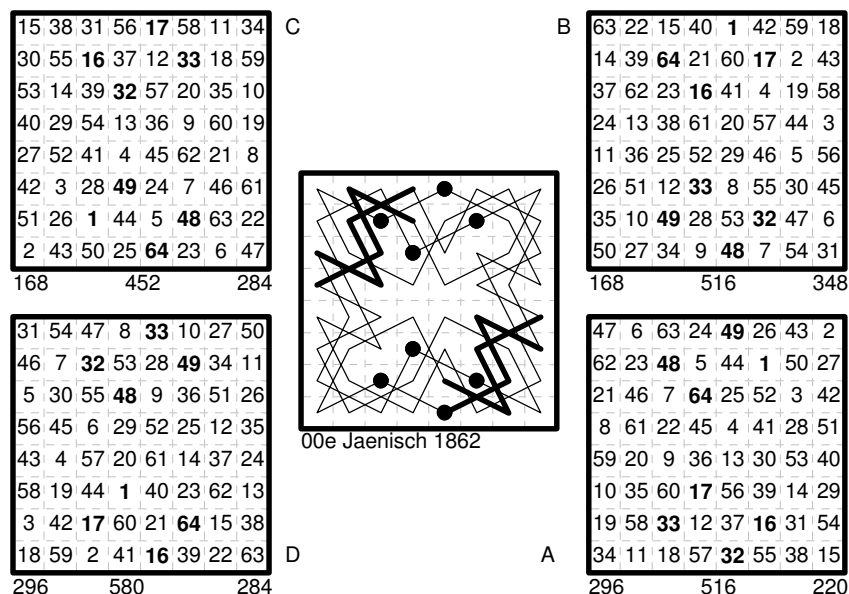


Tour (00a) Fig.49 has diametrically opposite cells differing by 8, and is especially interesting since it is magic when numbered from ten different origins, the most ever achieved. The numberings occur in five pairs, each the reversal of the other, since the reverse of a magic tour is also always magic. Jaenisch in fact only shows four pairs of magic numberings (A to D). The fifth pair (E) was noted by Exner (1876). Also, of course, each square can be presented in eight different orientations by rotation and reflection. All have sum of diagonals 520. The individual diagonal sums are 256+264 (twice), 136+384, 128+392 or 200+320. (The ten cases are 14, 56, 62, 81, 112, 121, 226, 253, 263, 271 in the arithmetical list.)



KNIGHT'S TOUR NOTES

Tour (00e) Fig.62 has Beverley quartes in opposite quarters, as do two by Wenzelides. The eight Figures on p.235-6 of the Treatise are the numberings of (00e). The eight entries in our arithmetical list are 22, 28, 44, 45, 228, 231, 244, 250.



Murray wrote: “Jaenisch’s contemporaries regarded his treatment of magic tours as definitive, and no new magic tours were discovered during the next fourteen years.”

The Age of Magic Tours 1876-1886

1876: A. Feisthamel in *Le Siècle*

A series with the title ‘Un Problème Par Jour’ edited by A. Feisthamel begins in the French newspaper *Le Siècle* {The Age} on 30 Oct 1876. The first knight’s tour that appears is the solution to problem 4 published on 2 Nov 1876 under the heading ‘Polygraphie du Cavalier’. It is presented in the form of a cryptotour with two or three letters to a cell, which when read in the correct knight-move sequence spells out an acrostic verse whose solution is a 4×4 word square. Thus in effect three problems in one! The solution, together with another tour problem appears in the issue for 10 Nov, and subsequent tours appear weekly. The series continued under Feisthamel’s editorship until Problem 5656 on 30 Apr 1894, and then for a short while under Emile Franck. *Le Siècle* can now be accessed online through the Gallica BnF (Bibliothèque nationale Française) website.

Murray (1951) wrote: “A period of great activity in the composition of magic tours opened in France in 1880, largely stimulated by M[onsieur] A. Feisthamel in his chess column in *Le Siècle* 1876–1885, in which he published all the known magic tours and new ones as they were produced.”

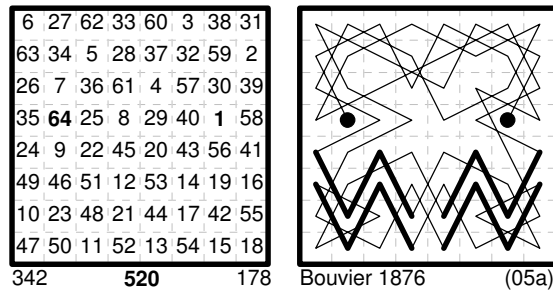
In the following sections I show the tours in the sequence and orientation that they appeared in *Le Siècle*. A few however I could not locate there. Many tours also appeared in other French newspaper columns. There were also contributions from Exner in Germany and Caldwell in England.

Apart from A. **Béline** (2 tours) and A. **Feisthamel** (1 tour) who used their own names the other contributors of tours adopted pseudonyms, which Murray identifies as: ‘Adsum’ = **Charles Bouvier**, ‘Céline’ = **Edouard Francony**, ‘Paul de Hijo’ = **Abbé Jolivald**, ‘Palamède’ = **Count Ligondès** of Orleans; ‘X à Belfort’ = **Prof C. E. Reuss**. The only one of these actually identified by Feisthamel is Reuss whose first name he gives as Emil. I’m not sure where Murray got his information about these pen-names, possibly from the Parmentier catalogues. Murray says: “Altogether this group added 63 geometrical and 80 arithmetical magic tours. The majority are composed of quartes. None of these composers gave any indication of the methods used to obtain his tours.”

KNIGHT'S TOUR NOTES

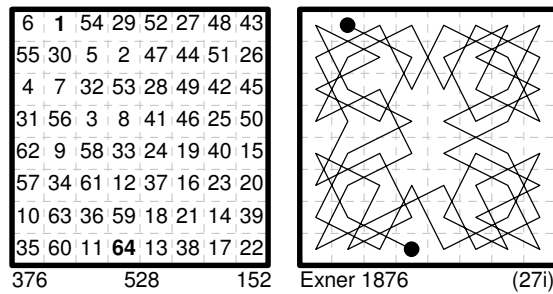
1876 Charles Bouvier (pen name 'Adsum').

Magic tour (05a) the first to make use of irregular 'Bouvier' quartes. I am not sure where this appeared. It is assigned to this date in Murray's catalogue. Only two others use this type of irregular quartre, namely (00j) and (27f) by Reuss 1880. (See also Bouvier 1882 and 1884).

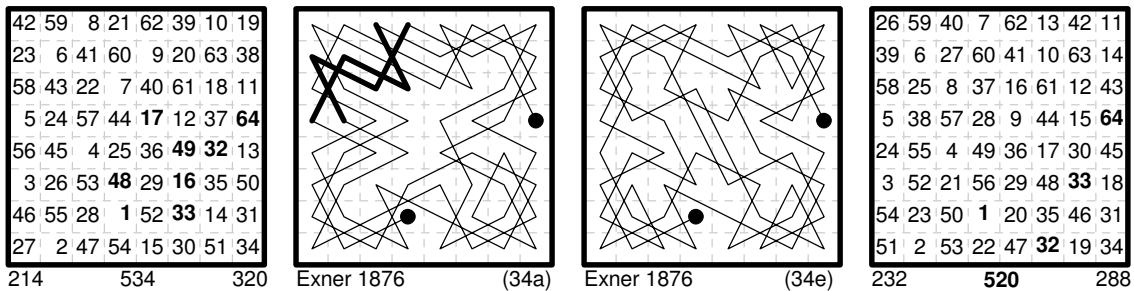


1876: (Dr) Heinrich Gustav Exner

Independent German work on magic tours was published in an Easter report of the *Königlichen Gymnasium zu Hirschberg*. (See Bibliography ☞ 12.). This contains 15 magic tours including the fifth arithmetical version of (00a) by Jaenisch. Three were new tours. One a semisymmetric (27i).



Two (34a, 34e) which are the first to have the {3,4} end-point separation. Also (34a) is the first to use only one pair of Beverley quartes. I have oriented these to correspond to the other four with the same end point position found later by Ligondes (1883).



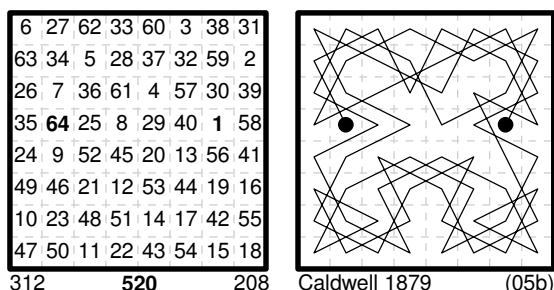
A magic tour in *L'Echiquier* (1925 p.44) and in Kraitichik (1927 p.37) attributed to **Mansion** (1876) is in fact (12a) by Wenzelides.

KNIGHT'S TOUR NOTES

1879: E. C. Caldwell

'The Knight's Tour on the Chessboard as a Magic Square' *English Mechanic and World of Science* vol. 29 (1879) p.317. Gives two magic knight's tours, one quoted from "a modern French Cyclopedia" *Larousse's Dictionnaire Universel*, this is Jaenisch (12o) in our catalogue, the other "is original and has never before been published" (05b). Both are of squares and diamonds type.

He notes "...the errors in excess in one section are made to compensate the errors in defect in another section, and vice versa."

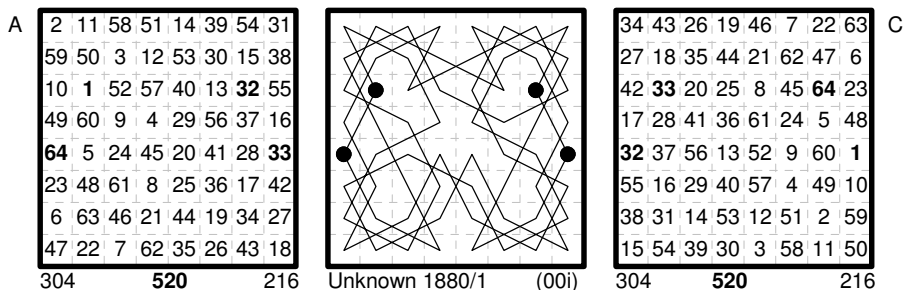


1879: Edouard Francony ('Mme Céline Fr.')

A tour attributed to this author, ¶730 in *Le Siècle* 7/14 Mar 1879 is (12b) by Wenzelides 1850.

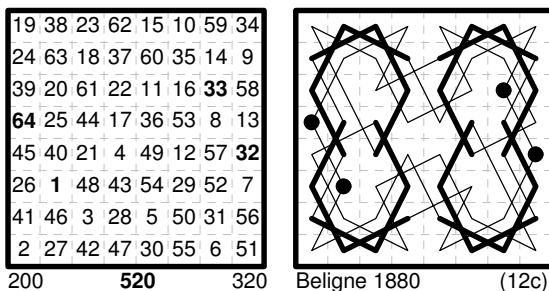
1879 — Unknown Author

This magic tour (00i) by an unknown composer appears as problem 772 in *Le Siècle* 25 Apr (sol 2 May) 1879. The word puzzle used with it is given as by M. Jacquemin-Molez but the composer of the design is not identified. This date is earlier than previously given. In the arithmetical catalogue the four forms are 12, 50, 223, 248. The second arithmetical form of the tour is labelled C to indicate that the cyclic shift is of 32 cells (not 16 as in 00m).



1880: A. Béligne

In *Le Siècle* magic tour symmetric (12c) = ¶1042 5/12 Mar 1880. This is the first with 'syncopated' quartes, spreading over the bar lines.

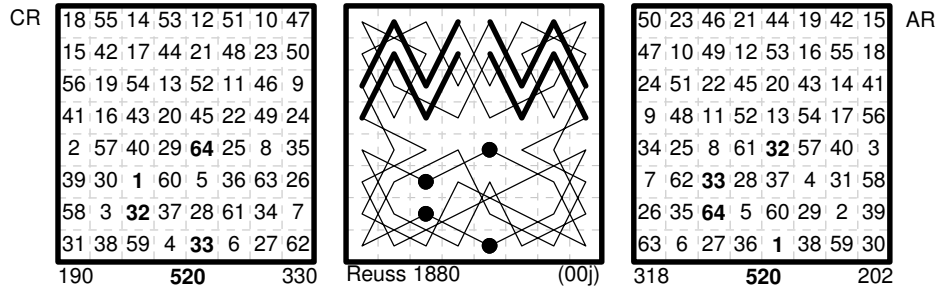


KNIGHT'S TOUR NOTES

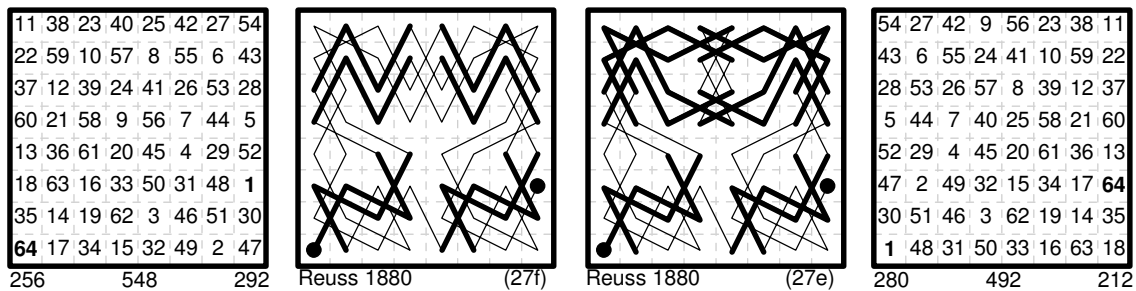
1880: C. E. Reuss ('X à Belfort')

He gives four new magic tours in *Le Siècle* Oct - Dec 1880. Three of these tours are given the later date of 1883 in Murray's catalogue. My 1986 catalogue attributed (27f) to Wihnyk who discovered it independently in 1885.

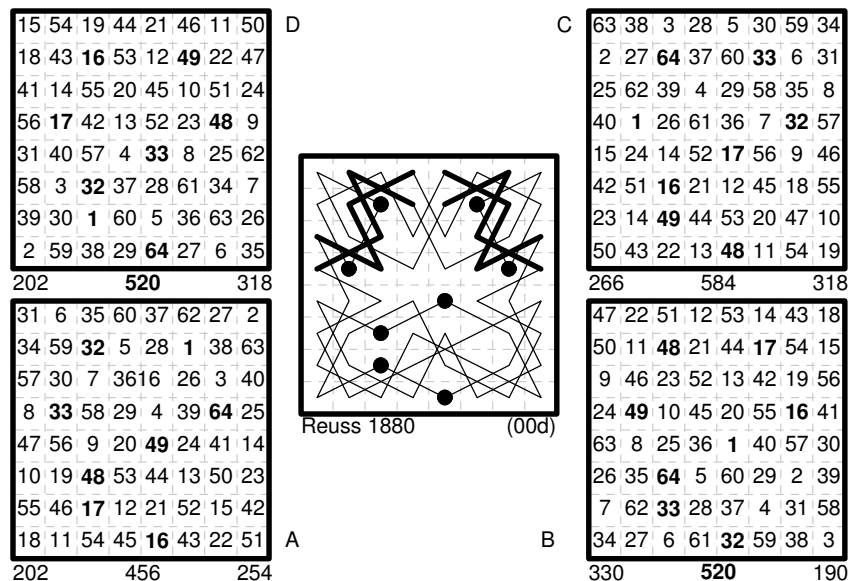
One is the two-fold magic (00j) ¶1252 *Le Siècle* 5 / 12 Nov 1880. (Our arithmetical catalogue entries are A 36, C 76, AR 209, CR 238.)



Two are modifications of Beverley's tour using irregular quartets in the other half. (27f) ¶1258 *Le Siècle* 12 / 19 Nov 1880 and (27e) ¶1270 *Le Siècle* 26 Nov / 3 Dec 1880.



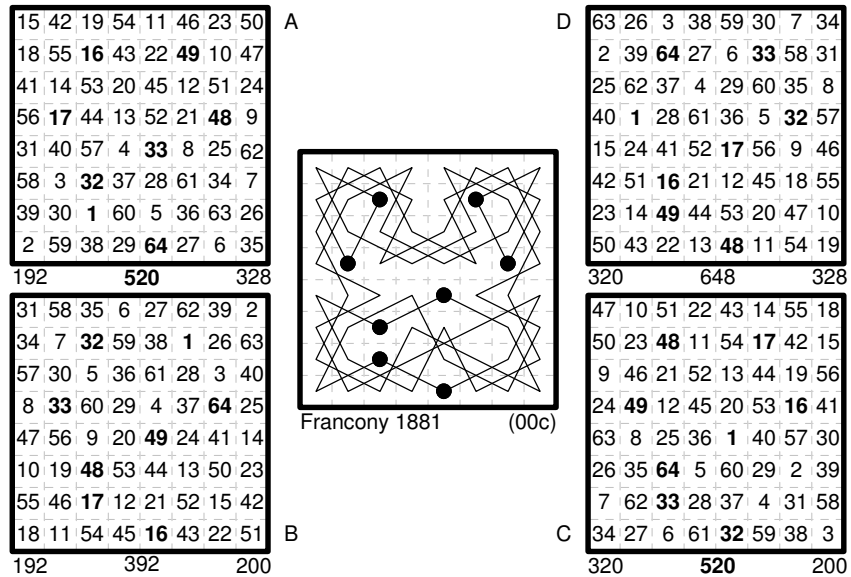
The fourth is fourfold cyclic (00d) ¶1276 *Le Siècle* 3/10 Dec 1880, though this is not stated. (The arithmetical versions are 30, 31, 34, 78, 204, 211, 236, 256.)



KNIGHT'S TOUR NOTES

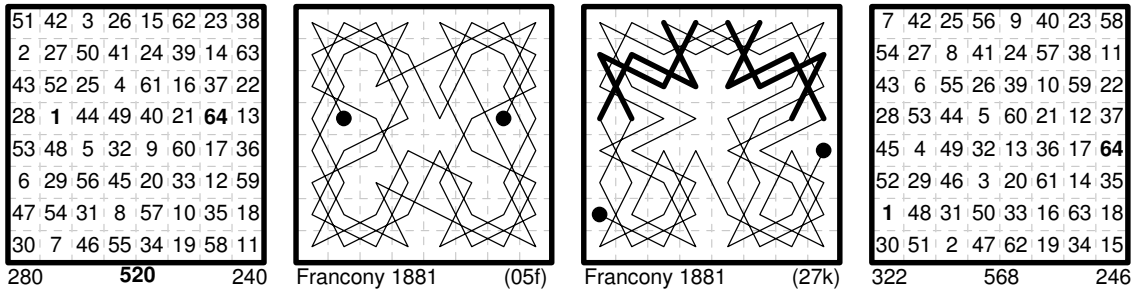
1881: Edouard Francony ('Céline')

Three tours, one (00c) fourfold magic is like (00b) which he found independently. The arithmetical catalogue entries are 35, 37, 38, 77, 205, 210, 237, 257.



The half with the three-move line is the same as (00d) and (00j) as well as (00b).

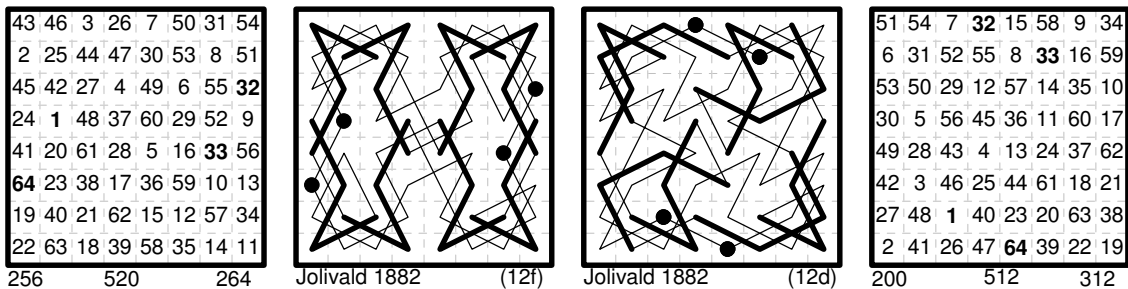
Another of squares and diamonds type (05f), and one with two pairs of Beverley quartes (27k).



Murray dates these three magic tours 1881 but I have not found were they were first published.

1882: 'Paul de Hijo' pen name of (Abbé) Philippe Jolivald

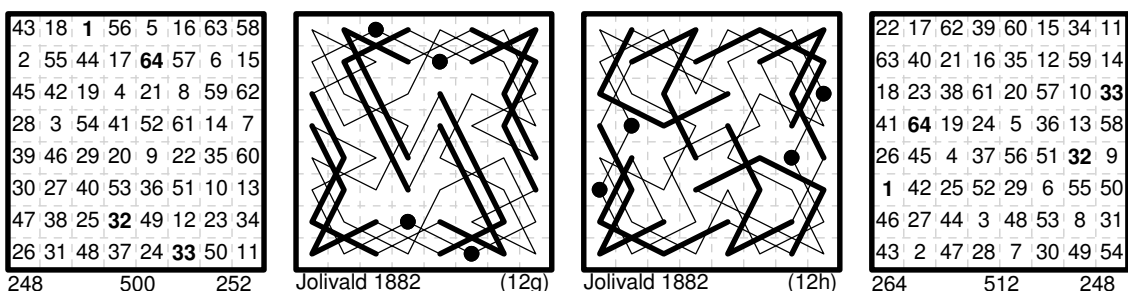
This author gave four magic tours, in *Le Siècle* presumably related to his work on enumerating circuits that appeared the same year.



These are: 12 May 1882 ¶1726 (12f), 19 May 1882 ¶1732 (12d), 26 May 1882 ¶1738 (12g), 2 Jun 1882 ¶1744 (12h). All symmetric with 8 extended quartes.

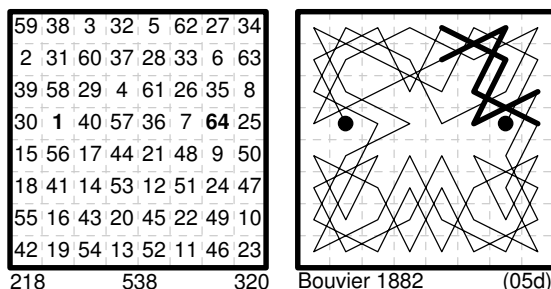
KNIGHT'S TOUR NOTES

The tour (12g) when reflected or rotated includes a striking letter N or Z outlined.



1882: Charles Bouvier (alias 'Adsum')

Magic knight tour (05d) with a pair of Beverley quartes. Oriented here to match Caldwell 1879.

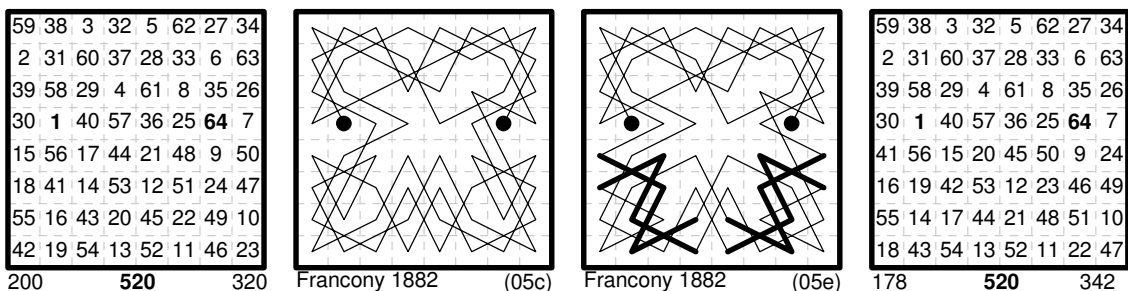


By deduction I suspect that this is one of two new magic tours noted by Feisthamel (in *Le Siècle* 18 Aug 1882) as being published in another French journal *Le Telegraphe* (Problem Nos 1232 and 1239) between May and August that year, but I've not seen this source.

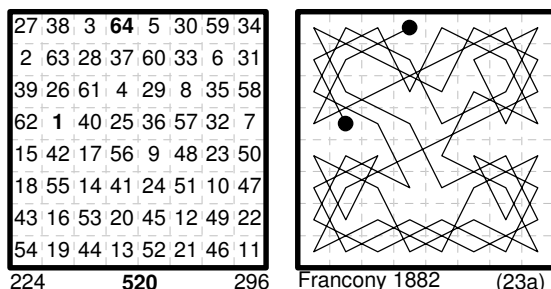
1882: Edouard Francony (alias 'Mme Céline')

The author of the other tour in *Le Telegraphe* recorded by Feisthamel 18 Aug 1882. My guess is that it was the Mysore tour. Murray records these further three magic tours from 1882, but where published is not known. They are similar to the Bouvier tour (05d) above.

Two with {0,5} end-separation: (05c) and (05e):



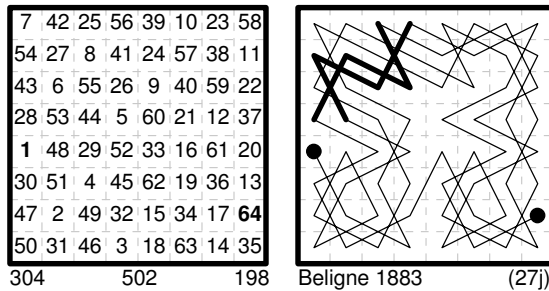
and (23a) shown here (see also 1884):



KNIGHT'S TOUR NOTES

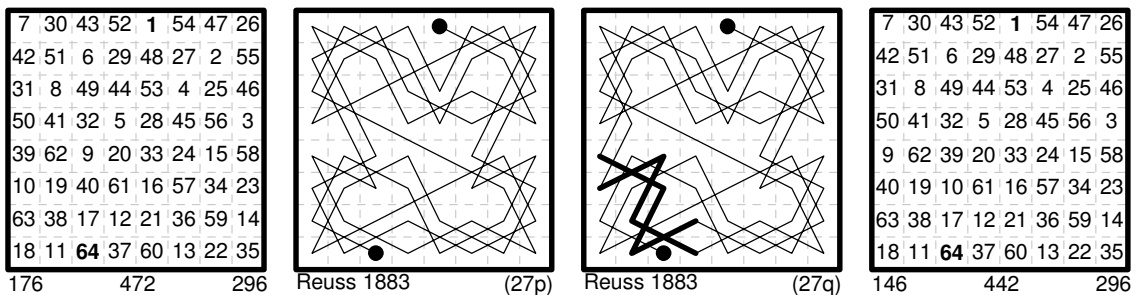
1883: A. Béligne

In *Le Siècle* magic tour (27j) = ¶1966 16/23 Feb 1883 (Murray dates it as 1881 so maybe it appeared earlier somewhere?).

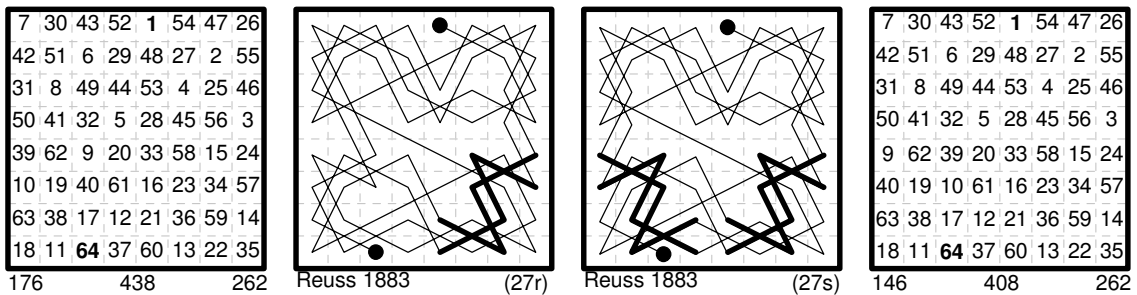


1883: C. E. Reuss ('X à Belfort')

Four new magic tours. These were found in 1883 according to Murray, but I have not found where they first appeared. See also 1880.

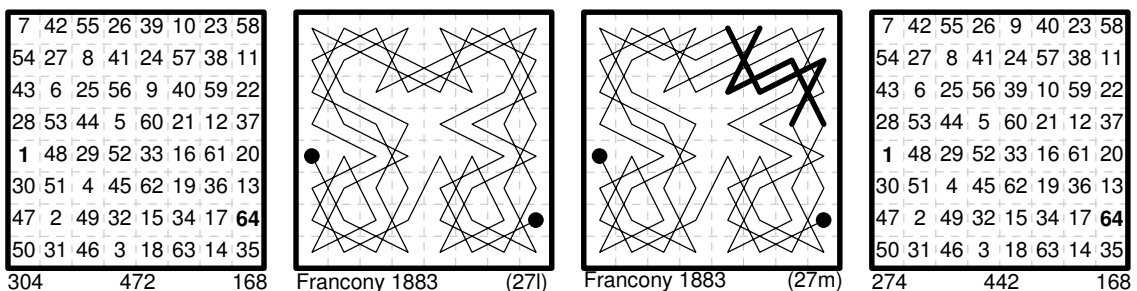


These four tours 27pqrs all include three three-unit lines forming a Z, and all have the same upper half both geometrically and numerically.



1883: Edouard Francony ('Mme Céline')

These two tours are the solutions to ¶1156 in *Gil Blas* (23 Feb / 2 Mar 1883).



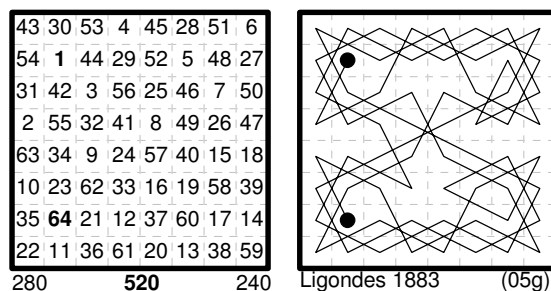
KNIGHT'S TOUR NOTES

[However tour (27m) also appears at the same dates as problem ¶1872 in *Le Siècle* where it is attributed to 'Adsum' (= **Bouvier**). The orientation is the same in each publication] Murray dated these tours 1881 and 1882, so maybe they appeared somewhere earlier.

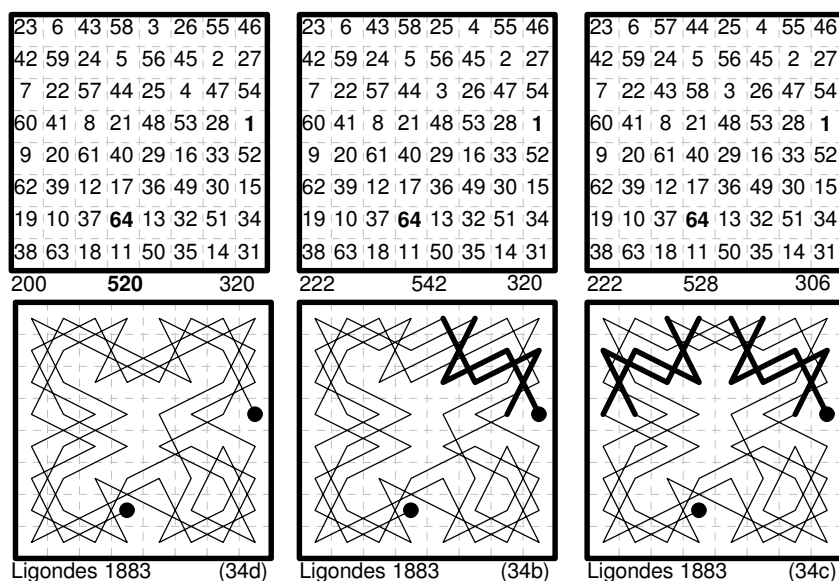
1883/4 (Vicomte) Raoul de Ligondès (alias 'Palamède')

Murray (1951) lists 32 tours as composed by Ligondès in the early 1880s. The following is a list of the problem numbers and dates of those that appeared in *Le Siècle* in 1883/5, which can now be consulted online. *Le Siècle* ¶1996 (05g) 23/30 Mar 1883, ¶2086 6/13 Jul 1883, ¶2092 (34b 34c 34d) 13/20 Jul 1883. [Three others in these sets (27i), (34a), (34e) were previously found by Exner (1876)] ¶2098 (14ab, 23 eghijkl) 20/27 Jul 1883, ¶2110 (23d, 25a) 3/10 Aug 1883 [and (05d) which is attributed by Murray to Bouvier 1882] ¶2116 (23fmn) 10/17 Aug 1883. ¶2122 (03f, 01b, 00g) 17/24 Aug 1883. ¶2128 (14c, 01c, 23c) 24/31 Aug 1883 [but he had published (23c) earlier in *Le Gaulois* on 6 Aug 1883]. ¶2134 (00f) 31 Aug/7 Sep 1883. ¶2242 (34f) 4/11 Jan 1884 ¶2248 (34g) 11/18 Jan 1884. ¶2254 (03e) 18/25 Jan 1884, ¶2260 (03a) 25 Jan/1 Feb 1884. ¶2266 (01a, 03cd) 1/8 Feb 1884. ¶2836 (23b) 4/11 December 1885. Ligondès (as Palamède) also has two magic tours in *Le Gaulois* 1883. One in ¶584 9/16 Jul 1883 is (34e) anticipated by Exner 1876. Another ¶608 in 6/13 Aug 1883 is (23c) also given in *Le Siècle*. Diagrams of all the above now follow.

The first is ¶1996 *Le Siècle* 23/30 Mar 1883 tour (05g) with a striking crossover in the middle.



In *Le Siècle* 6 and 13 July 1883 as ¶2086 and ¶2092 Ligondès (as Palamède) published two sets of three tours consisting of one of (27) type and five of (34) type. Three of these (27i), (34a), (34e) were previously found by Exner (1876). The other three (34b, 34c, 34d) are shown here.

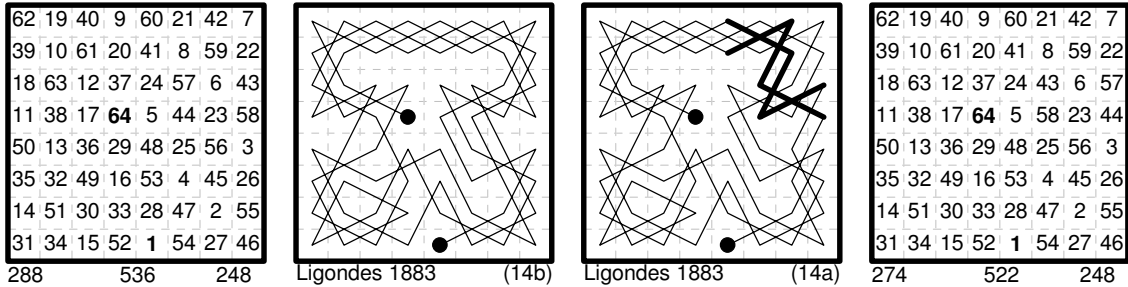


The second differs from the first only by the transposition of two pairs of numbers on ef68 (3, 26 and 25, 4) adding to 29. Similarly the third differs from the second only by a transposition on cd68 (43, 58 and 57, 44) adding to 101.

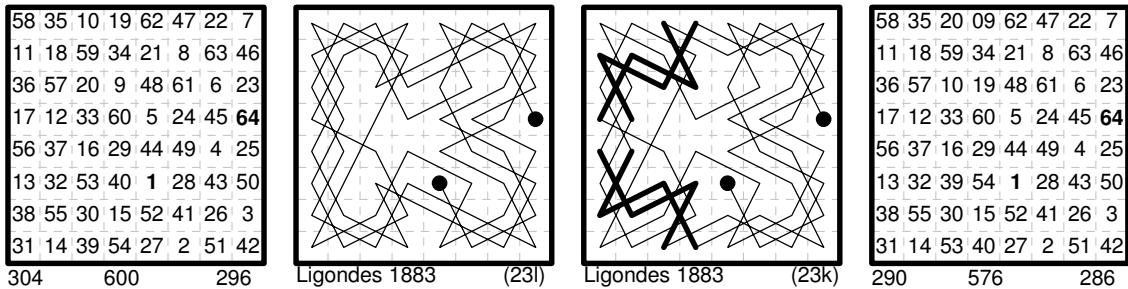
KNIGHT'S TOUR NOTES

In *Le Siècle* ¶2098 20 July 1883 Ligondes (as Palamède) published a set of nine magic tours: two of 14 type, seven of 23 type. They are presented as a cryptotour having a 3×3 array of letters in each cell of the chessboard. I show them in the orientation and sequence shown in this original source.

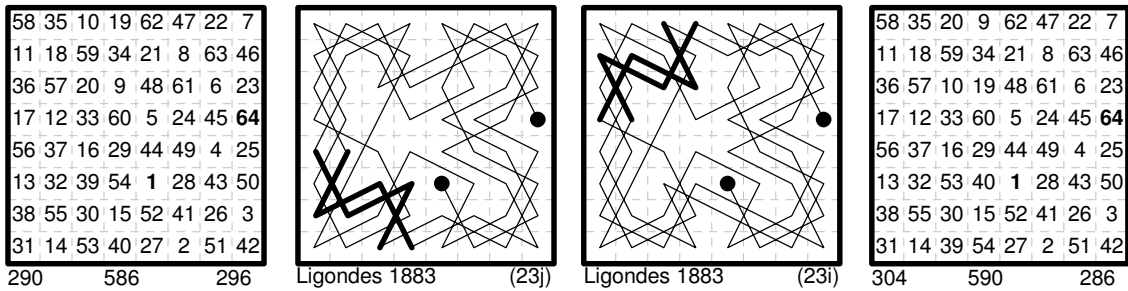
The 14a and 14b include extensive braids. They differ only by the transposition of two pairs of numbers in the f and h files (43, 58 and 44, 57) adding to 101.



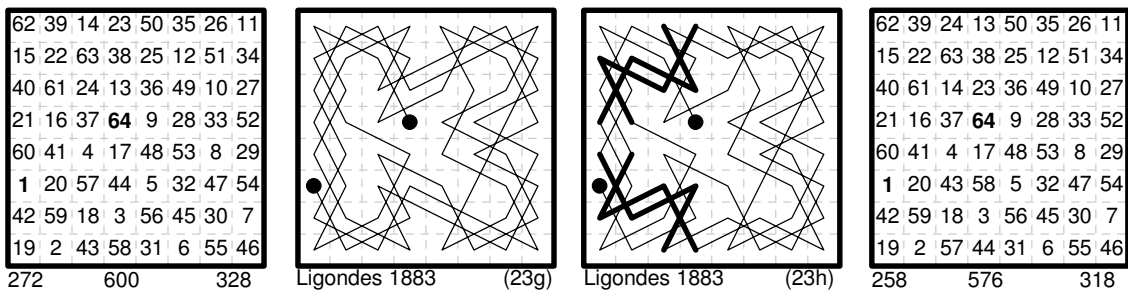
Eight of type 23. As with other examples the Beverley quartets serve to transpose two pairs of equal-summed numbers in the underlying squares and diamonds tour. 23l and 23k



23j and 23 i:

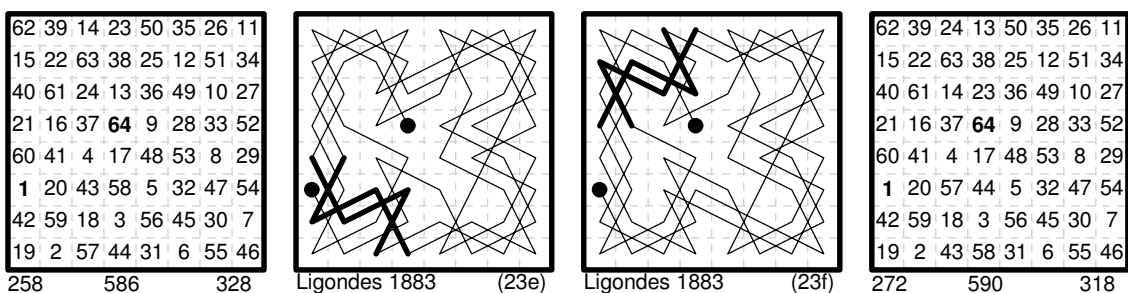


23g and 23h:

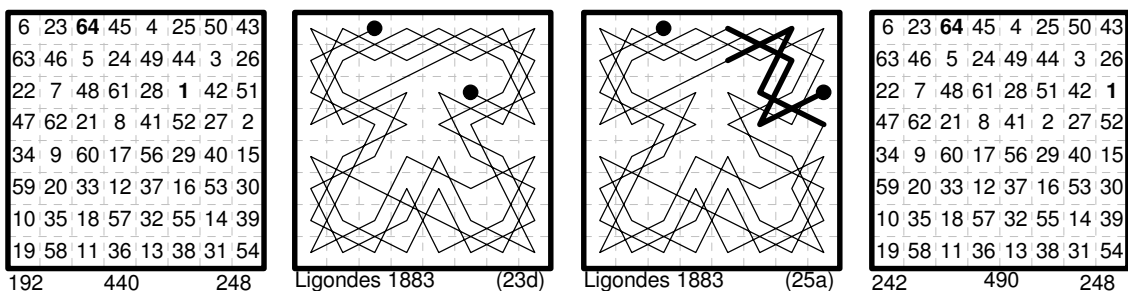


KNIGHT'S TOUR NOTES

23e and 23f (the tour 23f is from the later 10 Aug batch).

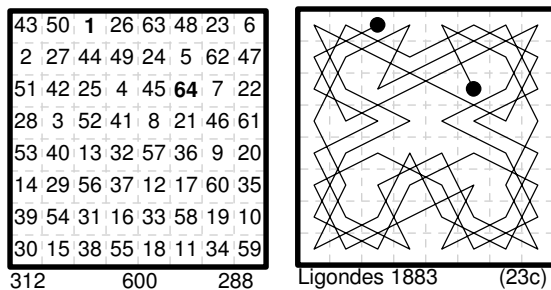


Le Siècle 3 Aug 1883 ¶2110 Ligondes gives three magic tours two are new (23d), (25a).

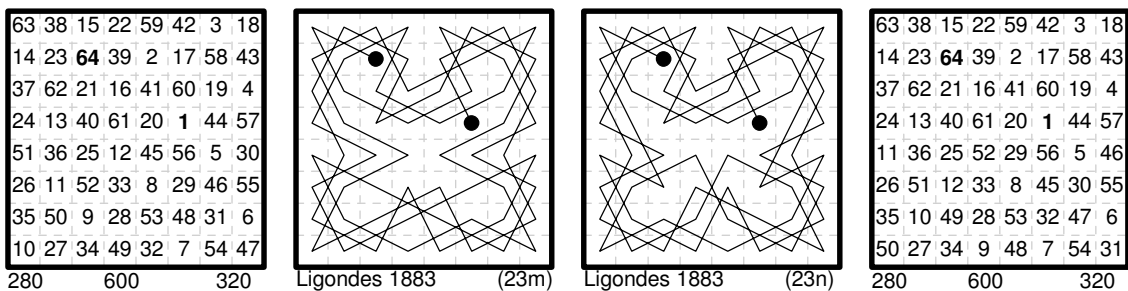


But the third is (05d) shown earlier is attributed by Murray to Bouvier 1882.

Count Ligondes (as Palamède) has two magic tours in *Le Gaulois* 1883. The one in ¶584 9/16 Jul 1883 is (34e) anticipated by Exner 1876. The other ¶608 in 6/13 Aug 1883 is (23c).

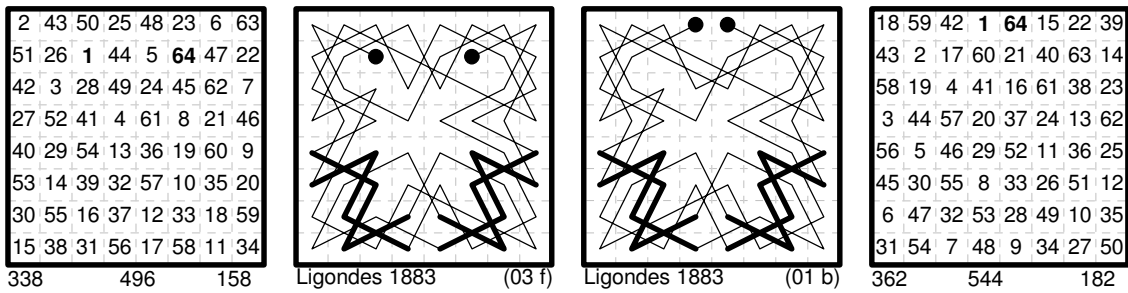


Le Siècle 10 Aug 1883 ¶2116 Ligondes gives three magic tours (23m), (23n) and (23f) which has been shown above, at the end of the 20 July group.

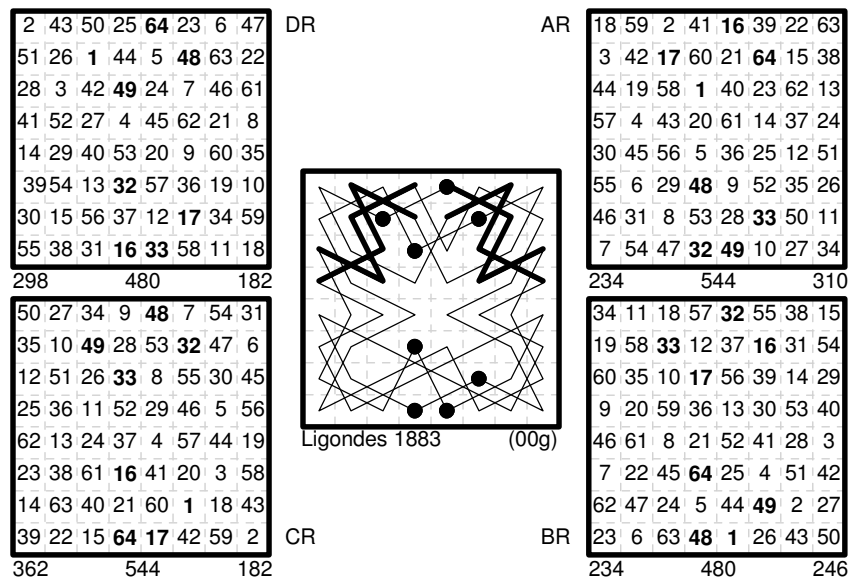


KNIGHT'S TOUR NOTES

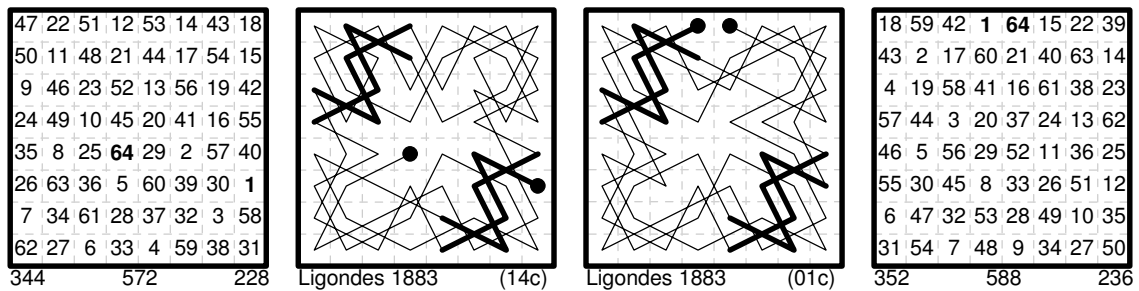
Le Siècle 17 Aug 1883 ¶2122 Ligondes gives three magic tours. The first two are (03f), (01b).



Third a closed tour of cyclic type (00g). (Arithmetic list 26, 42, 53, 148, 153, 216, 221, 224.)



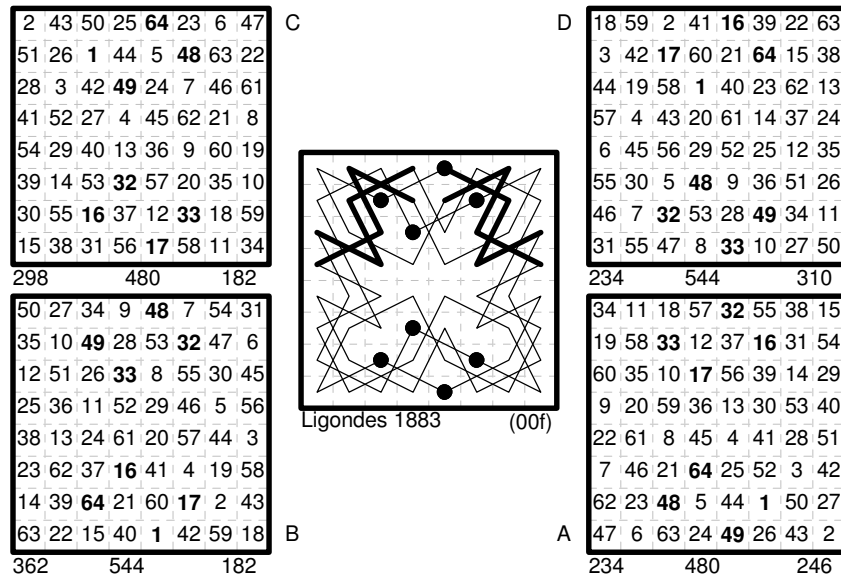
Le Siècle 24 Aug 1883 ¶2128 Ligondes gives three magic tours (14c), (01c), (23c)



but (23c) appeared earlier in *Le Gaulois* on 6 Aug 1883 (see above) reflected left to right.

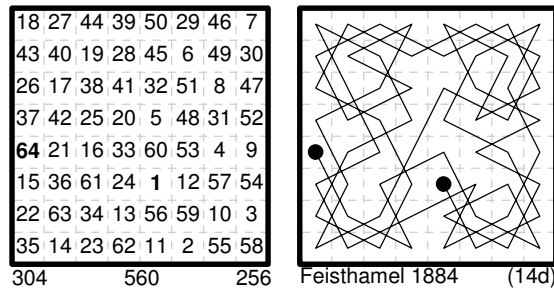
KNIGHT'S TOUR NOTES

This cyclic tour (00f) is ¶2134 *Le Siècle* 31 Aug 1883 (sol 7 Sep 1883). Arithmetic catalogue entries are 23, 27, 43, 46, 229, 230, 243, 251.



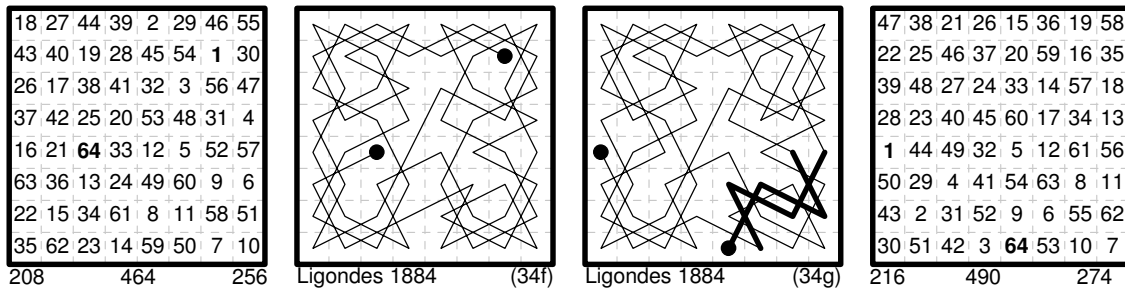
1884: A. Feisthamel

The editor of the chess puzzle column in *Le Siècle* {The Age} published a magic tour of his own (14d) on 4 Jan 1884 ¶2242 (solution in 11 Jan 1884). It is set as a double tour problem, the first solution being the very similar tour (34f) by Ligondes shown below.



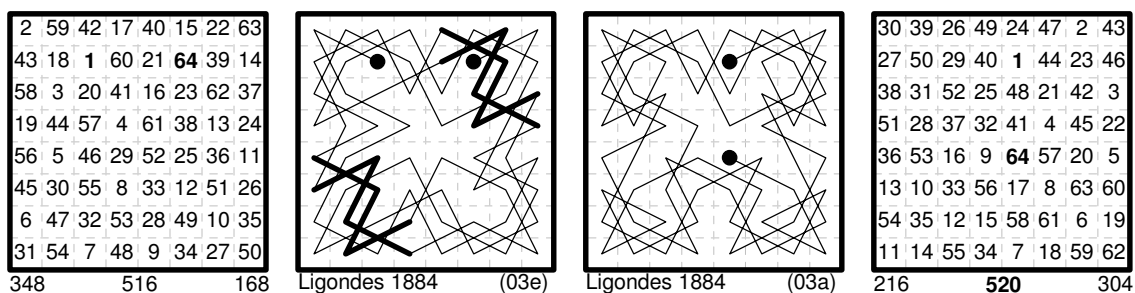
1884-5 (Vicomte) Raoul de Ligondès (alias Palamède).

Le Siècle ¶2242 (34f) 4 Jan 1884 and ¶2248 (34g) 11 Jan 1884 [both dated 1883 by Murray]. The first is given as part 1 of a problem which has the Feisthamel tour (14d) above as part 2. The paths of (34f) and (14d) are identical apart from two moves. The tour (34g) is also similar to (14d), but with a Beverley structure in one quarter.

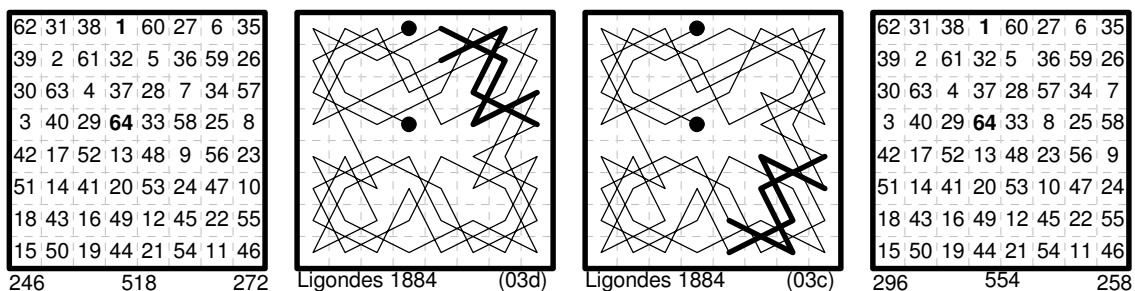


KNIGHT'S TOUR NOTES

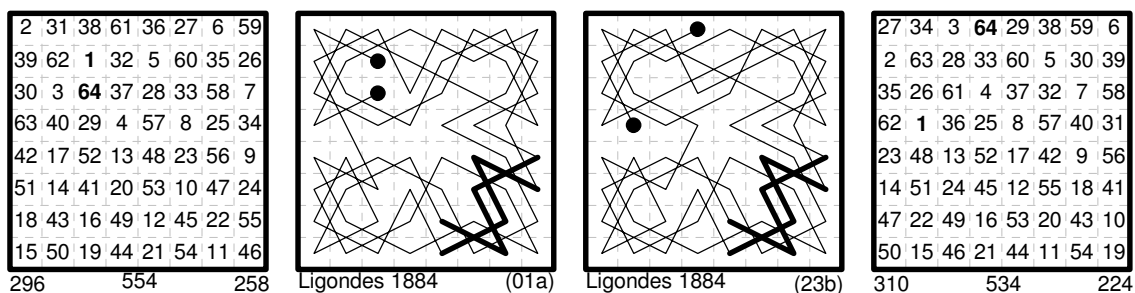
Ligondes *Le Siècle* ¶2254 (03e) 18 Jan 1884, ¶2260 (03a) 25 Jan 1884 [both 1883 in Murray].



Le Siècle ¶2266 1 Feb 1884 three magic tours, the first and third being (03d), (03c).



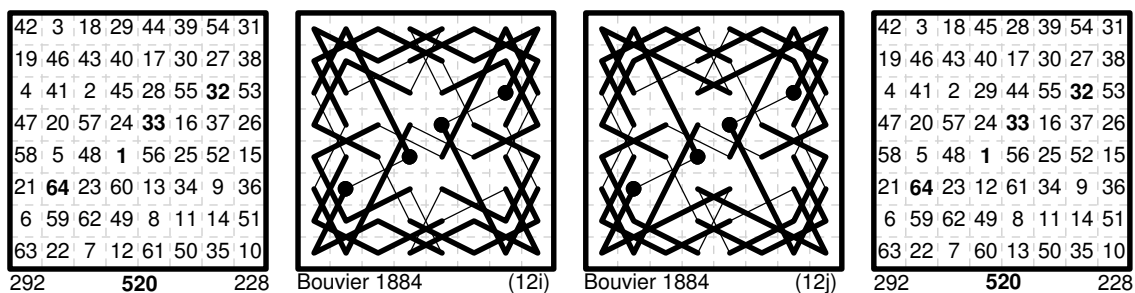
The middle one being (01a) [all three dated 1883 by Murray]. Ligondes (23b) *Le Siècle* ¶2836 4/11 December 1885 has similar structure [Murray dates it 1884].



See also Ligondès 1906-11.

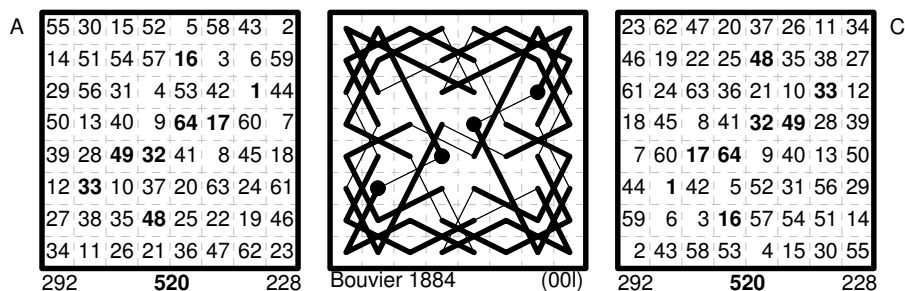
1884: Charles Bouvier (alias 'Adsum')

Le Siècle ¶2326 11 Apr 1884. Three magic tours, the first to make no use of squares, diamonds or Beverley quartes (12i), (12j), (00i), all 16 quartes are extended over cells in two quarters. The first two are symmetric and dated 1882 by Murray.



KNIGHT'S TOUR NOTES

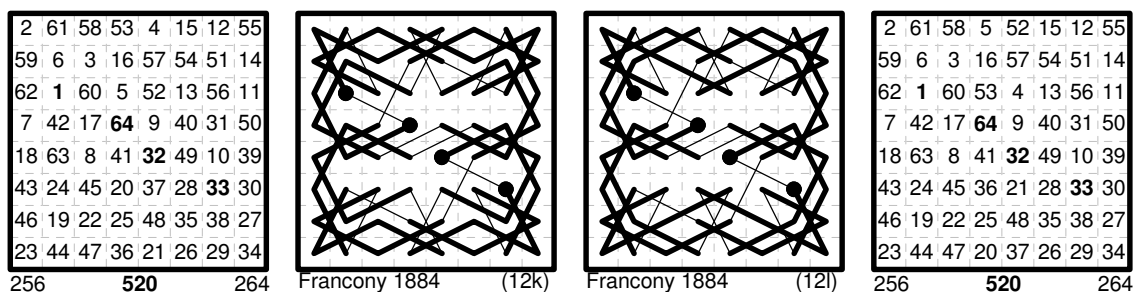
The third tour [dated 1883 by Murray] is asymmetric and cyclic with two magic numberings. The four arithmetic forms are 47, 55, 156, 159.



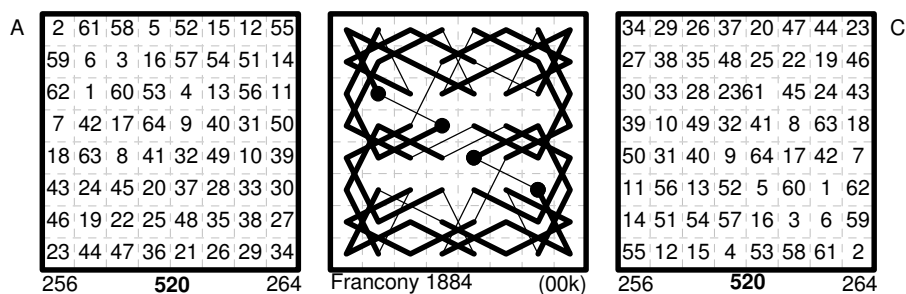
These tours are also Jaenischian, having the ends of the quarters in a knight path.

1884: Edouard Francony ('Mme Céline')

Three tours published as the three parts of ¶1582 in *Gil Blas* 10/17 May 1884. Parts 1 and 3 are symmetric tours (12k) and (12l) (dated 1882 by Murray).



Part 2, the middle tour, combines features of the first and third, but is asymmetric and has two different cyclic numberings. This is (00k) in my 1986 catalogue (dated 1883 in Murray). The arithmetical forms are 57, 60, 161, 164 in the catalogue.



Taking Stock 1886-1986

After the flurry of activity in the ten years from 1876, the next 100 years saw the discovery of only 13 new magic knight tours on the 8×8 board, and eight of those were found by H. J. R. Murray. Much work was done in gathering the existing tours into catalogue form.

1891-97: (General) J. C. T. Parmentier (1821-1910). Published catalogues of all known 8×8 magic knight tours to date in arithmetical form in a series of reports to the Congresses of the *Association Française pour l'Avancement des Sciences* held at Marseilles 1891, Pau 1892 and Caen 1894 (Aug 11). These being subsequently issued as: 'Chronologie des marches du cavalier aux échecs conduisant à des carrés semi-magiques' Au Secrétariat de l'Association (1897). For more biographical and publication details see the Bibliography (§ 12).

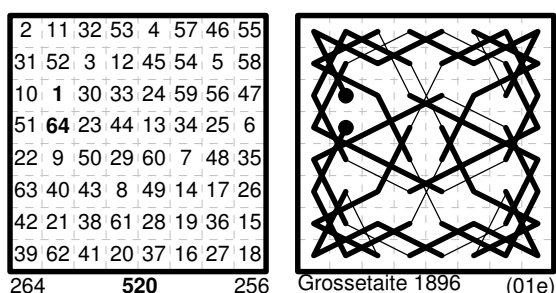
KNIGHT'S TOUR NOTES

1892: Edward Falkener (1814-1896) *Games Ancient and Oriental and How to Play Them*. This has pages dealing with magic squares and knight's tours, including magic knight's tours. However, the magic tours quoted are Beverley's, one by Wenzelides, seven by Jaenisch (three of which he claims as his own), eight by Palamède (Ligondès), and the one by Caldwell.

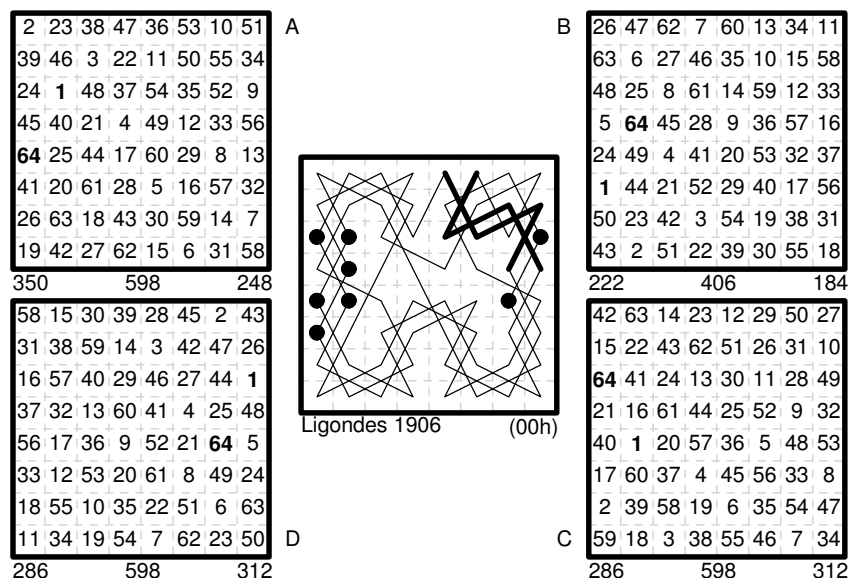
His story that Roget issued a card bearing a semi-magic tour of squares and diamonds type, overprinted with the pattern of 16 circuits and bearing the inscription 'Key to the Knight's Move as a Magic Square' is not confirmed by other sources, and the tour he shows is from Tomlinson (1845).

He also gives a collection of attractively patterned tours of the 8×8 board (some of which are from earlier sources such as Käfer and of the four-handed chessboard).

1896: E. Grossetaite The final 8×8 magic knight tour to be discovered in the 19th century the 84th in the historical sequence, was published by E. Grossetaite in the newspaper *Figaro*. In our catalogue it is tour (01e). The pattern is almost biaxial. This composer also contributed to the series of two-knight and four-knight tours in *Le Siècle* (see § 10).

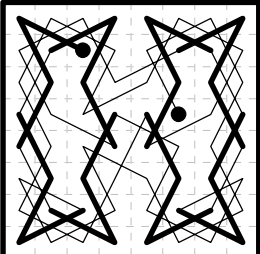
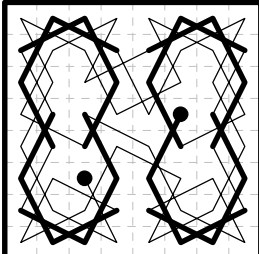


1906-1911 (Vicomte) Raoul de Ligondès (continued from 1884). In *La Mode du Petit Journal* 1906 (a fashion magazine) Count Ligondès published a new cyclic magic tour (00h), H85, with magic numbering from four origins (and their reversals). Unlike other cyclic tours the number of cells through which the tour is cycled starting from the A tour to B, C, D are 24, 40, 56 respectively instead of 16, 32, 48.



KNIGHT'S TOUR NOTES

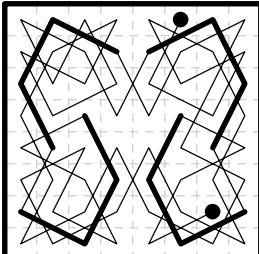
He returned to *La Mode du Petit Journal* in 1910 and 1911 with two new magic tours (23o) and (23p), 86 and 87 in the historical sequence, which were his last contributions to the subject.

<table style="width: 100%; border-collapse: collapse;"> <tr><td>2</td><td>27</td><td>42</td><td>47</td><td>18</td><td>23</td><td>62</td><td>39</td></tr> <tr><td>43</td><td>48</td><td>1</td><td>26</td><td>63</td><td>40</td><td>17</td><td>22</td></tr> <tr><td>28</td><td>3</td><td>46</td><td>41</td><td>24</td><td>19</td><td>38</td><td>61</td></tr> <tr><td>49</td><td>44</td><td>25</td><td>8</td><td>33</td><td>64</td><td>21</td><td>16</td></tr> <tr><td>4</td><td>29</td><td>56</td><td>45</td><td>20</td><td>9</td><td>60</td><td>37</td></tr> <tr><td>53</td><td>50</td><td>7</td><td>32</td><td>57</td><td>34</td><td>15</td><td>12</td></tr> <tr><td>30</td><td>5</td><td>52</td><td>55</td><td>10</td><td>13</td><td>36</td><td>59</td></tr> <tr><td>51</td><td>54</td><td>31</td><td>6</td><td>35</td><td>58</td><td>11</td><td>14</td></tr> </table>	2	27	42	47	18	23	62	39	43	48	1	26	63	40	17	22	28	3	46	41	24	19	38	61	49	44	25	8	33	64	21	16	4	29	56	45	20	9	60	37	53	50	7	32	57	34	15	12	30	5	52	55	10	13	36	59	51	54	31	6	35	58	11	14			<table style="width: 100%; border-collapse: collapse;"> <tr><td>6</td><td>31</td><td>50</td><td>55</td><td>10</td><td>15</td><td>58</td><td>35</td></tr> <tr><td>49</td><td>54</td><td>7</td><td>32</td><td>57</td><td>34</td><td>11</td><td>16</td></tr> <tr><td>30</td><td>5</td><td>56</td><td>51</td><td>14</td><td>9</td><td>36</td><td>59</td></tr> <tr><td>53</td><td>48</td><td>25</td><td>8</td><td>33</td><td>64</td><td>17</td><td>12</td></tr> <tr><td>4</td><td>29</td><td>52</td><td>41</td><td>24</td><td>13</td><td>60</td><td>37</td></tr> <tr><td>47</td><td>44</td><td>1</td><td>26</td><td>63</td><td>40</td><td>21</td><td>18</td></tr> <tr><td>28</td><td>3</td><td>42</td><td>45</td><td>20</td><td>23</td><td>38</td><td>61</td></tr> <tr><td>43</td><td>46</td><td>27</td><td>2</td><td>39</td><td>62</td><td>19</td><td>22</td></tr> </table>	6	31	50	55	10	15	58	35	49	54	7	32	57	34	11	16	30	5	56	51	14	9	36	59	53	48	25	8	33	64	17	12	4	29	52	41	24	13	60	37	47	44	1	26	63	40	21	18	28	3	42	45	20	23	38	61	43	46	27	2	39	62	19	22
2	27	42	47	18	23	62	39																																																																																																																												
43	48	1	26	63	40	17	22																																																																																																																												
28	3	46	41	24	19	38	61																																																																																																																												
49	44	25	8	33	64	21	16																																																																																																																												
4	29	56	45	20	9	60	37																																																																																																																												
53	50	7	32	57	34	15	12																																																																																																																												
30	5	52	55	10	13	36	59																																																																																																																												
51	54	31	6	35	58	11	14																																																																																																																												
6	31	50	55	10	15	58	35																																																																																																																												
49	54	7	32	57	34	11	16																																																																																																																												
30	5	56	51	14	9	36	59																																																																																																																												
53	48	25	8	33	64	17	12																																																																																																																												
4	29	52	41	24	13	60	37																																																																																																																												
47	44	1	26	63	40	21	18																																																																																																																												
28	3	42	45	20	23	38	61																																																																																																																												
43	46	27	2	39	62	19	22																																																																																																																												
216 424 208	Ligondes 1910 (23o)	Ligondes 1911 (23p)	176 424 248																																																																																																																																

1932-33: Max Bruno Lehmann in *Der Geometrische Aufbau Gleichsummiger Zahlenfiguren* 1932 (which was apparently part of a series with the title *Neue Mathematische Spiele*) catalogued all 87 magic 8×8 knight tours known at the time, showing them in geometrical form.

He also included magic knight tours by **E. Lange** of Hamburg on larger boards 12×12, 16×16, 24×24 and some magic king and queen tours.

He followed this in *Le Sphinx* August 1933 by publishing a magic 8×8 tour of his own construction. This is (16a) H88. The only one with this separation of end cells.

<table style="width: 100%; border-collapse: collapse;"> <tr><td>15</td><td>38</td><td>17</td><td>22</td><td>59</td><td>64</td><td>11</td><td>34</td></tr> <tr><td>20</td><td>23</td><td>14</td><td>37</td><td>12</td><td>35</td><td>58</td><td>61</td></tr> <tr><td>39</td><td>16</td><td>21</td><td>18</td><td>63</td><td>60</td><td>33</td><td>10</td></tr> <tr><td>24</td><td>19</td><td>44</td><td>13</td><td>36</td><td>5</td><td>62</td><td>57</td></tr> <tr><td>45</td><td>40</td><td>49</td><td>28</td><td>53</td><td>32</td><td>9</td><td>4</td></tr> <tr><td>50</td><td>25</td><td>46</td><td>43</td><td>6</td><td>3</td><td>56</td><td>31</td></tr> <tr><td>41</td><td>48</td><td>27</td><td>52</td><td>29</td><td>54</td><td>1</td><td>8</td></tr> <tr><td>26</td><td>51</td><td>42</td><td>47</td><td>2</td><td>7</td><td>30</td><td>55</td></tr> </table>	15	38	17	22	59	64	11	34	20	23	14	37	12	35	58	61	39	16	21	18	63	60	33	10	24	19	44	13	36	5	62	57	45	40	49	28	53	32	9	4	50	25	46	43	6	3	56	31	41	48	27	52	29	54	1	8	26	51	42	47	2	7	30	55	
15	38	17	22	59	64	11	34																																																										
20	23	14	37	12	35	58	61																																																										
39	16	21	18	63	60	33	10																																																										
24	19	44	13	36	5	62	57																																																										
45	40	49	28	53	32	9	4																																																										
50	25	46	43	6	3	56	31																																																										
41	48	27	52	29	54	1	8																																																										
26	51	42	47	2	7	30	55																																																										
336 520 184	Lehmann 1933 (16a)																																																																

1936-55: H. J. R. Murray. A biographical note from his 1951 ms *The Magic Knight's Tours*:

“My interest in the knight's problem was first aroused in the early years of the present century when I discovered that tours both on the 4×8 and 8×8 boards were included in the oldest works on chess, which were written by Arabian chess players, and I contributed my first article on the knight's tour to the *British Chess Magazine* in January 1902. For the next thirty years I found relief from the tedium of the many railway journeys necessitated by my professional work [schools inspector] in studying the general aspects of the problem in which John Keeble, Ernest Bergholt, G. L. Moore and T. R. Dawson gave me valuable help and encouragement.”

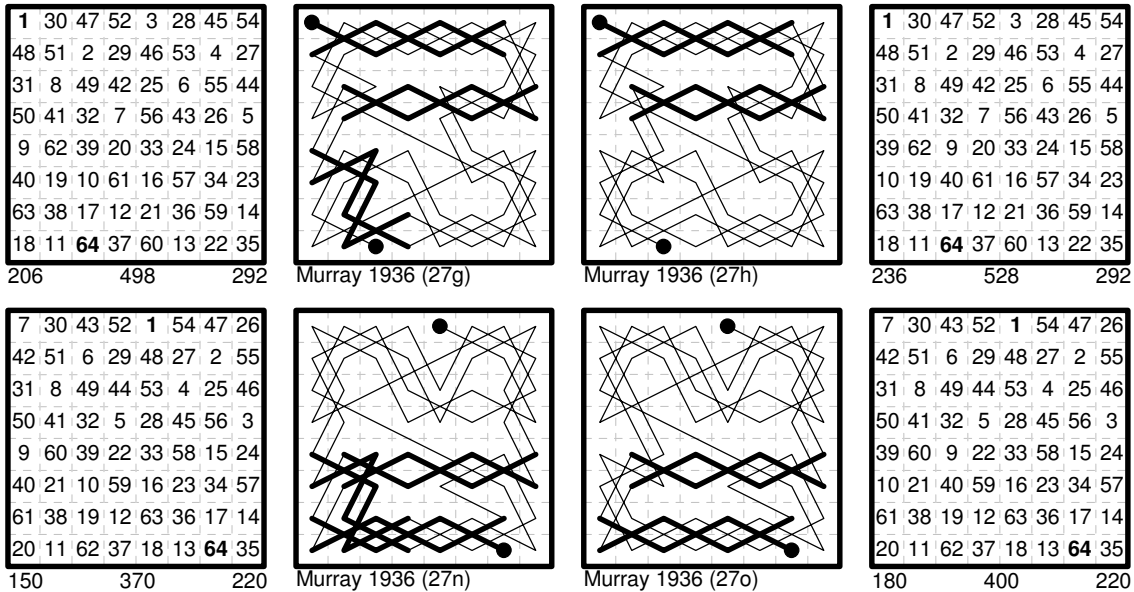
“The appearance of M. Kraitchik's *Le Problème du Cavalier*, 1927, in which the simpler aspects of the problem were discussed at length, ended this phase of activity, and I turned in the summer of 1935 to the special study of the magic tours, since Kraitchik had omitted to deal at any length with these tours. When I began the study of magic tours, the only information I had about them was the series of articles which Carl Wenzelides contributed to the *Schachzeitung* in its earlier years, and the chapters in E. Falkener's *Games Ancient and Oriental*, 1892, which I assumed to be a summary of all that was then known about magic tours.”

He also records: “The analysis was carried out by me on July 18 and 20, 1935 and was the first fruit of my examination of magic tours, and was given in summary in the *Fairy Chess Supplement to The Problemist* for February 1936.” By analysing Beverley's tour Murray formulated his idea of ‘contiguous contraparallel chains’ and found all 14 tours of this type. These tours appeared in every issue of *Problemist Fairy Chess Supplement* from Feb 1936 to Feb 1937 (the title having changed to *Fairy Chess Review* in Aug 1936). He later found that only four of these were new, not twelve as he had supposed. 27g, 27h, 27n, 27o.

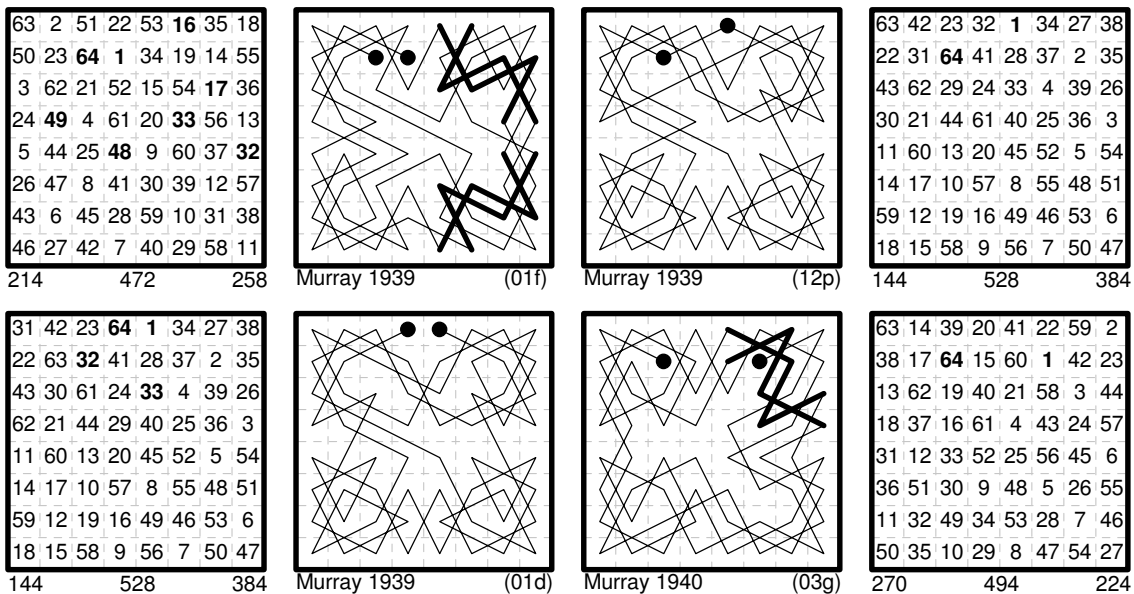
KNIGHT'S TOUR NOTES

The problem numbers are: *PFCS* #16 Feb 1936 p.166 ¶2107 = (27a Beverley 1848), ¶2108 = (27g), ¶2109 = (27m Francony 1882). *PFCS* #17 Apr 1936 p.177 ¶2239 = (27h), ¶2240 = (27s Reuss 1883), *PFCS* #18 Jun 1936 p.187 ¶2286 = (27q Reuss 1883), ¶2287 = (27r Reuss 1883). *Fairy Chess Review* vol.3 #1 Aug 1936 p.3 ¶2350 = (27n), ¶2351 = (27o). *FCR* #2 Oct 1936 p.18 ¶2466 = (27p Reuss 1883), ¶2467 = (27i Francony 1881). *FCR* #3 Dec 1936 p.29 ¶2541 = (27j Béliigne 1881), ¶2542 = (27k Francony 1881). *FCR* #4 Feb 1937 p.41 ¶2636 = (34d Ligondes 1883).

1936 *Problemist Fairy Chess Supplement* vol.2 #16 Feb 1936 p.166 'Beverley's Magic S-Tour and its Plan'. Here are diagrams of the four new tours, shown in the original orientation: In the historical sequence these are tours H89-92.



1939 *Fairy Chess Review* (vol.4 #3 Dec 1939, the TRDFCR dated Nov 28, p.43 ¶4132 = (01f), ¶4133 = (12p), ¶4134 = (01d), #6 Jun 1940 p.93 ¶4556 = (03g)). Four more new magic knight tours. Here are diagrams again shown in the original orientation: H93-96 in the historical sequence.



KNIGHT'S TOUR NOTES

In his 1951 ms Murray wrote: "It was not until the summer of 1938 that T. R. Dawson very generously put at my service the material he had collected about the knight's tours, and lent me Jaenisch's important work of 1862, and not until January 1939 that I succeeded in obtaining M. B. Lehmann's *Neue Mathematische Spiele* 1932 which included all the magic tours on the chessboard that were then known. By this time I had perfected my method of constructing quarte-tours, and had already obtained 49 of the 87 tours in Lehmann's work. By 1941 I had independently discovered by my methods all the tours given by Lehmann and brought the number of geometrical solutions to 96, of which eight were entirely new. So far from discovering these tours empirically, I had established the existence of every tour before I placed it on the chessboard."

Some details of Murray's methods are included in the Magic Methods section, p.373.

1951: Unpublished ms by H. J. R. Murray *The Magic Knight's Tours: A Mathematical Recreation* (1951). This is now in the Bodleian Library, Oxford. This includes the first 12×12 magic tour to have one diagonal magic, dated 1947, although Murray may not have realised this, since he does not mention the property. He constructed nondiagonal magic tours 12×12 to 32×32.

Murray's methods of construction of magic tours are described in detail in the 1951 ms which includes a complete catalogue of the 8×8 tours, as then known, including the eight he found himself, and both forward and reverse numberings. His numbering scheme for the tours is based on the underlying structure of the tours in terms of quartets. The chapter of the 1951 ms on *History of Magic Knight's Tours* was reproduced with commentary and new material in *The Games and Puzzles Journal* (vol.2 #14 Dec 1996 p.238-244 and #16 May 1999 p.291).

1955: Following Murray's death in 1955 (on 16th May at the age of 86) many of his unpublished papers on tours (26 boxes) were deposited in the Bodleian Library, Oxford [shelfmark; MSS H.J.MURRAY 101-126]. However three manuscripts (1930, 1942, 1951) were retained by his family until 1992. The *Fairy Chess Review* (vol.9 no.5 August 1955 p.44) gave a brief obituary. The same issue reported work by T. H. Willcocks and H. E. de Vasa in constructing magic tours on larger boards, though there was no space to show diagrams.

1960: R. C. Bell. The existence of some of the manuscripts left by Murray was mentioned at the end of *Board and Table Games From Many Civilizations*, by R. C. Bell, Oxford University Press 1960. This includes, p.198-200, a brief biography of H. J. R. Murray and lists three manuscripts. 1) *The Knight's Problem* (1942) which covers all aspects of the knight's tour on boards of all sizes and shapes, as well as magic tours. 2) The more specialised *Magic Knight's Tours. A Mathematical Recreation* (1951). 3) 'The Classification of Knight's Tours' which was in fact published in the *British Chess Magazine* (1949) and is about the straights and slants method.

1986: G. P. Jelliss: 'Magic Tours' Special Issue of *Chessics* #26 (Summer 1986). This special issue of *Chessics* was produced by me after studying Murray's 26 boxes of papers in the Bodleian Library. In only 16 small pages it covers work of de Vasa and Willcocks, 48×48 and 12×12 diamagic knight tours. Concepts of magic. Magic tours on 2, 3 and 4 rank boards. Magic tours 3×5 using four types of move. Diagonally magic king tours.

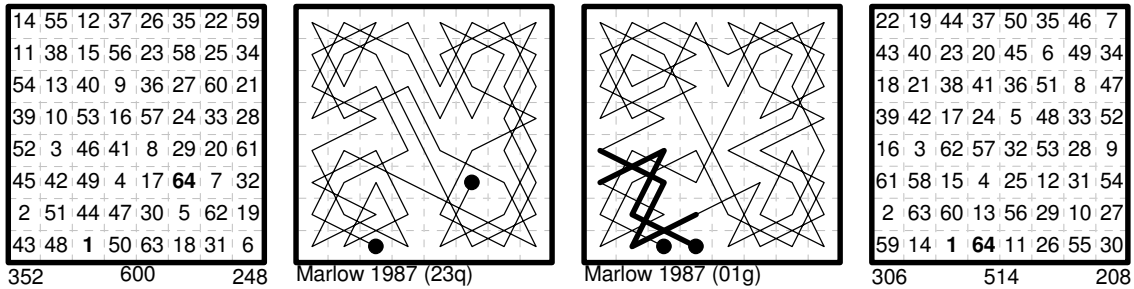
It also includes a complete catalogue of the 8×8 magic knight tours. The tours are classified by a two-digit code based on the separation of the end-points, closed tours being coded 12 and cyclic closed tours 00. (One diagram 03b included in error in the catalogue is not in fact magic.) Instead of using the Frénicle method to determine the first forward numbering of a tour an attempt was made to arrange them according to geometric criteria, having the initial cell as close to a corner as possible, and where necessary considering the subsequent cells.

The description of the methods of construction concluded "It should be possible to apply modern computer methods to ascertain whether all the magic 8×8 knight tours of this type [i.e. using regular quartets] have been discovered." This was immediately taken up by Tom Marlow.

Completing the Task 1987-2003

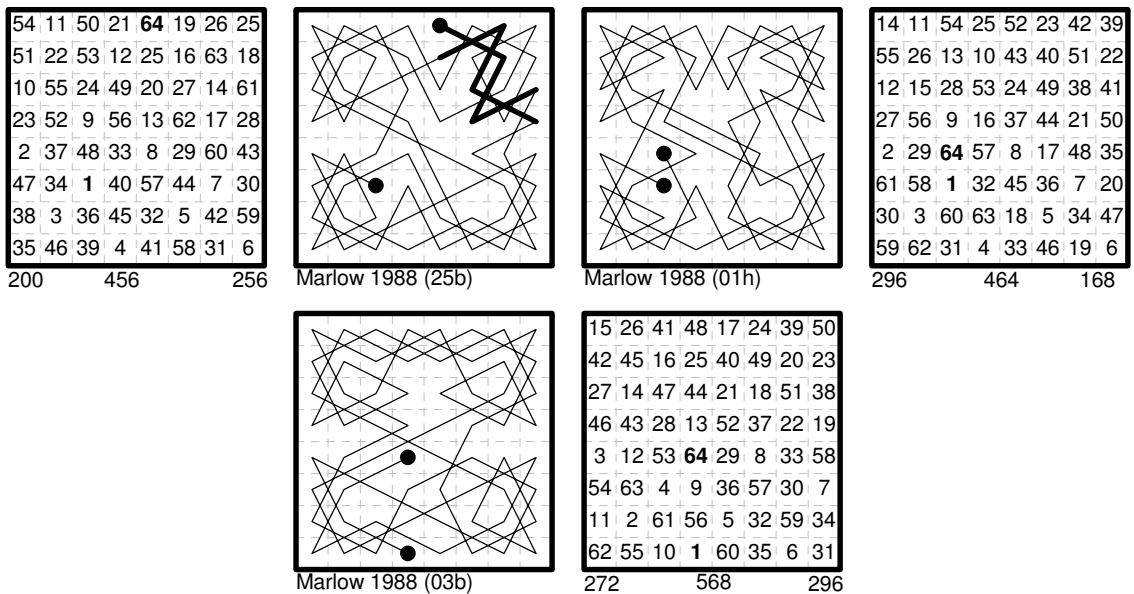
1987/8: Thomas William Marlow (1927-2011). Taking up the project suggested in *Chessics* 1986 Tom Marlow applied computer methods to enumerate all magic knight tours of regular quartes type and found five new tours. (01g, 01h, 03b, 23q, 25b).

Two (23q and 01g) were given in *The Games and Puzzles Journal* (issue 1 Sep-Oct 1987 p.11).



He wrote (30 Jul 1987): “I have been doing some more computer work on a check on S magic tours by the quartes method. ... I revised the logic of the programme and rewrote in machine code which runs much faster. The programme takes a specified starting square and looks for all tours from there, so it will be necessary to start from each of the 10 fundamentally different squares. So far I have tried a1, b1 and c1 and used over 170 hours of computer time. For example, starting from a1 the programme found 8926565 tours, not counting reflections in a1/h8. Of these only 4 were magic and were 27a,b,c and d in your catalogue.” From c1 it found two new magic tours (shown above in their original orientation, starting at c1).

All five tours appeared in ‘Magic Knight Tours’ *The Problemist* (vol.12 #19 Jan 1988 p.379). Here are the others. The conclusion was that there are 78 such tours in total.



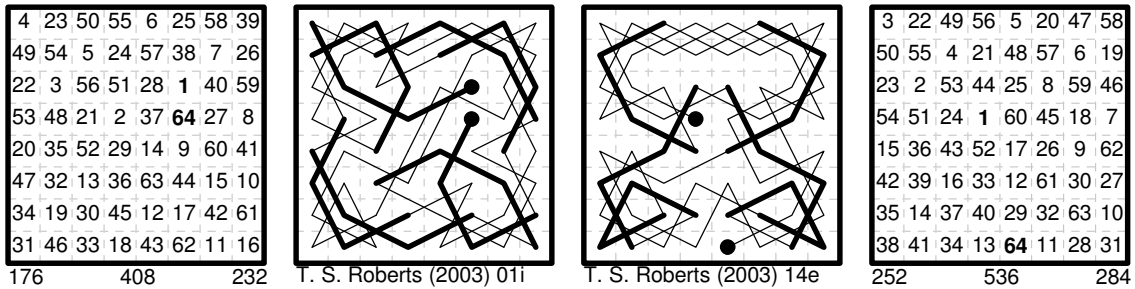
Tom wrote: “My investigation found that it was just within the capacity of a home computer [to test for further magic tours of quartes type]. It is interesting to know that the computer was finding complete tours and checking whether they were magic at a rate of about 55 per second. The total number of tours examined was about 55 million. There remains the general problem of how many other magic tours exist (not using the ‘method of quartes’) and whether a fully magic tour is possible. This may be beyond the capacity of any computer yet available.” It took another 15 years (2003).

KNIGHT'S TOUR NOTES

One of the tours found by Marlow (03b) fortunately replaced the erroneous tour in my catalogue. This tour and tour (01h) are particularly symmetric. The others are rather irregular regular tours!

This work has been independently confirmed using more powerful computer methods, first by Michael Gilpin of Michigan Technological University, USA (1997), later by Tim Roberts as reported in *G&P Journal* #25 (2003).

2003: Timothy S. Roberts (Central Queensland University, Australia) *The Games and Puzzles Journal* #25 (online) Jan-Feb 2003 'A New 8×8 Magic Knight's Tour' and 'Another New 8×8 Magic Knight's Tour'. These tours (01i) and (14e), of very different patterns, were found after encountering the catalogue of magic tours on the KTN web pages which led Tim to devise a program to search for new magic tours of irregular type, though they are particularly regular irregular tours!



These were also published in the *Mathematical Gazette* (Mar 2005) p.22 'The discovery of two new magic knight's tours'.

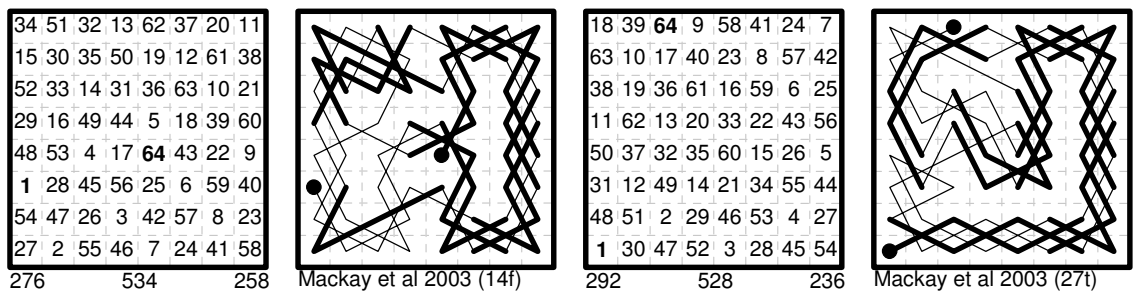
2003: Hugues Mackay, Jean-Charles Meyrignac and Günther Stertenbrink. *Computing Magic Knight Tours* [<http://magictour.free.fr/>].

At the beginning of 2003 I was contacted by Günther Stertenbrink who had a plan for searching for magic knight's tours using "distributed programming". Not knowing anything about this, and no actual tours being shown, I was sceptical, but subsequently his plan was developed into an actual program by Jean-Charles Meyrignac (in France) and was run simultaneously on a number of computers, principally by Hugues Mackay (in Canada). This international project, which involved a separate search for each of the 136 distinct possible positions for the end-cells of the tour, was carried out during Jun-Aug 2003, and found a further five (geometrically distinct) magic tours, in the sequence: (00n) 18/19 Jun, (14f) 21 Jun, (27t) 24 Jun, (00o) 1 Jul, (07a) 1 Aug 2003.

In tour (14f) the four corner diamonds, like b3-a1-c2/d4, are all broken in two and the bits distributed in between the other quartes, syncopating the numbering on some of them.

The tour (27t) has the same end-point separation as the first found by Beverley in 1848 but is of very different structure. It is more like a Collini tour, with border braid and central rhombs.

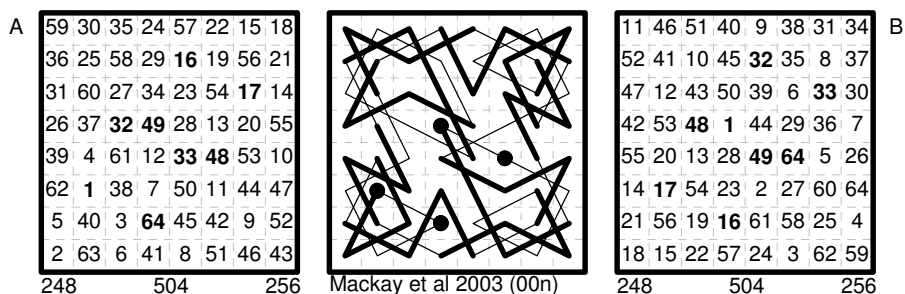
(14f) and (27t)



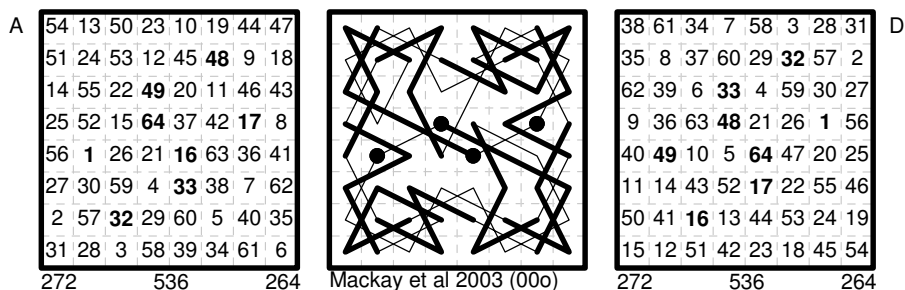
KNIGHT'S TOUR NOTES

Two of the new tours (00n) and (00o) have a second arithmetical solution by cyclic renumbering. (My sole contribution to the project was to notice the second numbering of 00n).

(00n) Tour B is related to tour A here by a cyclic shift of 16.

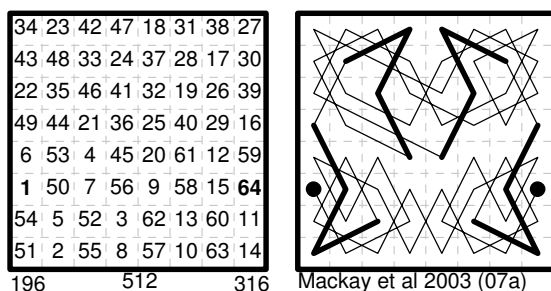


(00o) Tour D is related to tour A here by a cyclic shift of 48.



In my catalogue of magic tours (1986) I wrote “It is surprising that no tour of type 07 has been found”. The tour (07a), the last to be found, is in fact the only one of this type.

(07a)



The project also regenerated all previously known tours, bringing the total to 108 geometrically distinct. So it now seems safe to say that the chapter on 8x8 magic knight's tours has at last been brought to a conclusion, and there are no more to be found.

This grand achievement was enough to have been given a headline in *MathWorld*, though disappointingly this emphasised the negative aspect, that none of the tours is magic in the diagonals, rather than the positive side of the completion of a task that has occupied a whole army of researchers for over 150 years! [<http://mathworld.wolfram.com/news/2003-08-06/magictours/>]

The results were also summarised in *Variant Chess* articles (See entries for J. D. Beasley and G. P. Jelliss in the Bibliography).

Catalogue of 8×8 Magic Knight Tours

The tours are given here in three lists: (1) according to their historical sequence of publication, (2) according to their geometrical form and (3) according to their arithmetical forms. Further biographical details of the composers, where known, and full bibliographic titles of the sources are given in the Bibliography. Further information is also given in the History text, where the diagrams show the Beverley and irregular quartes and also diagonal sums.

This catalogue of tours derives from the successive work of Parmentier (1890s), Lehmann (1932), Murray (ms 1951), Jelliss (*Chessics* #26, 1986) and Beasley (2008). It includes the twelve new tours found since 1986. Recent online research (2017) into French newspaper columns has enabled me to give more precise publication citations for some tours, but there are still 14 tours whose sources remain unknown, and one with an unknown composer.

Thanks to work completed in 2003 there are now known to be 280 arithmetically distinct magic knight tours of the 8×8 board. Each can be oriented in 8 ways, making $8 \times 280 = 2240$ in all. Since the reverse of a magic tour is magic, and since an open knight tour of the 8×8 board cannot be symmetric, these 280 tours occur in pairs that are reversals of each other, and can be produced by numbering 140 geometrical open tours from either end. In the Beasley catalogue these are individually numbered and one arithmetical form of each is shown.

Of these 140 tours 76 are non-reentrant and 64 are reentrant (i.e. the two end-points are a knight move apart). By joining up the end-points of these reentrant tours we get not 64 but only 32 distinct closed tours. This is because some closed tours can be numbered from two or more different starting points to produce distinct arithmetical magic tours. Thus we can say that there are $76 + 32 = 108$ geometrically different magic knight tours on the 8×8 board.

The tours fall into 12 classes coded according to end-point separation, as introduced in my 1986 catalogue. The numbers of geometrically distinct tours in each class are as follows: (00) 15, (01) 9, (03) 7, (05) 7, (07) 1, (12) 16, (14) 6, (16) 1, (23) 16, (25) 3, (27) 20, (34) 7. Total 108. The (00) type are tours that can be cyclically renumbered from various different starting points and remain magic. This adds a further 32 arithmetical forms, making 47 of (00) type and the total 140.

Although 8×8 open tours are always asymmetric, closed tours can show 180° rotational symmetry. There are 16 symmetric magic tours in all, one (00m) by Wenzelides in the (00) class and 15 in the (12) class (12a - 12o), the one exception being (12p) by Murray which is asymmetric.

The tours also fall into three classes A, B, C, according to the types of quartes used: (A) 34 tours of squares and diamonds type, (B) 44 tours that also employ Beverley quartes, (C) 30 tours that employ irregular quartes.

Historical Catalogue of 8×8 Magic Knight Tours

The numbering 1 to 108 in sequence of publication is still somewhat arbitrary in places, so should preferably not be used to identify the tours. Further research is needed to identify where 13 tours were first published. Sources reported by H. J. R. Murray but not seen by me (*) or cited at an earlier date (^). Problem numbers (¶). For fuller names and reference details see the Bibliography.

1. (B 27a) Beverley *Philosophical Magazine* #220 Aug 1848 p.101–105.
2. (A 12a) Wenzelides *Schachzeitung* Feb-Mar 1849 p.94-97.
3. (A 12b) Wenzelides *Schachzeitung* 1850 vol.5 Fig.99 p.246 (with explanation in §93 p.237).
4. (B 27b) Wenzelides *Schachzeitung* 1850 vol.5 Fig.107 p.247.
5. (A 00b) Rajah of Mysore (31 Jul 1852) on a silk panel now in the Crumiller collection.
6. (B 00m) Wenzelides *Schachzeitung* vol.13 May 1858 p.174–175, Figure A.
7. (B 12e) Wenzelides *Schachzeitung* vol.13 May 1858 p.174–175, Figure B.
8. (A 12m) Wenzelides *Schachzeitung* vol.13 May 1858 p.174–175, Figure C.
9. (A 12o) Jaenisch *Chess Monthly* (Apr-Jun 1859).
10. (A 12n) Jaenisch *Chess Monthly* (Apr-Jun 1859).
11. (A 00a) Jaenisch *Traité* vol.2 1862, Figure 49.

KNIGHT'S TOUR NOTES

12. (B 27c) Jaenisch *Traité* vol.2 1862, Figure 55.
13. (B 27d) Jaenisch *Traité* vol.2 1862, Figure 56.
14. (B 00e) Jaenisch *Traité* vol.2 1862, Figure 62.
15. (B 05a) Bouvier (Adsum) Source unknown (1876)*
16. (A 27i) Exner *Progress* (W. Pfund 1876).*
17. (A 34e) Exner *Progress* (W. Pfund 1876)*
18. (B 34a) Exner *Progress* (W. Pfund 1876)*
19. (A 05b) Caldwell *English Mechanic and World of Science* (1879)
20. (A 00i) Unknown composer* *Le Siècle* ¶772 25 Apr/2 May 1879
21. (C 12c) Béligne *Le Siècle* (¶1042 5/12 Mar 1880). The first with irregular quartes.
22. (B 00j) Reuss (X à Belfort) *Le Siècle* ¶1252 5/12 Nov 1880
23. (C 27e) Reuss (X à Belfort) *Le Siècle* ¶1258 12/19 Nov 1880
24. (C 27f) Reuss (X à Belfort) *Le Siècle* ¶1270 26 Nov/3 Dec 1880
25. (B 00d) Reuss (X à Belfort) *Le Siècle* ¶1276 3/10 Dec 1880
26. (A 00c) Francony (Celina) Source unknown 1881*
27. (A 05f) Francony (Celina) Source unknown 1881*
28. (B 27k) Francony (Celina) Source unknown 1881*
29. (A 05c) Francony (Celina) Source unknown 1882*
30. (B 05e) Francony (Celina) Source unknown 1882*
31. (A 23a) Francony (Celina) Source unknown 1882*
32. (B 05d) Bouvier (Adsum) *Le Telegraphe* ¶1232 or ¶1239) cited in *Le Siècle* 1882
33. (C 12f) Jolivald (Paul de Hijo) *Le Siècle* ¶1726 12/19 May 1882
34. (C 12d) Jolivald (Paul de Hijo) *Le Siècle* ¶1732 19/26 May 1882
35. (C 12g) Jolivald (Paul de Hijo) *Le Siècle* ¶1738 26 May/2 Jun 1882
36. (C 12h) Jolivald (Paul de Hijo) *Le Siècle* ¶1744 2/9 Jun 1882
37. (B 27j) Béligne *Le Siècle* ¶1966 16/23 Feb 1883^
38. (A 27l) Francony (Celina) *Gil Blas* ¶1156(1) 23 Feb / 2 Mar 1883^
39. (B 27m) Francony (Celina) *Gil Blas* ¶1156(2) 23 Feb / 2 Mar 1883^
40. (A 27p) Reuss (X a Belfort) Source unknown 1883*
41. (B 27q) Reuss (X a Belfort) Source unknown 1883*
42. (B 27r) Reuss (X a Belfort) Source unknown 1883*
43. (B 27s) Reuss (X a Belfort) Source unknown 1883*
44. (A 05g) Ligondes (Palamede) *Le Siècle* ¶1996 23/30 Mar 1883
45. (B 34b) Ligondes (Palamede) *Le Siècle* ¶2086(1) 6/13 Jul 1883
46. (B 34c) Ligondes (Palamede) *Le Siècle* ¶2086(2) 6/13 Jul 1883
47. (A 34d) Ligondes (Palamede) *Le Siècle* ¶2092(2) 13/20 Jul 1883
48. (A 14b) Ligondes (Palamede) *Le Siècle* ¶2098(1) 20/27 Jul 1883^
49. (B 14a) Ligondes (Palamede) *Le Siècle* ¶2098(2) 20/27 Jul 1883^
50. (A 23l) Ligondes (Palamede) *Le Siècle* ¶2098(3) 20/27 Jul 1883
51. (B 23k) Ligondes (Palamede) *Le Siècle* ¶2098(4) 20/27 Jul 1883
52. (B 23j) Ligondes (Palamede) *Le Siècle* ¶2098(5) 20/27 Jul 1883
53. (B 23i) Ligondes (Palamede) *Le Siècle* ¶2098(6) 20/27 Jul 1883
54. (A 23g) Ligondes (Palamede) *Le Siècle* ¶2098(7) 20/27 Jul 1883
55. (B 23h) Ligondes (Palamede) *Le Siècle* ¶2098(8) 20/27 Jul 1883
56. (B 23e) Ligondes (Palamede) *Le Siècle* ¶2098(9) 20/27 Jul 1883
57. (A 23d) Ligondes (Palamede) *Le Siècle* ¶2110(1) 3/10 Aug 1883
58. (B 25a) Ligondes (Palamede) *Le Siècle* ¶2110(2) 3/10 Aug 1883
59. (A 23c) Ligondes (Palamede) *Le Gaulois* ¶608 6/13 Aug 1883
60. (A 23m) Ligondes (Palamede) *Le Siècle* ¶2116(1) 10/17 Aug 1883
61. (A 23n) Ligondes (Palamede) *Le Siècle* ¶2116(2) 10/17 Aug 1883
62. (B 23f) Ligondes (Palamede) *Le Siècle* ¶2116(3) 10/17 Aug 1883
63. (B 03f) Ligondes (Palamede) *Le Siècle* ¶2122(1) 17/24 Aug 1883
64. (B 01b) Ligondes (Palamede) *Le Siècle* ¶2122(2) 17/24 Aug 1883

KNIGHT'S TOUR NOTES

65. (B 00g) Ligondes (Palamede) *Le Siècle* ¶2122(3) 17/24 Aug 1883
66. (B 14c) Ligondes (Palamede) *Le Siècle* ¶2128(1) 24/31 Aug 1883
67. (B 01c) Ligondes (Palamede) *Le Siècle* ¶2128(2) 24/31 Aug 1883
68. (B 00f) Ligondes (Palamede) *Le Siècle* ¶2134 31 Aug/7 Sep 1883
69. (A 34f) Ligondes (Palamede) *Le Siècle* ¶2242(1) 4/11 Jan 1884^
70. (A 14d) A. Feisthamel *Le Siècle* ¶2242(2) 4/11 Jan 1884
71. (B 34g) Ligondes (Palamede) *Le Siècle* ¶2248 11/18 Jan 1884^
72. (B 03e) Ligondes (Palamede) *Le Siècle* ¶2254 18/25 Jan 1884^
73. (A 03a) Ligondes (Palamede) *Le Siècle* ¶2260 25 Jan/1 Feb 1884^
74. (B 03d) Ligondes (Palamede) *Le Siècle* ¶2266(1) 1/8 Feb 1884^
75. (B 01a) Ligondes (Palamede) *Le Siècle* ¶2266(2) 1/8 Feb 1884^
76. (B 03c) Ligondes (Palamede) *Le Siècle* ¶2266(3) 1/8 Feb 1884^
77. (C 12i) Bouvier (Adsum) *Le Siècle* ¶2326(1) 11/18 Apr 1884^
78. (C 12j) Bouvier (Adsum) *Le Siècle* ¶2326(2) 11/18 Apr 1884^
79. (C 00l) Bouvier (Adsum) *Le Siècle* ¶2326(3) 11/18 Apr 1884^
80. (C 12k) Francony (Celina) *Gil Blas* ¶1582(1) 10/17 May 1884^
81. (C 00k) Francony (Celina) *Gil Blas* ¶1582(2) 10/17 May 1884^
82. (C 12l) Francony (Celina) *Gil Blas* ¶1582(3) 10/17 May 1884^
83. (B 23b) Ligondes (Palamede) *Le Siècle* ¶2836 4/11 Dec 1885^
84. (C 01e) Grossetaite *Figaro* 1896*
85. (B 00h) Ligondes (Palamede) *La Mode Du Petit Journal* 1906*
86. (C 23o) Ligondes (Palamede) *La Mode Du Petit Journal* 1910*
87. (C 23p) Ligondes (Palamede) *La Mode Du Petit Journal* 1911*
88. (C 16a) Lehmann *Le Sphinx* August 1933*
89. (C 27g) Murray *The Problemist Fairy Chess Supplement* (¶2108 #16 Feb 1936 p.166)
90. (C 27h) Murray *The Problemist Fairy Chess Supplement* (¶2239 #17 Apr 1936 p.177)
91. (C 27n) Murray *Fairy Chess Review* (vol.3 ¶2350 #1 Aug 1936 p.3)
92. (C 27o) Murray *Fairy Chess Review* (vol.3 ¶2351 #1 Aug 1936 p.3)
93. (B 01f) Murray *Fairy Chess Review* (vol.4 ¶4132 #3 Dec 1939 p.43)
94. (A 12p) Murray *Fairy Chess Review* (vol.4 ¶4133 #3 Dec 1939 p.43)
95. (A 01d) Murray *Fairy Chess Review* (vol.4 ¶4134 #3 Dec 1939 p.43)
96. (B 03g) Murray *Fairy Chess Review* (vol.4 ¶4132 #6 Jun 1940 p.93)
97. (A 23q) Marlow *The Games and Puzzles Journal* #1 Sep-Oct 1987 p.11
98. (B 01g) Marlow *The Games and Puzzles Journal* #1 Sep-Oct 1987 p.11
99. (B 25b) Marlow *The Problemist* Vol.12 #19 Jan 1988 p.379
100. (A 01h) Marlow *The Problemist* Vol.12 #19 Jan 1988 p.379
101. (A 03b) Marlow *The Problemist* Vol.12 #19 Jan 1988 p.379
102. (C 01i) Roberts *The Games and Puzzles Journal* #25 (online) Jan-Feb 2003
103. (C 14e) Roberts *The Games and Puzzles Journal* #25 (online) Jan-Feb 2003
104. (C 00n) Mackay, Meyrignac & Stertenbrink *Computing Magic Knight Tours* 18/19 Jun 2003
105. (C 14f) Mackay, Meyrignac & Stertenbrink *Computing Magic Knight Tours* 21 Jun 2003
106. (C 27t) Mackay, Meyrignac & Stertenbrink *Computing Magic Knight Tours* 24 Jun 2003
107. (C 00o) Mackay, Meyrignac & Stertenbrink *Computing Magic Knight Tours* 1 Jul 2003
108. (C 07a) Mackay, Meyrignac & Stertenbrink *Computing Magic Knight Tours* 1 Aug 2003

Geometrical Catalogue of 8×8 Magic Knight Tours

The tours are listed here in the three classes A, B, C, determined by the types of quartes used: (A) 34 tours of squares and diamonds type, (B) 44 tours that also employ Beverley quartes, (C) 30 tours that employ irregular quartes. In the regular types (A) and (B) we list the Quads used.

The grey background pattern behind each diagram shows where numbers of the form $4 \cdot x$ and $4 \cdot x + 1$ lie, i.e. the ends of the quartes. Each rank and file must contain an even number of these 32 shaded cells. This pattern helps to show the general symmetry or irregularity of the tour and remains the same for the reversed or cyclical renumberings.

Subclassification according to symmetry of the background pattern: LYNX.: L = asymmetric (38), Y = axial (35), N = rotary 180 degree rotation (10), X = biaxial (25). These patterns can be further sub-classified according to the number of dark edge cells.

(A) Squares and Diamonds Type

The quads used are 1, 9, 13, 15, 17, 27, 28, 33, 35 printed here in bold.

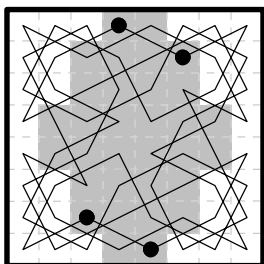
One type of quad. **(9)**: 00a, 01h, 12n,o, 34e; **(13)**: 00i, 05f, 12a,b,m; **(15)**: 27i; **(33)**: 05g; **(35)**: 34d; 27l. Two types of quad. **(1, 35)**: 14b, 23d,g,l, 27p; **(13, 17)**: 01d, 05c, 12p; **(13, 27)**: 23c; **(13, 33)**: 00b, 03b, 23a; 00c, 03a, 05b, 23m,n; **(13, 28)**: 14d, 34f; **(28, 33)**: 23q. Total 34.

AX Biaxial Background: 5 shapes and 13 tours

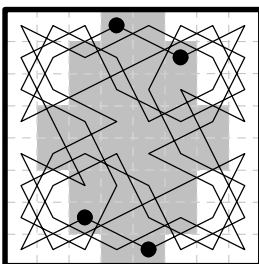
(1 with 5 tours and 4 dark edge cells)

(1 with 5 tours and 8 dark edge cells)

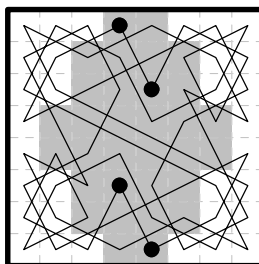
12a Wenzelides 1849



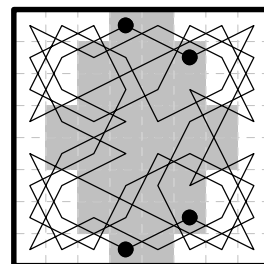
12b Wenzelides 1850



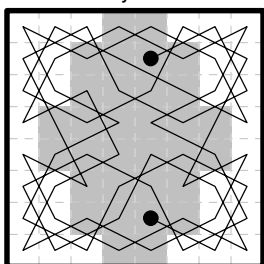
12m Wenzelides 1858



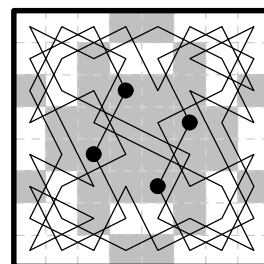
00i Unknown 1880/81



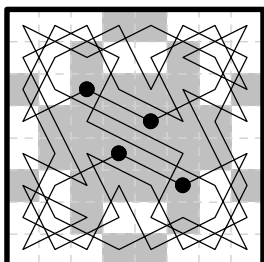
05f Francony 1881



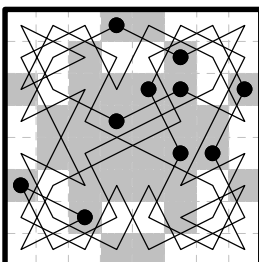
12n Jaenisch 1859



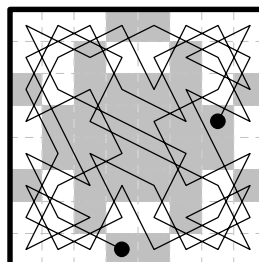
12o Jaenisch 1859



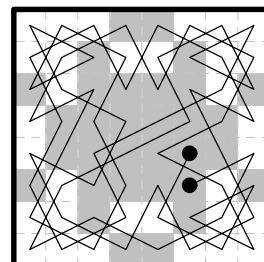
00a Jaenisch 1862



34e Exner 1876

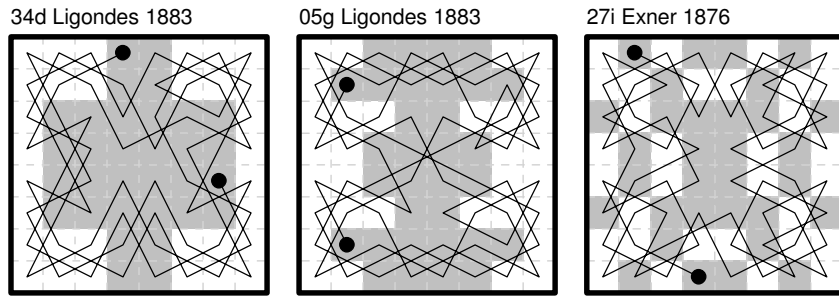


01h Marlow 1988

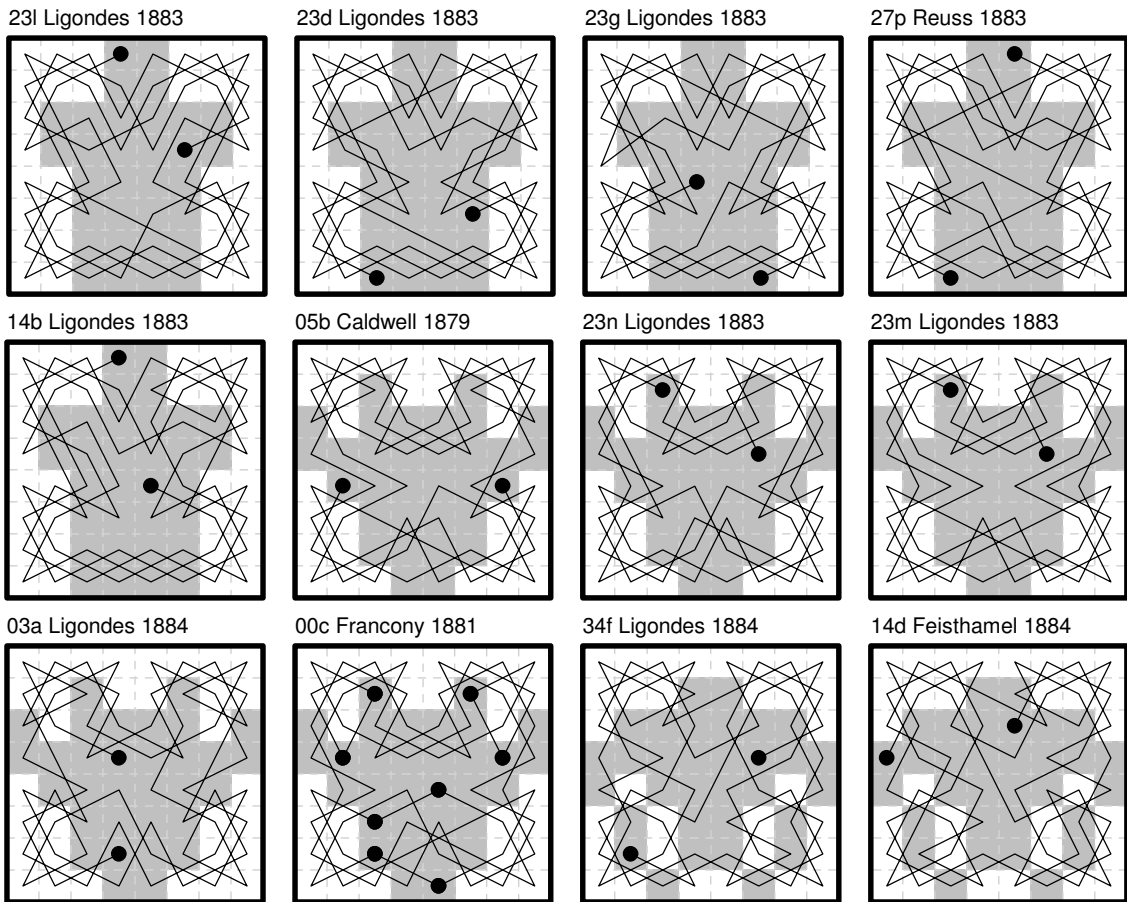


KNIGHT'S TOUR NOTES

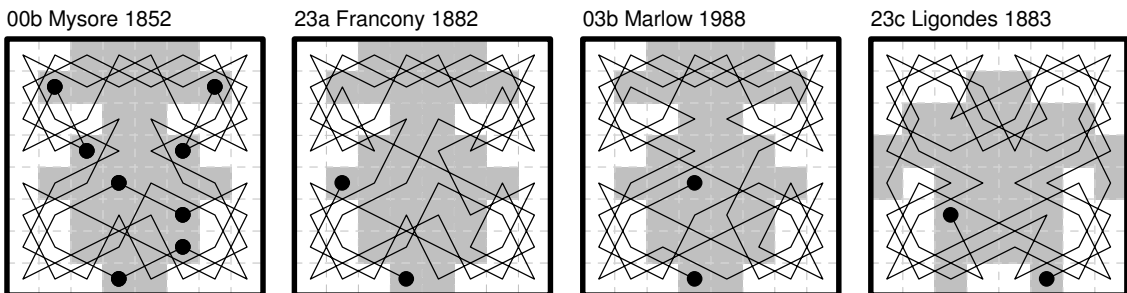
(3 individuals, with 4, 8 and 12 dark edge cells)



AY Axial Background: 6 shapes 19 tours
(2 with 5 tours, 1 with two tours)



(1 with 3 tours, and 1 individual)

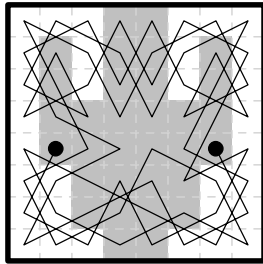


(The above 16 all have 6 dark edge cells)

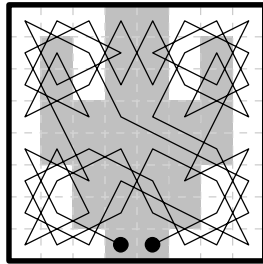
KNIGHT'S TOUR NOTES

(1 other with 3 tours, these have 4 dark edge cells)

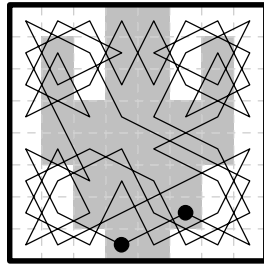
05c Francony 1882



01d Murray 1939

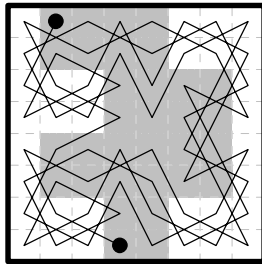


12p Murray 1939

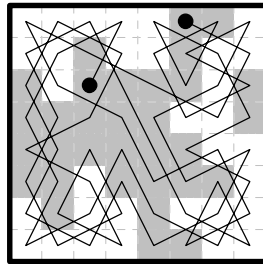


AL Asymmetric Background: 2 shapes (1 with 6 dark edge cells, 1 with 10).

27l Francony 1883



23q Marlow 1988



(B) Beverley Type

The quads used are 1, 8, 13, 17, 21, 24, 35, 37, 41, 44 printed here in bold.

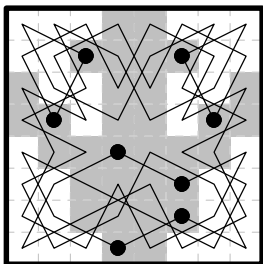
Two types of quad. **(1, 44)**: 00h; 27b; 27a; 12e; **(13, 41)**: 01a, 03c,d, 23b, 25b; 00d,f,g, 01b, 03f, 05e; 00e,m, 01c, 03e, 14c; **(35, 44)**: 01f, 23h,k, 27s; 27c; **(35, 37)**: 34a,b; 27m; 27j; 34c; 27k; **(37, 44)**: 27d. Three types of quad. **(1, 35, 44)**: 14a, 23e,f,i,j, 25a, 27q,r; **(13, 17, 41)**: 05d; **(13, 24, 41)**: 03g, 34g. Four types of quad. **(8, 21, 35, 44)**: 01g. Total 44.

BX Biaxial Background: none.

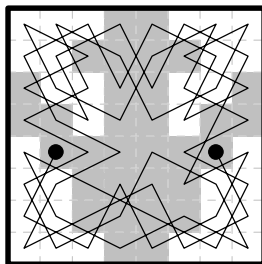
BY Axial Background: 3 shapes with 11 tours

(1 with 6 tours and 8 dark edge cells) (1 individual with 8 dark edge cells).

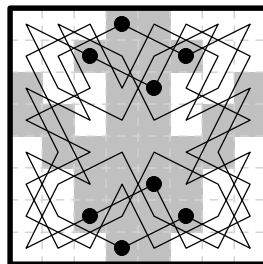
00d Reuss 1880



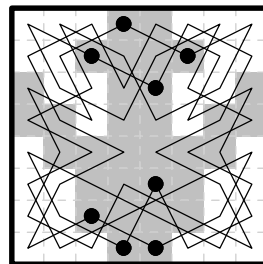
05e Francony 1882



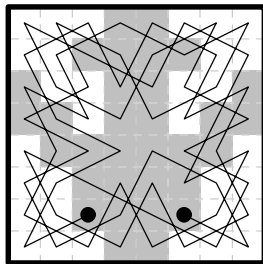
00f Ligondes 1883



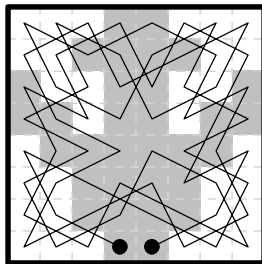
00g Ligondes 1883



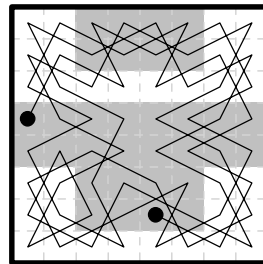
03f Ligondes 1883



01b Ligondes 1883



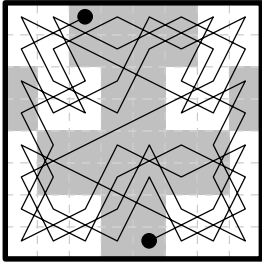
34c Ligondes 1883



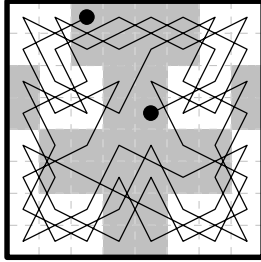
KNIGHT'S TOUR NOTES

(1 with 4 tours and 10 dark edge cells)

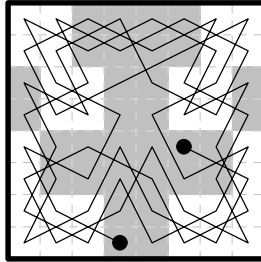
27s Reuss 1883



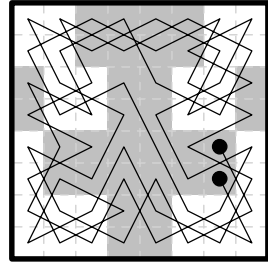
23h Ligondes 1883



23k Ligondes 1883



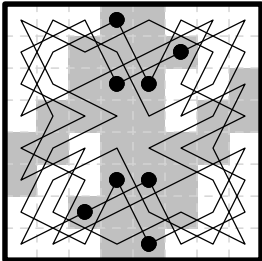
01f Murray 1939



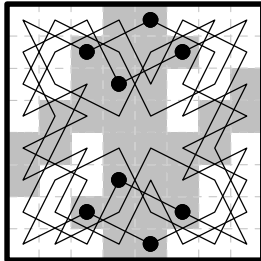
BN Rotary Background: 2 shapes with 5 tours.

(1 with 4 tours and 8 dark edge cells) (1 individual with 12 dark edge cells)

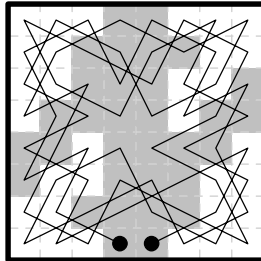
00m Wenzelides 1858



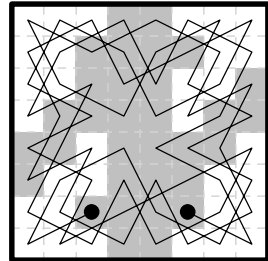
00e Jaenisch 1862



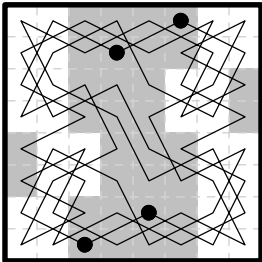
01c Ligondes 1883



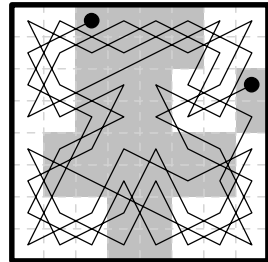
03e Ligondes 1884



12e Wenzelides 1858



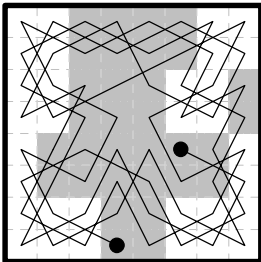
25a Ligondes 1883



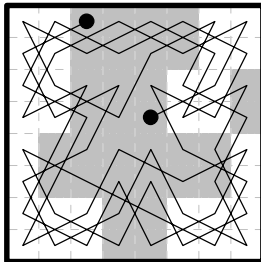
BL Asymmetric Background: 14 shapes in 28 tours.

(1 with 9 tours and 8 dark edge cells - one shown in the above panel)

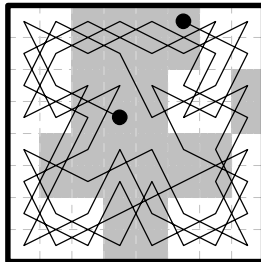
23j Ligondes 1883



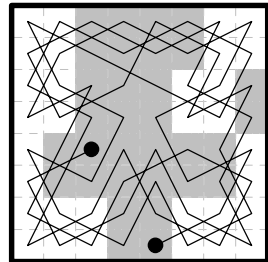
23f Ligondes 1883



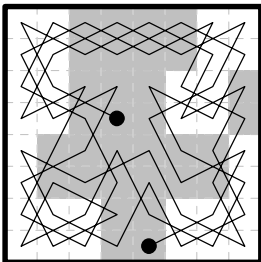
23e Ligondes 1883



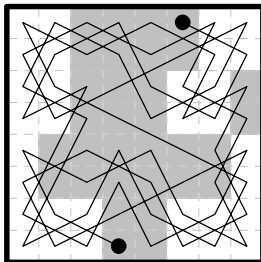
23i Ligondes 1883



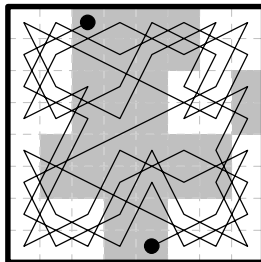
14a Ligondes 1883



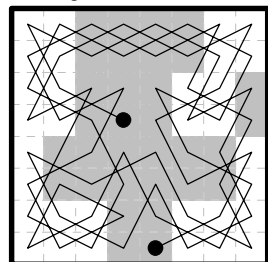
27q Reuss 1883



27r Reuss 1883



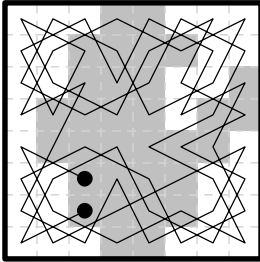
14c Ligondes 1883



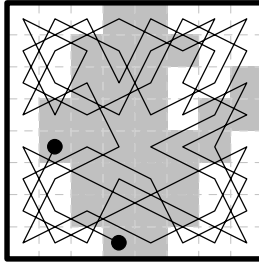
KNIGHT'S TOUR NOTES

(1 with 5 tours and 6 edge cells) (1 with 2 tours and 6 edge cells)

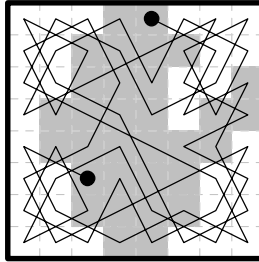
01a Ligondes 1884



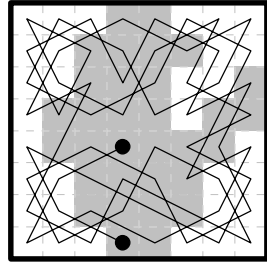
23b Ligondes 1884



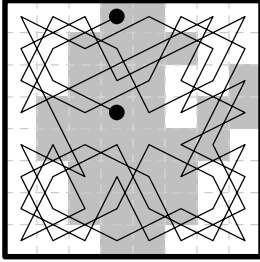
25b Marlow 1988



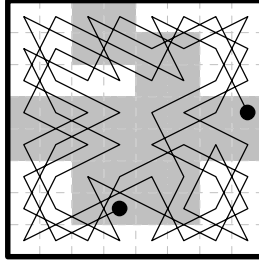
03c Ligondes 1884



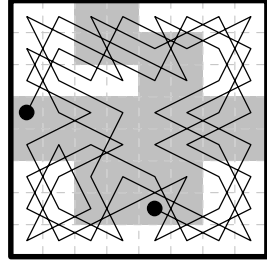
03d Ligondes 1884



34a Exner 1876

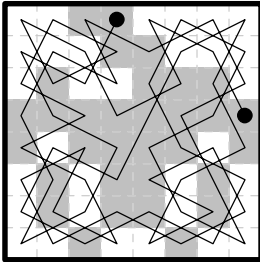


34b Ligondes 1883

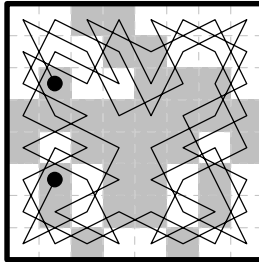


(1 with 2 tours and 8 edge cells) (10 individual patterns, these first two have 8 edge cells)

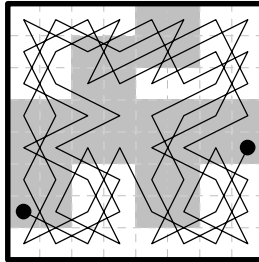
34g Ligondes 1884



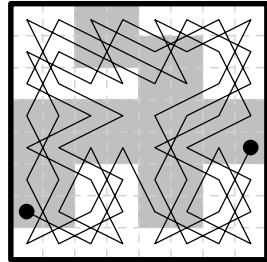
03g Murray 1940



27j Beligne 1883

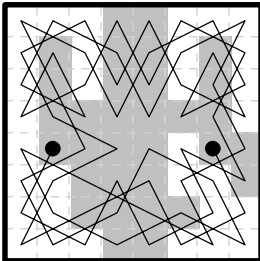


27m Francony 1883

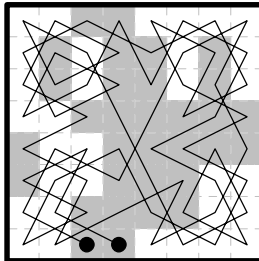


(these four have 6, 8, 10 and 10 edge cells). 01g is the only tour with four different quads.

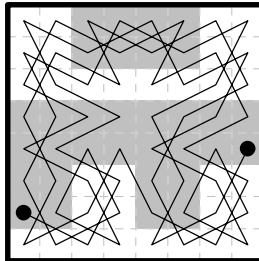
05d Bouvier 1882



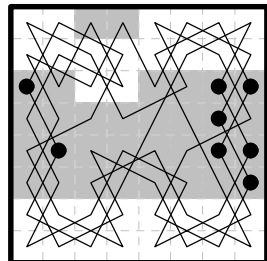
01g Marlow 1988



27k Francony 1881

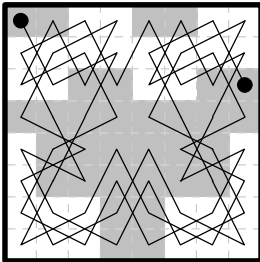


00h Ligondes 1906

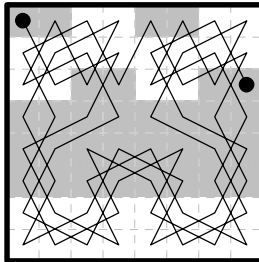


(This last batch all have odd numbers of edge cells, 9, 11, 11, 13)

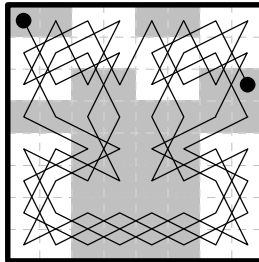
27c Jaenisch 1862



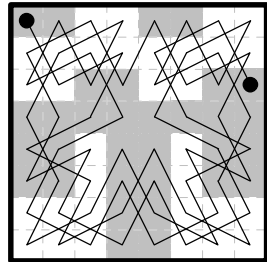
27a Beverley 1848



27b Wenzelides 1850



27d Jaenisch 1862



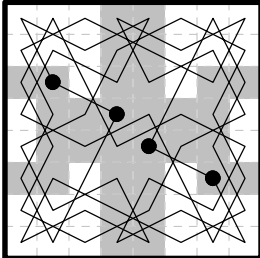
KNIGHT'S TOUR NOTES

(C) Irregular Type

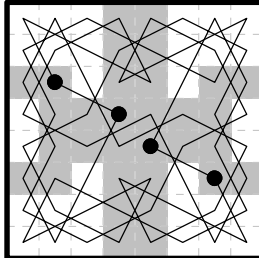
Tours 00j, 00k, 00l, 00n, 00o, 01e, 01i, 05a, 07a, 12c, 12d, 12f, 12g, 12h, 12i, 12j, 12k, 12l, 14e, 14f, 16a, 23o, 23p, 27e, 27f, 27g, 27h, 27n, 27o, 27t. Total 30.

CX Biaxial Background: 6 shapes in 12 tours, all have 8 dark edge cells
(2 with 3 tours, 2 individuals and, bottom row, 2 with 2 tours)

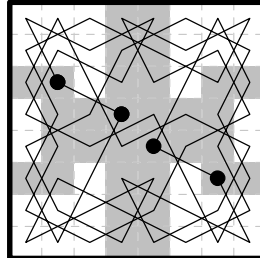
12i Bouvier 1884



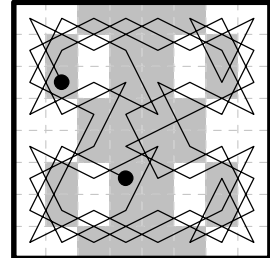
12j Bouvier 1884



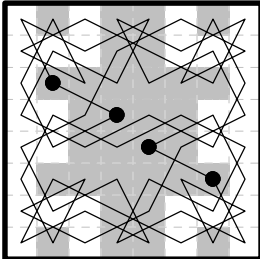
00l Bouvier 1884



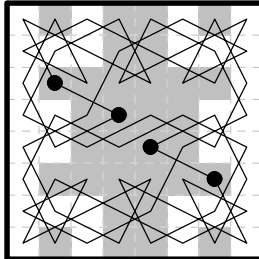
23o Ligondes 1910



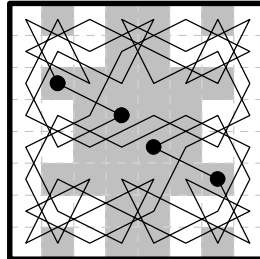
12k Francony 1884



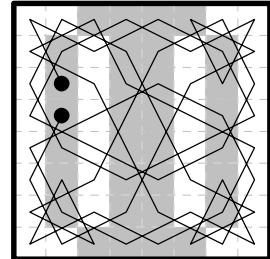
12l Francony 1884



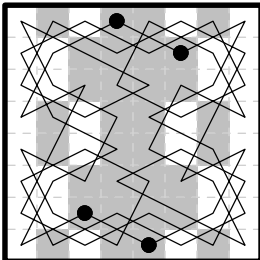
00k Francony 1884



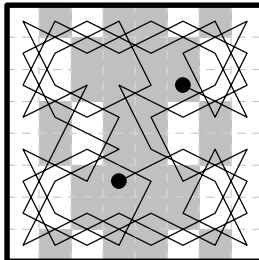
01e Grossetaite 1896



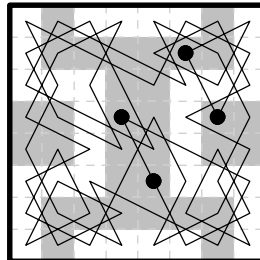
12c Beligne 1880



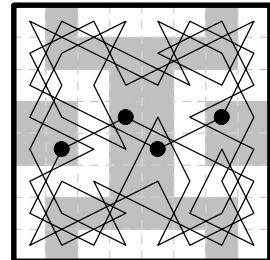
23p Ligondes 1911



00n Mackay et al

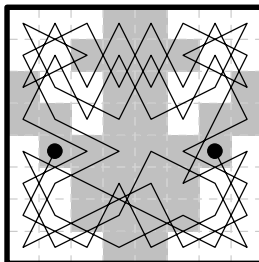


00o Mackay et al

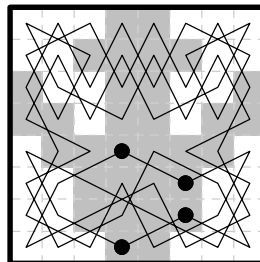


CY Axial Background: 4 shapes in 5 tours. (1 with 2 tours and 3 individuals)

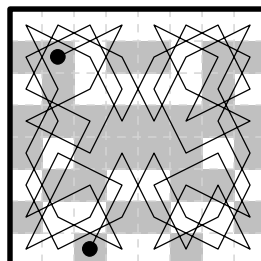
05a Bouvier 1876



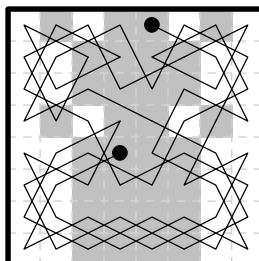
00j Reuss 1880



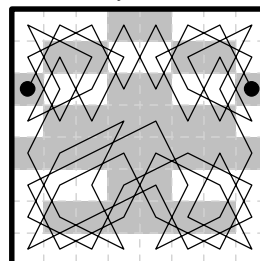
16a Lehmann 1933



14e Roberts 2003

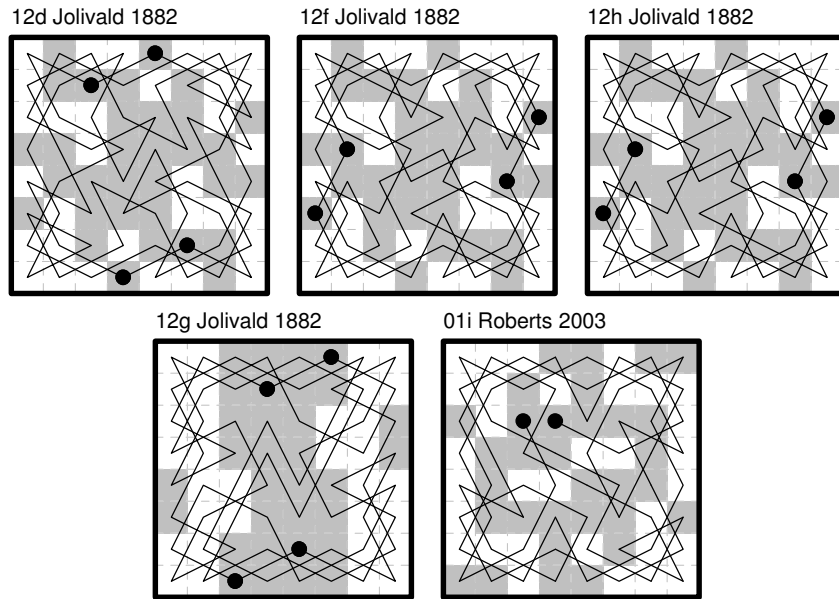


07a Mackay et al 2003

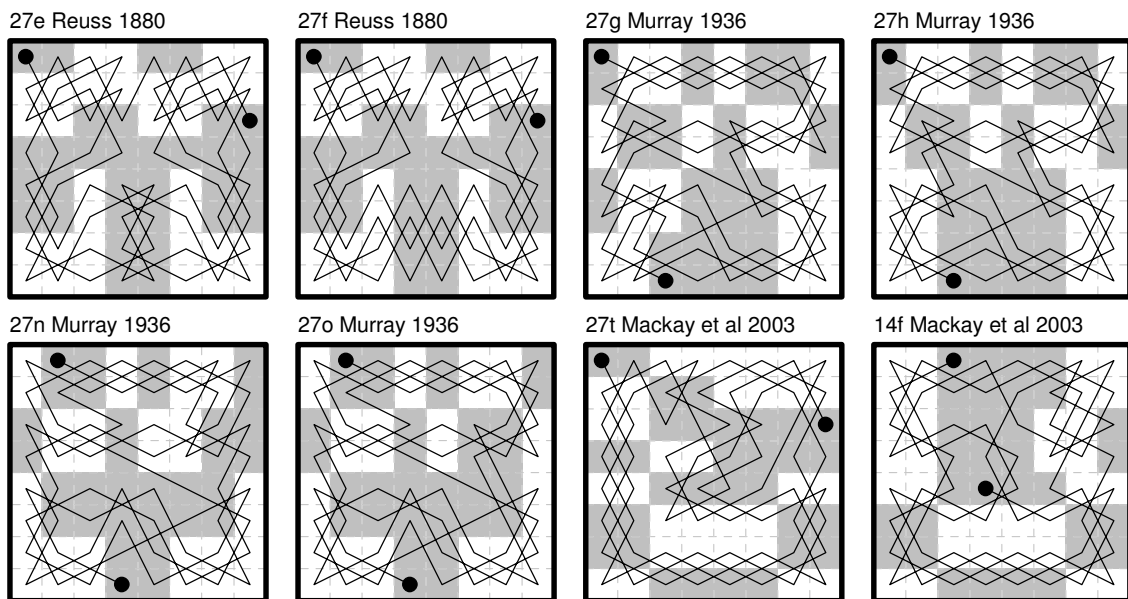


KNIGHT'S TOUR NOTES

CN Rotary Background: 3 shapes in 5 tours
 (1 with 3 tours) (2 individuals)



CL Asymmetric Background Patterns: 7 shapes in 8 tours (2 with one pattern, 6 individual)



Roberts (01i) is the only tour with two dark corners or with two white cells in the centre.
 There are five with a single white cell in the centre.(27g, h, n, o, t)

Arithmetical Catalogue of 8×8 Magic Knight Tours

Each arithmetical magic tour is oriented according to the Frénicle convention, that is with the smallest corner number at the top left, and the number to its right smaller than the number below it. The tours are arranged in numerical sequence when read rank by rank from the top left corner and are numbered from 1 to 280, in batches of 20 per page. Alongside the number we give the class of the tour (A, B, C) and its end-separation code (00 to 34) and arbitrary distinguishing letter (a to p). The composer and date of publication is also listed. The first appearance of a tour is taken to be its 'forward' numbering. The reverse of a tour is labelled R. In the case of a cyclic tour the first case is labelled A and a cyclic shift of 16 cells B, of 32 cells C, and of 48 cells D. However in 00h the shifts are 24, 40, 56, and 00a has a fifth numbering, E. Each of these has its own reversal, AR to ER.

Index of Arithmetical Tours. Given the two-figure and letter code of a tour, this index shows the first two numbers in the top row of its first appearance in the catalogue. For example, the Beverley tour 27a has the first two entries 1,30 and the first occurrence of the Jaenisch tour 00a (denoted 00aA) is under 2,15. It may be noticed that apart from the newly discovered 00o the cyclic tours, type 00, all begin with 2, i.e. they all have an origin a knight move from a corner. **00a:** 2,15 **00bcj:** 2,39 **00defg:** 2,27 **00h:** 2,23 **00i:** 2,11 **00k:** 2,59 **00lm:** 2,43 **00n:** 2,5 **00o:** 6,35 **01a:** 2,31 **01bc:** 15,30 **01d:** 18,15 **01e:** 2,11 **01f:** 2,15 **01g:** 6,51 **01h:** 6,3 **01i:** 4,23 **03a:** 3,6 **03b:** 3,10 **03cd:** 3,26 **03ef:** 2,43 **03g:** 2,23 **05abcdeg:** 6,27 **05f:** 11,18 **07a:** 14,11 **12a:** 2,11 **12bij:** 2,43 **12cd:** 2,27 **12e:** 11,34 **12fh:** 11,14 **12g:** 7,2 **12kl:** 2,59 **12mn:** 6,3 **12o:** 3,6 **12p:** 2,23 **14ab:** 3,26 **14c:** 3,38 **14d:** 7,10 **14e:** 3,22 **14f:** 7,24 **16a:** 10,35 **23a:** 11,22 **23b:** 6,39 **23cd:** 6,23 **23efgh:** 3,26 **23ijkl:** 7,22 **23mno:** 2,27 **23p:** 6,31 **23q:** 6,19 **25a:** 6,23 **25b:** 6,31 **27abcdegh:** 1,30 **27i:** 6,1 **27jklm:** 7,42 **27nopqrs:** 7,30 **34abcd:** 19,10 **34e:** 11,14 **34f:** 10,7 **34g:** 7,10

Reverse Index. Given the first two numbers in the top row this index gives the two-figure and letter code of tours with these values. **1,30:** 27abcdegh **2,5:** 00n **2,11:** 00i,01e,12a **2,15:** 00a,01f **2,23:** 00h,03g,12p **2,27:** 00defg,12cd,23mno **2,31:** 01a **2,39:** 00bcj **2,43:** 00lm,03ef,12bij **2,59:** 00k,12kl **3,6:** 03a,12o **3,10:** 03b **3,22:** 14e **3,26:** 03cd,14ab,23efgh **3,38:** 14c **4,23:** 01i **6,1:** 27i **6,3:** 01h,12mn **6,19:** 23q **6,23:** 23cd,25a **6,27:** 05abcdeg **6,31:** 23p,25b **6,35:** 00o **6,39:** 23b **6,51:** 01g **7,2:** 12g **7,10:** 14d,34g **7,22:** 23ijkl **7,24:** 14f **7,30:** 27nopqrs **7,42:** 27jklm **10,7:** 34f **10,35:** 16a **11,14:** 12fh,34e **11,18:** 05f **11,22:** 23a **11,34:** 12e **14,11:** 07a **15,30:** 01bc **18,15:** 01d **19,10:** 34abcd

Diagonal Sums. There are 41 of the 140 arithmetic magic tours in which the diagonals together sum to 520. These are: 00a(5), 00b(2), 00c(2), 00d(2), 00i(2), 00j(2), 00k(2), 00l(2), 01e, 03a, 05a, 05b, 05c, 05e, 05f, 05g, 12a, 12b, 12c, 12f, 12i, 12j, 12k, 12l, 12n, 12o, 16a, 23a, 34d, 34e. Their reverse numberings of course have the same property. The 00b, 00c, 00d cases each also have two other cyclic forms that do not add to 520. These 6 and the other 93 deviate from 520 by various amounts as follows: (± 2) 03d, 14a; (± 4) 00eAB, 03e; (± 6) 01g; (± 8) 01d, 07a, 12d, 12h, 12p, 27h, 27i, 27t, 34c; (± 14) 14f, 23b, 34a; (± 16) 00n(2), 00o(2), 12m, 14b, 14e; (± 18) 05d, 27j; (± 20) 12g; (± 22) 27g, 34b; (± 24) 00fBD, 00gAC, 01b, 03f; (± 26) 03g; (± 28) 00mB, 27a, 27b, 27c, 27d, 27e, 27f; (± 30) 25a, 34g; (± 34) 01a, 03c; (± 36) 00mA, 12e; (± 40) 00gBD, 00fAC, 14d; (± 48) 01f, 03b, 27k, 27l, 27p; (± 52) 14c; (± 56) 01h, 23h, 23k, 34f; (± 60) 00eD; (± 64) 00dAC, 25b; (± 66) 23e, 23j; (± 68) 00eC, 01c; (± 70) 23f, 23i; (± 78) 00hACD, 27m, 27q; (± 80) 23c, 23d, 23g, 23l, 23m, 23n, 23q; (± 82) 27r; (± 96) 23o, 23p; (± 112) 01i, 27s; (± 114) 00hB; (± 120) 27o; (± 128) 00bCD, 00cBD; (± 150) 27n.

Jaenischian magic tours. There are 22 of the magic tours that are Jaenischian, having the numbers 1, 16, 17, 32, 33, 48, 49, 64 forming a knight path. These are: 00a, 00k, 00l, 01f, 12i, 12j, 12k, 12l, 14a, 14b, 23e, 23f, 23g, 23h, 23i, 23j, 23k, 23l, 34a, 34b, 34c, 34d. There are also 11 in which the numbers 1, 32, 33, 64 form a knight path. These are: 00n, 00o, 01d, 01e, 03a, 05f, 05g, 12n, 12o, 16a, 34e.

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 1 - 20

<p>1 C 27g Murray 1936 01 30 47 52 03 28 45 54 48 51 02 29 46 53 04 27 31 08 49 42 25 06 55 44 50 41 32 07 56 43 26 05 09 62 39 20 33 24 15 58 40 19 10 61 16 57 34 23 63 38 17 12 21 36 59 14 18 11 64 37 60 13 22 35</p>	<p>2 C 27h Murray 1936 01 30 47 52 03 28 45 54 48 51 02 29 46 53 04 27 31 08 49 42 25 06 55 44 50 41 32 07 56 43 26 05 39 62 09 20 33 24 15 58 10 19 40 61 16 57 34 23 63 38 17 12 21 36 59 14 18 11 64 37 60 13 22 35</p>	<p>3 C 27t Mackay et al 2003 01 30 47 52 03 28 45 54 48 51 02 29 46 53 04 27 31 12 49 14 21 34 55 44 50 37 32 35 60 15 26 05 11 62 13 20 33 22 43 56 38 19 36 61 16 59 06 25 63 10 17 40 23 08 57 42 18 39 64 09 58 41 24 07</p>	<p>4 C 27e Reuss 1880 01 30 47 52 05 28 43 54 48 51 02 29 44 53 06 27 31 46 49 04 07 26 55 42 50 03 32 45 40 57 24 09 33 62 15 20 25 08 41 56 16 19 34 61 58 39 10 23 63 14 17 36 21 12 59 38 18 35 64 13 60 37 22 11</p>
<p>5 C 27f Reuss 1880 01 30 47 52 05 28 43 54 48 51 02 29 44 53 06 27 31 46 49 04 07 26 55 42 50 03 32 45 56 41 08 25 33 62 15 20 09 24 57 40 16 19 34 61 58 39 10 23 63 14 17 36 21 12 59 38 18 35 64 13 60 37 22 11</p>	<p>6 B 27a Beverley 1848 01 30 47 52 05 28 43 54 48 51 02 29 44 53 06 27 31 46 49 04 25 08 55 42 50 03 32 45 56 41 26 07 33 62 15 20 09 24 39 58 16 19 34 61 40 57 10 23 63 14 17 36 21 12 59 38 18 35 64 13 60 37 22 11</p>	<p>7 B 27d Jaenisch 1862 01 30 47 52 05 28 43 54 48 51 02 29 44 53 26 07 31 46 49 04 27 06 55 42 50 03 32 45 56 41 08 25 33 62 15 20 09 24 57 40 16 19 34 61 38 59 10 23 63 14 17 36 21 12 39 58 18 35 64 13 60 37 22 11</p>	<p>8 B 27c Jaenisch 1862 01 30 47 52 27 54 43 06 48 51 02 29 44 05 26 55 31 46 49 04 53 28 07 42 50 03 32 45 08 41 56 25 33 62 15 20 57 24 09 40 16 19 34 61 12 37 58 23 63 14 17 36 21 60 39 10 18 35 64 13 38 11 22 59</p>
<p>9 B 27b Wenzelides 1850 01 30 47 52 43 54 07 26 48 51 02 29 06 27 42 55 31 46 49 04 53 44 25 08 50 03 32 45 28 05 56 41 33 62 15 20 37 60 09 24 16 19 34 61 12 21 40 57 63 14 17 36 59 38 23 10 18 35 64 13 22 11 58 39</p>	<p>10 C 00nA Mackay et al 2003 02 05 62 39 26 31 36 59 63 40 01 04 37 60 25 30 06 03 38 61 32 27 58 35 41 64 07 12 49 34 29 24 08 45 50 33 28 23 16 57 51 42 11 48 13 54 19 22 46 09 44 53 20 17 56 15 43 52 47 10 55 14 21 18</p>	<p>11 C 01e Grossetaite 1896 02 11 32 53 04 57 46 55 31 52 03 12 45 54 05 58 10 01 30 33 24 59 56 47 51 64 23 44 13 34 25 06 22 09 50 29 60 07 48 35 63 40 43 08 49 14 17 26 42 21 38 61 28 19 36 15 39 62 41 20 37 16 27 18</p>	<p>12 A 00iA Unknown 1879 02 11 58 51 14 39 54 31 59 50 03 12 53 30 15 38 10 01 52 57 40 13 32 55 49 60 09 04 29 56 37 16 64 05 24 45 20 41 28 33 23 48 61 08 25 36 17 42 06 63 46 21 44 19 34 27 47 22 07 62 35 26 43 18</p>
<p>13 A 12a Wenzelides 1849 02 11 58 51 30 39 54 15 59 50 03 12 53 14 31 38 10 01 52 57 40 29 16 55 49 60 09 04 13 56 37 32 64 05 24 45 36 41 28 17 23 48 61 08 25 20 33 42 06 63 46 21 44 35 18 27 47 22 07 62 19 26 43 34</p>	<p>14 A 00aA Jaenisch 1862 02 15 50 29 64 27 38 35 51 30 01 14 39 36 63 26 16 03 32 49 28 61 34 37 31 52 13 04 33 40 25 62 54 17 48 41 12 05 60 23 45 42 53 20 57 24 11 08 18 55 44 47 06 09 22 59 43 46 19 56 21 58 07 10</p>	<p>15 B 01f Murray 1939 02 15 62 41 60 39 22 19 63 42 03 16 21 18 59 38 14 01 44 61 40 57 20 23 43 64 13 04 17 24 37 58 12 31 50 45 56 35 06 25 49 46 11 32 05 26 55 36 30 51 48 09 28 53 34 07 47 10 29 52 33 08 27 54</p>	<p>16 B 00hA Ligondes 1906 02 23 38 47 36 53 10 51 39 46 03 22 11 50 55 34 24 01 48 37 54 35 52 9 45 40 21 04 49 12 33 56 64 25 44 17 60 29 08 13 41 20 61 28 05 16 57 32 26 63 18 43 30 59 14 07 19 42 27 62 15 06 31 58</p>
<p>17 A 12p Murray 1939 02 23 42 33 64 31 38 27 43 34 01 24 37 28 63 30 22 03 36 41 32 61 26 39 35 44 21 04 25 40 29 62 54 05 52 45 20 13 60 11 51 48 55 08 57 10 17 14 06 53 46 49 16 19 12 59 47 50 07 56 09 58 15 18</p>	<p>18 B 03g Murray 1940 02 23 44 57 06 55 46 27 59 42 03 24 45 26 07 54 22 01 58 43 56 05 28 47 41 60 21 04 25 48 53 08 20 15 40 61 52 09 34 29 39 64 19 16 33 30 49 10 14 17 62 37 12 51 32 35 63 38 13 18 31 36 11 50</p>	<p>19 C 23o Ligondes 1910 02 27 42 47 18 23 62 39 43 48 01 26 63 40 17 22 28 03 46 41 24 19 38 61 49 44 25 08 33 64 21 16 04 29 56 45 20 09 60 37 53 50 07 32 57 34 15 12 30 05 52 55 10 13 36 59 51 54 31 06 35 58 11 14</p>	<p>20 C 12c Béligne 1880 02 27 42 47 30 55 06 51 41 46 03 28 05 50 31 56 26 01 48 43 54 29 52 07 45 40 21 04 49 12 57 32 64 25 44 17 36 53 08 13 39 20 61 22 11 16 33 58 24 63 18 37 60 35 14 09 19 38 23 62 15 10 59 34</p>

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 21 - 40

21 C 12d Jolivald 1882 02 27 42 49 30 53 06 51 41 48 03 28 05 50 31 54 26 01 46 43 56 29 52 07 47 40 25 04 45 12 55 32 64 23 44 13 36 57 08 15 39 20 61 24 11 14 33 58 22 63 18 37 60 35 16 09 19 38 21 62 17 10 59 34	22 B 00eA Jaenisch 1862 02 27 42 51 40 29 54 15 43 50 03 28 53 14 31 38 26 01 52 41 30 39 16 55 49 44 25 04 13 56 37 32 24 05 64 45 36 17 12 57 63 48 07 22 09 60 33 18 06 23 46 61 20 35 58 11 47 62 21 08 59 10 19 34	23 B 00fA Ligondes 1883 02 27 42 51 40 29 54 15 43 50 03 28 53 14 31 38 26 01 52 41 30 39 16 55 49 44 25 04 13 56 37 32 24 05 64 45 36 17 12 57 63 48 21 08 59 10 33 18 06 23 46 61 20 35 58 11 47 62 07 22 09 60 19 34	24 A 23m Ligondes 1883 02 27 50 43 06 23 62 47 51 42 01 26 63 48 07 22 28 03 44 49 24 05 46 61 41 52 25 04 45 64 21 08 14 29 40 53 20 09 60 35 39 54 13 32 57 36 19 10 30 15 56 37 12 17 34 59 55 38 31 16 33 58 11 18
25 A 23n Ligondes 1883 02 27 50 43 06 23 62 47 51 42 01 26 63 48 07 22 28 03 44 49 24 05 46 61 41 52 25 04 45 64 21 08 54 29 40 13 36 09 60 19 39 14 53 32 57 20 35 10 30 55 16 37 12 33 18 59 15 38 31 56 17 58 11 34	26 B 00gA Ligondes 1883 02 27 52 41 14 39 54 31 43 50 03 28 53 30 15 38 26 01 42 51 40 13 32 55 49 44 25 04 29 56 37 16 24 05 64 45 60 17 12 33 63 48 07 22 09 36 57 18 06 23 46 61 20 59 34 11 47 62 21 08 35 10 19 58	27 B 00fDR Ligondes 1883 02 27 52 41 30 39 54 15 43 50 03 28 53 14 31 38 26 01 42 51 40 29 16 55 49 44 25 04 13 56 37 32 24 05 64 45 36 17 12 57 63 48 07 22 09 60 33 18 06 23 46 61 20 35 58 11 47 62 21 08 59 10 19 34	28 B 00eDR Jaenisch 1862 02 27 52 41 30 39 54 15 43 50 03 28 53 14 31 38 26 01 42 51 40 29 16 55 49 44 25 04 13 56 37 32 24 05 64 45 36 17 12 57 63 48 21 08 59 10 33 18 06 23 46 61 20 35 58 11 47 62 07 22 09 60 19 34
29 B 03gR Murray 1940 02 27 52 47 34 29 54 15 51 48 03 28 53 14 33 30 26 01 46 49 32 35 16 55 45 50 25 04 13 56 31 36 24 05 44 61 40 17 12 57 43 64 07 22 09 60 37 18 06 23 62 41 20 39 58 11 63 42 21 08 59 10 19 38	30 B 00dA Reuss 1880 02 27 62 37 60 35 06 31 63 38 01 28 05 32 59 34 40 03 26 61 36 07 30 57 25 64 39 04 29 58 33 08 14 41 24 49 20 09 56 47 23 50 13 44 53 48 19 10 42 15 52 21 12 17 46 55 51 22 43 16 45 54 11 18	31 B 00dCR Reuss 1880 02 27 62 37 60 35 06 31 63 38 01 28 05 32 59 34 40 03 26 61 36 07 30 57 25 64 39 04 29 58 33 08 50 41 24 13 48 09 56 19 23 14 49 44 53 20 47 10 42 51 16 21 12 45 18 55 15 22 43 52 17 54 11 46	32 B 01a Ligondes 1884 02 31 38 61 36 27 06 59 39 62 01 32 05 60 35 26 30 03 64 37 28 33 58 07 63 40 29 04 57 08 25 34 42 17 52 13 48 23 56 09 51 14 41 20 53 10 47 24 18 43 16 49 12 45 22 55 15 50 19 44 21 54 11 46
33 A 00bA Mysore 1852 02 39 58 31 18 15 54 43 59 30 03 40 55 42 17 14 38 01 32 57 16 19 44 53 29 60 37 04 41 56 13 20 64 05 28 33 24 09 52 45 27 36 61 08 49 46 21 12 06 63 34 25 10 23 48 51 35 26 07 62 47 50 11 22	34 B 00dD Reuss 1880 02 39 58 31 56 41 18 15 59 30 03 40 17 14 43 54 38 01 32 57 42 55 16 19 29 60 37 04 13 20 53 44 64 05 28 33 52 45 12 21 27 36 61 08 23 10 49 46 06 63 34 25 48 51 22 11 35 26 07 62 09 24 47 50	35 A 00cA Francony 1881 02 39 58 31 56 41 18 15 59 30 03 40 17 14 55 42 38 01 32 57 44 53 16 19 29 60 37 04 13 20 43 54 64 05 28 33 52 45 22 11 27 36 61 08 21 12 49 46 06 63 34 25 48 51 10 23 35 26 07 62 09 24 47 50	36 C 00jA Reuss 1880 02 39 58 31 56 41 18 15 59 30 03 40 17 14 55 42 38 01 32 57 54 43 16 19 29 60 37 04 13 20 53 44 64 05 28 33 52 45 12 21 27 36 61 08 11 22 49 46 06 63 34 25 48 51 10 23 35 26 07 62 09 24 47 50
37 A 00cB Francony 1881 02 39 62 27 06 35 58 31 63 26 01 38 59 32 07 34 40 03 28 61 36 05 30 57 25 64 37 04 29 60 33 08 14 41 24 49 20 09 56 47 23 50 13 44 53 48 19 10 42 15 52 21 12 17 46 55 51 22 43 16 45 54 11 18	38 A 00cDR Francony 1881 02 39 62 27 06 35 58 31 63 26 01 38 59 32 07 34 40 03 28 61 36 05 30 57 25 64 37 04 29 60 33 08 50 41 24 13 48 09 56 19 23 14 49 44 53 20 47 10 42 51 16 21 12 45 18 55 15 22 43 52 17 54 11 46	39 B 03e Ligondes 1884 02 43 50 25 48 23 06 63 51 26 01 44 05 64 47 22 28 03 42 49 24 45 62 07 41 52 27 04 61 08 21 46 54 29 40 13 36 19 60 09 39 14 53 32 57 10 35 20 30 55 16 37 12 33 18 59 15 38 31 56 17 58 11 34	40 B 03f Ligondes 1883 02 43 50 25 48 23 06 63 51 26 01 44 05 64 47 22 42 03 28 49 24 45 62 07 27 52 41 04 61 08 21 46 40 29 54 13 36 19 60 09 53 14 39 32 57 10 35 20 30 55 16 37 12 33 18 59 15 38 31 56 17 58 11 34

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 41 - 60

41 B 00mA Wenzelides 1858 02 43 50 25 48 63 06 23 51 26 01 44 05 24 47 62 28 03 42 49 64 45 22 07 41 52 27 04 21 08 61 46 14 29 40 53 36 59 20 09 39 54 13 32 17 10 35 60 30 15 56 37 12 33 58 19 55 38 31 16 57 18 11 34	42 B 00gDR Ligondes 1883 02 43 50 25 64 23 06 47 51 26 01 44 05 48 63 22 28 03 42 49 24 07 46 61 41 52 27 04 45 62 21 08 14 29 40 53 20 09 60 35 39 54 13 32 57 36 19 10 30 15 56 37 12 17 34 59 55 38 31 16 33 58 11 18	43 B 00fC Ligondes 1883 02 43 50 25 64 23 06 47 51 26 01 44 05 48 63 22 28 03 42 49 24 07 46 61 41 52 27 04 45 62 21 08 54 29 40 13 36 09 60 19 39 14 53 32 57 20 35 10 30 55 16 37 12 33 18 59 15 38 31 56 17 58 11 34	44 B 00eBR Jaenisch 1862 02 43 50 25 64 23 06 47 51 26 01 44 05 48 63 22 28 03 42 49 24 61 46 07 41 52 27 04 45 08 21 62 54 29 40 13 36 19 60 09 39 14 53 32 57 10 35 20 30 55 16 37 12 33 18 59 15 38 31 56 17 58 11 34
45 B 00eC Jaenisch 1862 02 43 50 25 64 23 06 47 51 26 01 44 05 48 63 22 42 03 28 49 24 07 46 61 27 52 41 04 45 62 21 08 40 29 54 13 36 09 60 19 53 14 39 32 57 20 35 10 30 55 16 37 12 33 18 59 15 38 31 56 17 58 11 34	46 B 00fBR Ligondes 1883 02 43 50 25 64 23 06 47 51 26 01 44 05 48 63 22 42 03 28 49 24 61 46 07 27 52 41 04 45 08 21 62 40 29 54 13 36 19 60 09 53 14 39 32 57 10 35 20 30 55 16 37 12 33 18 59 15 38 31 56 17 58 11 34	47 C 001A Bouvier 1884 02 43 58 05 52 15 30 55 59 06 03 16 57 54 51 14 44 01 42 53 04 31 56 29 07 60 17 64 09 40 13 50 18 45 08 41 32 49 28 39 61 24 63 20 37 10 33 12 46 19 22 25 48 35 38 27 23 62 47 36 21 26 11 34	48 C 12j Bouvier 1884 02 43 58 05 52 15 30 55 59 06 03 16 57 54 51 14 44 01 42 53 04 31 56 29 07 60 17 64 09 40 13 50 18 45 08 41 32 49 28 39 61 24 63 36 21 10 33 12 46 19 22 25 48 35 38 27 23 62 47 20 37 26 11 34
49 A 12b Wenzelides 1850 02 43 58 19 30 39 22 47 59 18 03 44 21 46 31 38 42 01 20 57 40 29 48 23 17 60 41 04 45 24 37 32 64 05 56 13 36 09 28 49 55 16 61 08 25 52 33 10 06 63 14 53 12 35 50 27 15 54 07 62 51 26 11 34	50 A 00iCR Unknown 1879 02 43 58 19 46 39 22 31 59 18 03 44 21 30 47 38 42 01 20 57 40 45 32 23 17 60 41 04 29 24 37 48 64 05 56 13 52 09 28 33 55 16 61 08 25 36 49 10 06 63 14 53 12 51 34 27 15 54 07 62 35 26 11 50	51 B 03eR Ligondes 1884 02 43 58 19 56 45 06 31 59 18 03 44 05 30 47 54 42 01 20 57 46 55 32 07 17 60 41 04 29 08 53 48 40 21 16 61 52 33 28 09 15 64 23 38 25 12 49 34 22 39 62 13 36 51 10 27 63 14 37 24 11 26 35 50	52 B 03fR Ligondes 1883 02 43 58 19 56 45 06 31 59 18 03 44 05 30 47 54 42 01 20 57 46 55 32 07 17 60 41 04 29 08 53 48 40 21 16 61 52 33 28 09 15 64 37 24 11 26 49 34 22 39 62 13 36 51 10 27 63 14 23 38 25 12 35 50
53 B 00gCR Ligondes 1883 02 43 58 19 56 45 06 31 59 18 03 44 05 30 47 54 42 01 20 57 46 55 32 07 17 60 41 04 29 08 53 48 64 21 16 37 52 33 28 09 15 40 61 24 11 26 49 34 22 63 38 13 36 51 10 27 39 14 23 62 25 12 35 50	54 C 12i Bouvier 1884 02 43 58 53 04 15 30 55 59 06 03 16 57 54 51 14 44 01 42 05 52 31 56 29 07 60 17 64 09 40 13 50 18 45 08 41 32 49 28 39 61 24 63 20 37 10 33 12 46 19 22 25 48 35 38 27 23 62 47 36 21 26 11 34	55 C 001C Bouvier 1884 02 43 58 53 04 15 30 55 59 06 03 16 57 54 51 14 44 01 42 05 52 31 56 29 07 60 17 64 09 40 13 50 18 45 08 41 32 49 28 39 61 24 63 36 21 10 33 12 46 19 22 25 48 35 38 27 23 62 47 20 37 26 11 34	56 A 00aE Jaenisch 1862 02 51 64 15 54 29 18 27 63 14 03 52 17 26 55 30 50 01 16 61 32 53 28 19 13 62 49 04 25 20 31 56 48 39 12 33 60 41 06 21 11 36 45 40 05 24 57 42 38 47 34 09 44 59 22 07 35 10 37 46 23 08 43 58
57 C 00kA Francony 1884 02 59 62 07 18 43 46 23 61 06 01 42 63 24 19 44 58 03 60 17 08 45 22 47 05 16 53 64 41 20 25 36 52 57 04 09 32 37 48 21 15 54 13 40 49 28 35 26 12 51 56 31 10 33 38 29 55 14 11 50 39 30 27 34	58 C 12l Francony 1884 02 59 62 07 18 43 46 23 61 06 01 42 63 24 19 44 58 03 60 17 08 45 22 47 05 16 53 64 41 36 25 20 52 57 04 09 32 21 48 37 15 54 13 40 49 28 35 26 12 51 56 31 10 33 38 29 55 14 11 50 39 30 27 34	59 C 12k Francony 1884 02 59 62 07 18 43 46 23 61 06 01 42 63 24 19 44 58 03 60 17 08 45 22 47 53 16 05 64 41 20 25 36 04 57 52 09 32 37 48 21 15 54 13 40 49 28 35 26 12 51 56 31 10 33 38 29 55 14 11 50 39 30 27 34	60 C 00kC Francony 1884 02 59 62 07 18 43 46 23 61 06 01 42 63 24 19 44 58 03 60 17 08 45 22 47 53 16 05 64 41 36 25 20 04 57 52 09 32 21 48 37 15 54 13 40 49 28 35 26 12 51 56 31 10 33 38 29 55 14 11 50 39 30 27 34

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 61 - 80

<p>61 A 03a Ligondes 1884 03 06 47 58 31 10 51 54 46 59 04 07 50 53 30 11 05 02 57 48 09 32 55 52 60 45 08 01 56 49 12 29 43 20 61 24 33 28 37 14 62 23 44 17 40 13 34 27 19 42 21 64 25 36 15 38 22 63 18 41 16 39 26 35</p>	<p>62 A 00aC Jaenisch 1862 03 06 59 32 61 18 47 34 58 31 04 07 46 33 62 19 05 02 29 60 17 64 35 48 30 57 08 01 36 45 20 63 55 28 37 44 09 16 49 22 40 43 56 25 52 21 10 13 27 54 41 38 15 12 23 50 42 39 26 53 24 51 14 11</p>	<p>63 A 12o Jaenisch 1859 03 06 59 48 61 10 23 50 58 47 04 07 22 49 62 11 05 02 45 60 09 64 51 24 46 57 08 01 52 21 12 63 31 44 53 20 33 40 25 14 56 19 32 41 28 13 34 37 43 30 17 54 39 36 15 26 18 55 42 29 16 27 38 35</p>	<p>64 A 03b Marlow 1988 03 10 55 64 05 30 59 34 54 63 04 09 60 33 06 31 11 02 61 56 29 08 35 58 62 53 12 01 36 57 32 07 19 22 37 52 13 28 43 46 38 51 18 21 44 47 14 27 23 20 49 40 25 16 45 42 50 39 24 17 48 41 26 15</p>
<p>65 C 14e Roberts 2003 03 22 49 56 05 20 47 58 50 55 04 21 48 57 06 19 23 02 53 44 25 08 59 46 54 51 24 01 60 45 18 07 15 36 43 52 17 26 09 62 42 39 16 33 12 61 30 27 35 14 37 40 29 32 63 10 38 41 34 13 64 11 28 31</p>	<p>66 B 03c Ligondes 1884 03 26 35 62 23 14 47 50 34 63 02 25 48 51 22 15 27 04 61 36 13 24 49 46 64 33 28 01 52 45 16 21 05 60 37 32 17 12 53 44 38 29 08 57 42 55 20 11 59 06 31 40 09 18 43 54 30 39 58 07 56 41 10 19</p>	<p>67 B 03d Ligondes 1884 03 26 35 62 23 14 47 50 34 63 02 25 48 51 22 15 27 04 61 36 13 24 49 46 64 33 28 01 52 45 16 21 05 60 37 32 17 12 53 44 38 29 58 07 56 41 20 11 59 06 31 40 09 18 43 54 30 39 08 57 42 55 10 19</p>	<p>68 B 23f Ligondes 1883 03 26 41 52 15 30 39 54 50 43 02 27 40 53 14 31 25 04 51 42 29 16 55 38 44 49 28 01 56 37 32 13 05 24 61 48 17 12 57 36 64 45 08 21 60 33 18 11 23 06 47 62 09 20 35 58 46 63 22 07 34 59 10 19</p>
<p>69 B 23h Ligondes 1883 03 26 41 52 15 30 39 54 50 43 02 27 40 53 14 31 25 04 51 42 29 16 55 38 44 49 28 01 56 37 32 13 05 24 61 48 17 12 57 36 64 45 22 07 60 33 18 11 23 06 47 62 09 20 35 58 46 63 08 21 34 59 10 19</p>	<p>70 A 14b Ligondes 1883 03 26 47 54 15 30 51 34 46 55 02 27 52 33 14 31 25 04 53 48 29 16 35 50 56 45 28 01 36 49 32 13 05 24 41 60 17 12 37 64 44 57 08 21 40 61 18 11 23 06 59 42 09 20 63 38 58 43 22 07 62 39 10 19</p>	<p>71 B 14a Ligondes 1883 03 26 47 54 15 30 51 34 46 55 02 27 52 33 14 31 25 04 53 48 29 16 35 50 56 45 28 01 36 49 32 13 05 24 41 60 17 12 37 64 44 57 22 07 40 61 18 11 23 06 59 42 09 20 63 38 58 43 08 21 62 39 10 19</p>	<p>72 A 23g Ligondes 1883 03 26 51 42 15 30 39 54 50 43 02 27 40 53 14 31 25 04 41 52 29 16 55 38 44 49 28 01 56 37 32 13 05 24 61 48 17 12 57 36 64 45 08 21 60 33 18 11 23 06 47 62 09 20 35 58 46 63 22 07 34 59 10 19</p>
<p>73 B 23e Ligondes 1883 03 26 51 42 15 30 39 54 50 43 02 27 40 53 14 31 25 04 41 52 29 16 55 38 44 49 28 01 56 37 32 13 05 24 61 48 17 12 57 36 64 45 22 07 60 33 18 11 23 06 47 62 09 20 35 58 46 63 08 21 34 59 10 19</p>	<p>74 B 14c Ligondes 1883 03 38 59 32 61 06 27 34 58 31 04 37 28 33 62 07 39 02 29 60 05 26 35 64 30 57 40 01 36 63 08 25 41 16 55 20 45 24 49 10 56 19 42 13 52 09 46 23 15 54 17 44 21 48 11 50 18 43 14 53 12 51 22 47</p>	<p>75 A 00bC Mysore 1852 03 38 59 32 61 06 27 34 58 31 04 37 28 33 62 07 39 02 29 60 05 64 35 26 30 57 40 01 36 25 08 63 15 42 17 56 09 48 23 50 18 55 14 41 24 51 10 47 43 16 53 20 45 12 49 22 54 19 44 13 52 21 46 11</p>	<p>76 C 00jC Reuss 1880 03 38 59 32 61 06 27 34 58 31 04 37 28 33 62 07 39 02 29 60 05 64 35 26 30 57 40 01 36 25 08 63 41 16 43 20 45 22 49 24 56 19 54 13 52 11 46 09 15 42 17 44 21 48 23 50 18 55 14 53 12 51 10 47</p>
<p>77 A 00cC Francony 1881 03 38 59 32 61 06 27 34 58 31 04 37 28 33 62 07 39 02 29 60 05 64 35 26 30 57 40 01 36 25 08 63 41 16 53 20 45 12 49 24 56 19 44 13 52 21 46 09 15 42 17 54 11 48 23 50 18 55 14 43 22 51 10 47</p>	<p>78 B 00dB Reuss 1880 03 38 59 32 61 06 27 34 58 31 04 37 28 33 62 07 39 02 29 60 05 64 35 26 30 57 40 01 36 25 08 63 41 16 55 20 45 10 49 24 56 19 42 13 52 23 46 09 15 54 17 44 21 48 11 50 18 43 14 53 12 51 22 47</p>	<p>79 C 01i Roberts 2003 04 23 50 55 06 25 58 39 49 54 05 24 57 38 07 26 22 03 56 51 28 01 40 59 53 48 21 02 37 64 27 08 20 35 52 29 14 09 60 41 47 32 13 36 63 44 15 10 34 19 30 45 12 17 42 61 31 46 33 18 43 62 11 16</p>	<p>80 A 27i Exner 1876 06 01 54 29 52 27 48 43 55 30 05 02 47 44 51 26 04 07 32 53 28 49 42 45 31 56 03 08 41 46 25 50 62 09 58 33 24 19 40 15 57 34 61 12 37 16 23 20 10 63 36 59 18 21 14 39 35 60 11 64 13 38 17 22</p>

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 81 - 100

81 A 00aBR

Jaenisch 1862

06 03 30 57 28 55 42 39
31 58 05 02 43 40 27 54
04 07 60 29 56 25 38 41
59 32 01 08 37 44 53 26
18 61 36 45 16 09 24 51
33 46 17 64 21 52 15 12
62 19 48 35 10 13 50 23
47 34 63 20 49 22 11 14

82 A 01h

Marlow 1988

06 03 34 61 32 19 46 59
35 62 05 02 47 60 31 18
04 07 64 33 20 29 58 45
63 36 01 08 57 48 17 30
38 09 56 49 28 21 44 15
53 50 37 12 41 16 27 24
10 39 52 55 22 25 14 43
51 54 11 40 13 42 23 26

83 A 12n

Jaenisch 1859

06 03 46 57 44 31 18 55
47 58 05 02 19 56 43 30
04 07 60 45 32 41 54 17
59 48 01 08 53 20 29 42
10 61 52 21 40 33 16 27
49 22 09 64 13 28 39 36
62 11 24 51 34 37 26 15
23 50 63 12 25 14 35 38

84 A 12m

Wenzelides 1858

06 03 58 51 30 43 54 15
59 50 07 04 53 14 31 42
02 05 52 57 44 29 16 55
49 60 01 08 13 56 41 32
64 09 24 45 40 33 28 17
23 48 61 12 25 20 37 34
10 63 46 21 36 39 18 27
47 22 11 62 19 26 35 38

85 C 00nBR

Mackay et al 2003

06 03 62 41 08 43 50 47
61 40 07 04 49 46 09 44
02 05 38 63 42 11 48 51
39 60 01 16 37 52 45 10
58 29 36 21 64 17 12 23
35 32 59 26 15 22 53 18
28 57 30 33 20 55 24 13
31 34 27 56 25 14 19 54

86 A 23q

Marlow 1988

06 19 32 61 28 21 34 59
31 62 07 20 33 60 25 22
18 05 64 29 24 27 58 35
63 30 17 08 57 36 23 26
50 47 04 41 16 09 56 37
01 44 49 46 53 40 15 12
48 51 42 03 10 13 38 55
43 02 45 52 39 54 11 14

87 A 01hR

Marlow 1988

06 19 46 33 04 31 62 59
47 34 05 18 63 60 03 30
20 07 36 45 32 01 58 61
35 48 17 08 57 64 29 02
50 21 44 37 16 09 56 27
41 38 49 24 53 28 15 12
22 51 40 43 10 13 26 55
39 42 23 52 25 54 11 14

88 A 23c

Ligondes 1883

06 23 48 63 26 01 50 43
47 62 05 24 49 44 27 02
22 07 64 45 04 25 42 51
61 46 21 08 41 52 03 28
20 09 36 57 32 13 40 53
35 60 17 12 37 56 29 14
10 19 58 33 16 31 54 39
59 34 11 18 55 38 15 30

89 A 23d

Ligondes 1883

06 23 64 45 04 25 50 43
63 46 05 24 49 44 03 26
22 07 48 61 28 01 42 51
47 62 21 08 41 52 27 02
34 09 60 17 56 29 40 15
59 20 33 12 37 16 53 30
10 35 18 57 32 55 14 39
19 58 11 36 13 38 31 54

90 B 25a

Ligondes 1883

06 23 64 45 04 25 50 43
63 46 05 24 49 44 03 26
22 07 48 61 28 51 42 01
47 62 21 08 41 02 27 52
34 09 60 17 56 29 40 15
59 20 33 12 37 16 53 30
10 35 18 57 32 55 14 39
19 58 11 36 13 38 31 54

91 A 05g

Ligondes 1883

06 27 50 47 18 39 14 59
51 48 07 26 15 58 17 38
28 05 46 49 40 19 60 13
45 52 25 08 57 16 37 20
04 29 56 41 24 33 12 61
53 44 03 32 09 62 21 36
30 01 42 55 34 23 64 11
43 54 31 02 63 10 35 22

92 A 05gR

Ligondes 1883

06 27 52 45 04 29 54 43
51 48 05 28 53 44 01 30
26 07 46 49 32 03 42 55
47 50 25 08 41 56 31 02
18 39 16 57 24 33 10 63
15 58 19 40 09 62 23 34
38 17 60 13 36 21 64 11
59 14 37 20 61 12 35 22

93 B 05d

Bouvier 1882

06 27 62 33 60 03 38 31
63 34 05 28 37 32 59 02
26 07 36 61 04 39 30 57
35 64 25 08 29 58 01 40
50 09 48 21 44 17 56 15
47 24 51 12 53 14 41 18
10 49 22 45 20 43 16 55
23 46 11 52 13 54 19 42

94 C 05a

Bouvier 1876

06 27 62 33 60 03 38 31
63 34 05 28 37 32 59 02
26 07 36 61 04 57 30 39
35 64 25 08 29 40 01 58
24 09 22 45 20 43 56 41
49 46 51 12 53 14 19 16
10 23 48 21 44 17 42 55
47 50 11 52 13 54 15 18

95 B 05e

Francony 1882

06 27 62 33 60 03 38 31
63 34 05 28 37 32 59 02
26 07 36 61 04 57 30 39
35 64 25 08 29 40 01 58
24 09 50 45 20 15 56 41
49 46 23 12 53 42 19 16
10 51 48 21 44 17 14 55
47 22 11 52 13 54 43 18

96 A 05b

Caldwell 1879

06 27 62 33 60 03 38 31
63 34 05 28 37 32 59 02
26 07 36 61 04 57 30 39
35 64 25 08 29 40 01 58
24 09 52 45 20 13 56 41
49 46 21 12 53 44 19 16
10 23 48 51 14 17 42 55
47 50 11 22 43 54 15 18

97 A 05c

Francony 1882

06 27 62 33 60 03 38 31
63 34 05 28 37 32 59 02
26 07 36 61 04 57 30 39
35 64 25 08 29 40 01 58
50 09 48 21 44 17 56 15
47 24 51 12 53 14 41 18
10 49 22 45 20 43 16 55
23 46 11 52 13 54 19 42

98 C 00nAR

Mackay et al 2003

06 29 34 39 26 03 60 63
35 40 05 28 61 64 25 02
30 07 38 33 04 27 62 59
41 36 31 16 53 58 01 24
08 49 42 37 32 15 20 57
43 46 11 52 17 54 23 14
50 09 48 45 12 21 56 19
47 44 51 10 55 18 13 22

99 A 23qR

Marlow 1988

06 31 44 37 04 33 46 59
43 40 05 32 45 58 03 34
30 07 38 41 36 01 60 47
39 42 29 08 57 48 35 02
28 09 56 49 24 61 18 15
53 50 25 12 19 16 21 64
10 27 52 55 62 23 14 17
51 54 11 26 13 20 63 22

100 C 23p

Ligondes 1911

06 31 50 55 10 15 58 35
49 54 07 32 57 34 11 16
30 05 56 51 14 09 36 59
53 48 25 08 33 64 17 12
04 29 52 41 24 13 60 37
47 44 01 26 63 40 21 18
28 03 42 45 20 23 38 61
43 46 27 02 39 62 19 22

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 101 - 120

101 A 23cR

Ligondes 1883

06 31 54 47 10 27 50 35
55 46 07 32 49 34 11 26
30 05 48 53 28 09 36 51
45 56 29 08 33 52 25 12
04 19 44 57 24 13 62 37
43 58 01 20 61 40 23 14
18 03 60 41 16 21 38 63
59 42 17 02 39 64 15 22

102 B 25b

Marlow 1988

06 31 58 41 04 39 46 35
59 42 05 32 45 36 03 38
30 07 44 57 40 01 34 47
43 60 29 08 33 48 37 02
28 17 62 13 56 09 52 23
61 14 27 20 49 24 55 10
18 63 16 25 12 53 22 51
15 26 19 64 21 50 11 54

103 A 00bB

Mysore 1852

06 35 60 29 04 37 62 27
59 32 05 36 61 28 01 38
34 07 30 57 40 03 26 63
31 58 33 08 25 64 39 02
18 09 56 45 16 41 24 51
55 46 17 12 21 52 15 42
10 19 48 53 44 13 50 23
47 54 11 20 49 22 43 14

104 A 00bDR

Mysore 1852

06 35 60 29 04 37 62 27
59 32 05 36 61 28 01 38
34 07 30 57 40 03 26 63
31 58 33 08 25 64 39 02
46 09 56 17 52 41 24 15
55 18 45 12 21 16 51 42
10 47 20 53 44 49 14 23
19 54 11 48 13 22 43 50

105 C 00oA

Mackay et al 2003

06 35 62 41 08 43 18 47
61 40 07 36 17 46 09 44
34 05 38 63 42 11 48 19
39 60 33 16 37 20 45 10
58 29 04 21 64 49 12 23
03 32 59 26 15 22 53 50
28 57 30 01 52 55 24 13
31 02 27 56 25 14 51 54

106 B 01aR

Ligondes 1884

06 39 58 31 56 41 10 19
59 30 07 40 09 18 43 54
38 05 32 57 42 55 20 11
29 60 37 08 17 12 53 44
04 33 28 61 52 45 16 21
27 64 01 36 13 24 49 46
34 03 62 25 48 51 22 15
63 26 35 02 23 14 47 50

107 B 23b

Ligondes 1885

06 39 58 31 56 41 10 19
59 30 07 40 09 18 43 54
38 05 32 57 42 55 20 11
29 60 37 08 17 12 53 44
64 33 04 25 52 45 16 21
03 28 61 36 13 24 49 46
34 63 26 01 48 51 22 15
27 02 35 62 23 14 47 50

108 B 27cR

Jaenisch 1862

06 43 54 27 52 01 30 47
55 26 05 44 29 48 51 02
42 07 28 53 04 31 46 49
25 56 41 08 45 50 03 32
40 09 24 57 20 33 62 15
23 58 37 12 61 16 19 34
10 39 60 21 36 63 14 17
59 22 11 38 13 18 35 64

109 B 00hCR

Ligondes 1906

06 47 62 27 10 19 58 31
63 26 07 46 59 30 11 18
48 05 28 61 20 09 32 57
25 64 45 08 29 60 17 12
44 49 04 21 40 13 56 33
01 24 41 52 35 54 37 16
50 43 22 03 14 39 34 55
23 02 51 42 53 36 15 38

110 B 01g

Marlow 1988

06 51 64 01 54 39 10 35
63 02 05 52 09 36 55 38
04 07 50 61 40 53 34 11
49 62 03 08 33 12 37 56
26 23 48 41 60 17 32 13
47 44 27 24 29 14 57 18
22 25 42 45 20 59 16 31
43 46 21 28 15 30 19 58

111 C 12g

Jolivald 1882

07 02 49 60 09 64 47 22
50 59 08 01 48 21 10 63
03 06 57 44 61 46 23 20
58 51 04 13 24 11 62 37
05 30 43 56 45 36 19 26
52 55 14 29 12 25 38 35
31 42 53 16 33 40 27 18
54 15 32 41 28 17 34 39

112 A 00aDR

Jaenisch 1862

07 10 23 60 25 62 35 38
22 59 08 11 34 37 26 63
09 06 57 24 61 28 39 36
58 21 12 05 40 33 64 27
19 56 41 48 13 04 29 50
44 47 20 53 32 49 14 01
55 18 45 42 03 16 51 30
46 43 54 17 52 31 02 15

113 B 34g

Ligondes 1884

07 10 53 64 03 42 51 30
62 55 06 09 52 31 02 43
11 08 63 54 41 04 29 50
56 61 12 05 32 49 44 01
13 34 17 60 45 40 23 28
18 57 14 33 24 27 48 39
35 16 59 20 37 46 25 22
58 19 36 15 26 21 38 47

114 A 14d

Feisthamel 1884

07 10 63 54 03 42 51 30
62 55 06 09 52 31 02 43
11 08 53 64 41 04 29 50
56 61 12 05 32 49 44 01
13 34 17 60 45 40 23 28
18 57 14 33 24 27 48 39
35 16 59 20 37 46 25 22
58 19 36 15 26 21 38 47

115 C 14eR

Roberts 2003

07 18 45 60 09 16 43 62
46 59 08 17 44 61 10 15
19 06 57 40 21 12 63 42
58 47 20 05 64 41 14 11
03 56 39 48 13 22 29 50
38 35 04 53 32 49 26 23
55 02 33 36 25 28 51 30
34 37 54 01 52 31 24 27

116 A 14bR

Ligondes 1883

07 22 43 58 03 26 55 46
42 59 06 23 56 45 02 27
21 08 57 44 25 04 47 54
60 41 24 05 48 53 28 01
09 20 37 64 29 16 33 52
40 61 12 17 36 49 30 15
19 10 63 38 13 32 51 34
62 39 18 11 50 35 14 31

117 B 23i

Ligondes 1883

07 22 47 62 09 20 35 58
46 63 08 21 34 59 18 11
23 06 61 48 19 10 57 36
64 45 24 05 60 33 12 17
25 04 49 44 29 16 37 56
50 43 28 01 40 53 32 13
03 26 41 52 15 30 55 38
42 51 02 27 54 39 14 31

118 B 23k

Ligondes 1883

07 22 47 62 09 20 35 58
46 63 08 21 34 59 18 11
23 06 61 48 19 10 57 36
64 45 24 05 60 33 12 17
25 04 49 44 29 16 37 56
50 43 28 01 54 39 32 13
03 26 41 52 15 30 55 38
42 51 02 27 40 53 14 31

119 A 23l

Ligondes 1883

07 22 47 62 19 10 35 58
46 63 08 21 34 59 18 11
23 06 61 48 09 20 57 36
64 45 24 05 60 33 12 17
25 04 49 44 29 16 37 56
50 43 28 01 40 53 32 13
03 26 41 52 15 30 55 38
42 51 02 27 54 39 14 31

120 B 23j

Ligondes 1883

07 22 47 62 19 10 35 58
46 63 08 21 34 59 18 11
23 06 61 48 09 20 57 36
64 45 24 05 60 33 12 17
25 04 49 44 29 16 37 56
50 43 28 01 54 39 32 13
03 26 41 52 15 30 55 38
42 51 02 27 40 53 14 31

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 121 - 140

121 A 00aER

Jaenisch 1862

07 22 57 42 19 28 55 30
58 43 06 21 56 31 18 27
23 08 41 60 25 20 29 54
44 59 24 05 32 53 26 17
09 34 45 40 61 16 03 52
46 37 12 33 04 49 64 15
35 10 39 48 13 62 51 02
38 47 36 11 50 01 14 63

122 B 14aR

Ligondes 1883

07 22 57 44 03 26 55 46
42 59 06 23 56 45 02 27
21 08 43 58 25 04 47 54
60 41 24 05 48 53 28 01
09 20 37 64 29 16 33 52
40 61 12 17 36 49 30 15
19 10 63 38 13 32 51 34
62 39 18 11 50 35 14 31

123 C 14f

Mackay et al 2003

07 24 41 58 19 10 63 38
42 57 08 23 62 39 18 11
25 06 59 40 09 20 37 64
56 43 22 01 48 61 12 17
05 26 47 60 21 16 49 36
44 55 02 29 34 51 32 13
27 04 53 46 15 30 35 50
54 45 28 03 52 33 14 31

124 C 27n

Murray 1936

07 30 43 52 01 54 47 26
42 51 06 29 48 27 02 55
31 08 49 44 53 04 25 46
50 41 32 05 28 45 56 03
09 60 39 22 33 58 15 24
40 21 10 59 16 23 34 57
61 38 19 12 63 36 17 14
20 11 62 37 18 13 64 35

125 B 27q

Reuss 1883

07 30 43 52 01 54 47 26
42 51 06 29 48 27 02 55
31 08 49 44 53 04 25 46
50 41 32 05 28 45 56 03
09 62 39 20 33 24 15 58
40 19 10 61 16 57 34 23
63 38 17 12 21 36 59 14
18 11 64 37 60 13 22 35

126 B 27s

Reuss 1883

07 30 43 52 01 54 47 26
42 51 06 29 48 27 02 55
31 08 49 44 53 04 25 46
50 41 32 05 28 45 56 03
09 62 39 20 33 58 15 24
40 19 10 61 16 23 34 57
63 38 17 12 21 36 59 14
18 11 64 37 60 13 22 35

127 C 27o

Murray 1936

07 30 43 52 01 54 47 26
42 51 06 29 48 27 02 55
31 08 49 44 53 04 25 46
50 41 32 05 28 45 56 03
39 60 09 22 33 58 15 24
10 21 40 59 16 23 34 57
61 38 19 12 63 36 17 14
20 11 62 37 18 13 64 35

128 A 27p

Reuss 1883

07 30 43 52 01 54 47 26
42 51 06 29 48 27 02 55
31 08 49 44 53 04 25 46
50 41 32 05 28 45 56 03
39 62 09 20 33 24 15 58
10 19 40 61 16 57 34 23
63 38 17 12 21 36 59 14
18 11 64 37 60 13 22 35

129 B 27r

Reuss 1883

07 30 43 52 01 54 47 26
42 51 06 29 48 27 02 55
31 08 49 44 53 04 25 46
50 41 32 05 28 45 56 03
39 62 09 20 33 58 15 24
10 19 40 61 16 23 34 57
63 38 17 12 21 36 59 14
18 11 64 37 60 13 22 35

130 B 23iR

Ligondes 1883

07 30 45 56 03 18 43 58
54 47 06 31 44 57 02 19
29 08 55 46 17 04 59 42
48 53 32 05 60 41 20 01
09 28 49 36 21 16 61 40
52 33 12 25 64 37 22 15
27 10 35 50 13 24 39 62
34 51 26 11 38 63 14 23

131 B 23kR

Ligondes 1883

07 30 45 56 03 18 43 58
54 47 06 31 44 57 02 19
29 08 55 46 17 04 59 42
48 53 32 05 60 41 20 01
09 28 49 36 21 16 61 40
52 33 26 11 64 37 22 15
27 10 35 50 13 24 39 62
34 51 12 25 38 63 14 23

132 B 34gR

Ligondes 1884

07 30 47 52 09 54 03 58
46 49 08 31 04 57 10 55
29 06 51 48 53 02 59 12
50 45 32 05 60 11 56 01
39 28 41 20 33 24 13 62
44 19 38 25 16 61 34 23
27 40 17 42 21 36 63 14
18 43 26 37 64 15 22 35

133 A 14dR

Feisthamel 1884

07 30 47 52 09 54 03 58
46 49 08 31 04 57 10 55
29 06 51 48 53 12 59 02
50 45 32 05 60 01 56 11
39 28 41 20 33 24 13 62
44 19 38 25 16 61 34 23
27 40 17 42 21 36 63 14
18 43 26 37 64 15 22 35

134 A 23iR

Ligondes 1883

07 30 55 46 03 18 43 58
54 47 06 31 44 57 02 19
29 08 45 56 17 04 59 42
48 53 32 05 60 41 20 01
09 28 49 36 21 16 61 40
52 33 12 25 64 37 22 15
27 10 35 50 13 24 39 62
34 51 26 11 38 63 14 23

135 B 23jR

Ligondes 1883

07 30 55 46 03 18 43 58
54 47 06 31 44 57 02 19
29 08 45 56 17 04 59 42
48 53 32 05 60 41 20 01
09 28 49 36 21 16 61 40
52 33 26 11 64 37 22 15
27 10 35 50 13 24 39 62
34 51 12 25 38 63 14 23

136 B 01gR

Marlow 1988

07 34 47 52 09 54 27 30
46 49 08 33 28 31 10 55
35 06 51 48 53 12 29 26
50 45 36 05 32 25 56 11
37 20 41 24 57 04 13 64
44 23 38 17 62 15 60 01
19 40 21 42 03 58 63 14
22 43 18 39 16 61 02 59

137 B 00hDR

Ligondes 1906

07 34 49 28 09 32 47 54
50 27 08 33 48 53 10 31
35 06 25 52 29 12 55 46
26 51 36 05 56 45 30 11
37 62 19 24 13 04 43 58
20 23 38 61 44 57 14 03
63 18 21 40 01 16 59 42
22 39 64 17 60 41 02 15

138 B 00hAR

Ligondes 1906

07 34 59 50 03 38 23 46
58 51 06 35 22 47 02 39
33 08 49 60 37 04 45 24
52 57 36 05 48 21 40 01
09 32 53 16 61 44 25 20
56 13 30 11 28 17 64 41
31 10 15 54 43 62 19 26
14 55 12 29 18 27 42 63

139 B 27k

Francony 1881

07 42 25 56 09 40 23 58
54 27 08 41 24 57 38 11
43 06 55 26 39 10 59 22
28 53 44 05 60 21 12 37
01 48 29 52 33 16 61 20
30 51 04 45 62 19 36 13
47 02 49 32 15 34 17 64
50 31 46 03 18 63 14 35

140 B 27kR

Francony 1881

07 42 25 56 09 40 23 58
54 27 08 41 24 57 38 11
43 06 55 26 39 10 59 22
28 53 44 05 60 21 12 37
45 04 49 32 13 36 17 64
52 29 46 03 20 61 14 35
01 48 31 50 33 16 63 18
30 51 02 47 62 19 34 15

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 141 - 160

<p>141 B 27j Béligne 1883 07 42 25 56 39 10 23 58 54 27 08 41 24 57 38 11 43 06 55 26 09 40 59 22 28 53 44 05 60 21 12 37 01 48 29 52 33 16 61 20 30 51 04 45 62 19 36 13 47 02 49 32 15 34 17 64 50 31 46 03 18 63 14 35</p>	<p>142 B 27m Francony 1883 07 42 25 56 39 10 23 58 54 27 08 41 24 57 38 11 43 06 55 26 09 40 59 22 28 53 44 05 60 21 12 37 45 04 49 32 13 36 17 64 52 29 46 03 20 61 14 35 01 48 31 50 33 16 63 18 30 51 02 47 62 19 34 15</p>	<p>143 B 27mR Francony 1883 07 42 55 26 09 40 23 58 54 27 08 41 24 57 38 11 43 06 25 56 39 10 59 22 28 53 44 05 60 21 12 37 01 48 29 52 33 16 61 20 30 51 04 45 62 19 36 13 47 02 49 32 15 34 17 64 50 31 46 03 18 63 14 35</p>	<p>144 B 27jR Béligne 1883 07 42 55 26 09 40 23 58 54 27 08 41 24 57 38 11 43 06 25 56 39 10 59 22 28 53 44 05 60 21 12 37 45 04 49 32 13 36 17 64 52 29 46 03 20 61 14 35 01 48 31 50 33 16 63 18 30 51 02 47 62 19 34 15</p>
<p>145 A 27I Francony 1883 07 42 55 26 39 10 23 58 54 27 08 41 24 57 38 11 43 06 25 56 09 40 59 22 28 53 44 05 60 21 12 37 01 48 29 52 33 16 61 20 30 51 04 45 62 19 36 13 47 02 49 32 15 34 17 64 50 31 46 03 18 63 14 35</p>	<p>146 A 27IR Francony 1883 07 42 55 26 39 10 23 58 54 27 08 41 24 57 38 11 43 06 25 56 09 40 59 22 28 53 44 05 60 21 12 37 45 04 49 32 13 36 17 64 52 29 46 03 20 61 14 35 01 48 31 50 33 16 63 18 30 51 02 47 62 19 34 15</p>	<p>147 B 00mB Wenzelides 1858 07 46 55 30 57 44 03 18 54 31 06 45 04 19 42 59 47 08 29 56 43 58 17 02 32 53 48 05 20 01 60 41 09 28 33 52 37 16 21 64 34 49 26 11 24 61 40 15 27 10 51 36 13 38 63 22 50 35 12 25 62 23 14 39</p>	<p>148 B 00gAR Ligondes 1883 07 46 55 30 57 44 03 18 54 31 06 45 04 19 42 59 47 08 29 56 43 58 17 02 32 53 48 05 20 01 60 41 49 28 09 36 61 40 21 16 10 33 52 25 14 23 64 39 27 50 35 12 37 62 15 22 34 11 26 51 24 13 38 63</p>
<p>149 C 01eR Grossetaite 1896 10 07 18 59 30 39 50 47 19 60 09 40 17 48 29 38 08 11 06 31 58 51 46 49 61 20 41 52 05 16 37 28 12 53 32 21 36 57 04 45 33 62 35 42 15 22 27 24 54 13 64 01 56 25 44 03 63 34 55 14 43 02 23 26</p>	<p>150 A 34f Ligondes 1884 10 07 50 59 14 23 62 35 51 58 11 08 61 34 15 22 06 09 60 49 24 13 36 63 57 52 05 12 33 64 21 16 04 31 48 53 20 25 42 37 47 56 03 32 41 38 17 26 30 01 54 45 28 19 40 43 55 46 29 02 39 44 27 18</p>	<p>151 A 34fR Ligondes 1884 10 19 36 63 26 21 38 47 35 64 11 20 37 46 25 22 18 09 62 33 24 27 48 39 61 34 17 12 45 40 23 28 08 13 60 53 32 01 44 49 59 56 05 16 41 52 29 02 14 07 54 57 04 31 50 43 55 58 15 06 51 42 03 30</p>	<p>152 B 00mAR Wenzelides 1858 10 27 34 49 08 47 54 31 35 50 09 28 53 32 07 46 26 11 52 33 48 55 30 05 51 36 25 12 29 06 45 56 24 13 38 61 44 57 04 19 37 62 23 16 01 20 43 58 14 39 64 21 60 41 18 03 63 22 15 40 17 02 59 42</p>
<p>153 B 00gD Ligondes 1883 10 27 34 49 32 07 54 47 35 50 09 28 53 48 31 06 26 11 52 33 08 29 46 55 51 36 25 12 45 56 05 30 24 13 38 61 20 03 44 57 37 62 23 16 41 58 19 04 14 39 64 21 60 17 02 43 63 22 15 40 01 42 59 18</p>	<p>154 A 23mR Ligondes 1883 10 27 34 49 32 07 54 47 35 50 09 28 53 48 31 06 26 11 52 33 08 29 46 55 51 36 25 12 45 56 05 30 24 13 40 61 20 01 44 57 37 62 21 16 41 60 19 04 14 23 64 39 02 17 58 43 63 38 15 22 59 42 03 18</p>	<p>155 A 12nR Jaenisch 1859 10 35 48 23 38 29 50 27 47 22 11 36 49 26 39 30 34 09 24 45 32 37 28 51 21 46 33 12 25 52 31 40 08 63 20 57 44 01 14 53 19 60 05 64 13 56 41 02 62 07 58 17 04 43 54 15 59 18 61 06 55 16 03 42</p>	<p>156 C 00IAR Bouvier 1884 10 35 50 13 60 07 22 63 51 14 11 08 49 62 59 06 36 09 34 61 12 23 64 21 15 52 25 56 01 48 05 58 26 37 16 33 24 57 20 47 53 32 55 28 45 02 41 04 38 27 30 17 40 43 46 19 31 54 39 44 29 18 03 42</p>
<p>157 C 12jR Bouvier 1884 10 35 50 13 60 07 22 63 51 14 11 08 49 62 59 06 36 09 34 61 12 23 64 21 15 52 25 56 01 48 05 58 26 37 16 33 24 57 20 47 53 32 55 44 29 02 41 04 38 27 30 17 40 43 46 19 31 54 39 28 45 18 03 42</p>	<p>158 C 12iR Bouvier 1884 10 35 50 61 12 07 22 63 51 14 11 08 49 62 59 06 36 09 34 13 60 23 64 21 15 52 25 56 01 48 05 58 26 37 16 33 24 57 20 47 53 32 55 28 45 02 41 04 38 27 30 17 40 43 46 19 31 54 39 44 29 18 03 42</p>	<p>159 C 00ICR Bouvier 1884 10 35 50 61 12 07 22 63 51 14 11 08 49 62 59 06 36 09 34 13 60 23 64 21 15 52 25 56 01 48 05 58 26 37 16 33 24 57 20 47 53 32 55 44 29 02 41 04 38 27 30 17 40 43 46 19 31 54 39 28 45 18 03 42</p>	<p>160 C 16a Lehmann 1933 10 35 58 63 18 23 14 39 57 64 11 36 13 38 17 24 34 09 62 59 22 19 40 15 61 56 33 12 37 16 25 20 08 03 60 29 52 21 46 41 55 32 05 02 47 44 49 26 04 07 30 53 28 51 42 45 31 54 01 06 43 48 27 50</p>

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 161 - 180

161 C 00kAR

Francony 1884

10 51 54 15 26 35 38 31
53 14 09 34 55 32 27 36
50 11 52 25 16 37 30 39
13 08 61 56 33 28 17 44
60 49 12 01 24 45 40 29
07 62 05 48 57 20 43 18
04 59 64 23 02 41 46 21
63 06 03 58 47 22 19 42

162 C 12IR

Francony 1884

10 51 54 15 26 35 38 31
53 14 09 34 55 32 27 36
50 11 52 25 16 37 30 39
13 08 61 56 33 44 17 28
60 49 12 01 24 29 40 45
07 62 05 48 57 20 43 18
04 59 64 23 02 41 46 21
63 06 03 58 47 22 19 42

163 C 12kR

Francony 1884

10 51 54 15 26 35 38 31
53 14 09 34 55 32 27 36
50 11 52 25 16 37 30 39
61 08 13 56 33 28 17 44
12 49 60 01 24 45 40 29
07 62 05 48 57 20 43 18
04 59 64 23 02 41 46 21
63 06 03 58 47 22 19 42

164 C 00kCR

Francony 1884

10 51 54 15 26 35 38 31
53 14 09 34 55 32 27 36
50 11 52 25 16 37 30 39
61 08 13 56 33 44 17 28
12 49 60 01 24 29 40 45
07 62 05 48 57 20 43 18
04 59 64 23 02 41 46 21
63 06 03 58 47 22 19 42

165 C 12f

Jolivald 1882

11 14 33 56 09 52 31 54
34 57 10 13 32 55 08 51
15 12 59 36 49 06 53 30
58 35 16 05 60 29 50 07
39 18 61 28 37 48 03 26
62 21 38 17 04 27 44 47
19 40 23 64 45 42 25 02
22 63 20 41 24 01 46 43

166 C 12h

Jolivald 1882

11 14 33 58 09 50 31 54
34 59 10 13 32 55 08 49
15 12 57 36 51 06 53 30
60 35 20 05 56 29 48 07
39 16 61 24 37 52 03 28
62 21 38 19 04 25 44 47
17 40 23 64 45 42 27 02
22 63 18 41 26 01 46 43

167 C 12fR

Jolivald 1882

11 14 35 58 39 18 63 22
34 57 12 15 62 21 40 19
13 10 59 36 17 38 23 64
56 33 16 05 28 61 20 41
09 52 29 60 37 48 01 24
32 55 06 49 04 27 42 45
51 08 53 30 47 44 25 02
54 31 50 07 26 03 46 43

168 A 34e

Exner 1876

11 14 43 64 45 18 31 34
42 63 12 15 30 33 46 19
13 10 61 44 17 48 35 32
62 41 16 09 36 29 20 47
07 60 37 28 49 56 01 22
40 27 08 57 04 21 50 53
59 06 25 38 55 52 23 02
26 39 58 05 24 03 54 51

169 C 00oAR

Mackay et al 2003

11 14 51 40 09 38 30 34
52 41 10 13 64 35 08 37
15 12 43 50 39 06 33 62
42 53 16 01 44 61 36 07
55 20 45 28 49 32 05 26
46 17 54 23 02 27 60 31
21 56 19 48 29 58 25 04
18 47 22 57 24 03 30 59

170 A 03aR

Ligondes 1884

11 14 55 34 07 18 59 62
54 35 12 15 58 61 06 19
13 10 33 56 17 08 63 60
36 53 16 09 64 57 20 05
51 28 37 32 41 04 45 22
38 31 52 25 48 21 42 03
27 50 29 40 01 44 23 46
30 39 26 49 24 47 02 43

171 B 25bR

Marlow 1988

11 14 55 42 63 18 27 30
54 43 10 13 28 31 62 19
15 12 41 56 17 64 29 26
44 53 16 09 32 25 20 61
01 40 45 52 57 08 33 24
46 49 38 03 36 21 60 07
39 02 51 48 05 58 23 34
50 47 04 37 22 35 06 59

172 C 12hR

Jolivald 1882

11 16 35 58 37 18 63 22
34 57 12 17 62 21 38 19
15 10 59 36 13 40 23 64
56 33 14 09 28 61 20 39
07 52 29 60 41 46 01 24
32 55 08 45 04 27 42 47
51 06 53 30 49 44 25 02
54 31 50 05 26 03 48 43

173 B 00hD

Ligondes 1906

11 18 33 56 37 16 31 58
34 55 12 17 32 57 38 15
19 10 53 36 13 40 59 30
54 35 20 09 60 29 14 39
07 22 61 52 41 46 03 28
62 51 08 21 04 27 42 45
23 06 49 64 25 44 47 02
50 63 24 05 48 01 26 43

174 A 05f

Francony 1881

11 18 59 36 13 22 63 38
58 35 12 17 64 37 14 23
19 10 33 60 21 16 39 62
34 57 20 09 40 61 24 15
55 08 45 32 49 04 41 26
46 31 56 05 44 25 50 03
07 54 29 48 01 52 27 42
30 47 06 53 28 43 02 51

175 C 27gR

Murray 1936

11 20 37 62 13 18 35 64
38 61 12 19 36 63 14 17
21 10 59 40 23 16 57 34
60 39 22 09 58 33 24 15
07 50 41 32 45 26 03 56
42 31 08 49 04 55 46 25
51 06 29 44 53 48 27 02
30 43 52 05 28 01 54 47

176 C 27tR

Mackay et al 2003

11 20 37 62 13 18 35 64
38 61 12 19 36 63 14 17
21 10 31 44 51 16 53 34
60 39 50 05 30 33 28 15
09 22 43 32 45 52 03 54
40 59 06 49 04 29 46 27
23 08 57 42 25 48 55 02
58 41 24 07 56 01 26 47

177 C 27hR

Murray 1936

11 20 37 62 13 18 35 64
38 61 12 19 36 63 14 17
21 10 59 40 23 16 57 34
60 39 22 09 58 33 24 15
07 50 41 32 45 56 03 26
42 31 08 49 04 25 46 55
51 06 29 44 53 48 27 02
30 43 52 05 28 01 54 47

178 C 14fR

Mackay et al 2003

11 20 37 62 13 32 51 34
38 61 12 19 50 35 30 15
21 10 63 36 31 14 33 52
60 39 18 05 44 49 16 29
09 22 43 64 17 04 53 48
40 59 06 25 56 45 28 01
23 08 57 42 03 26 47 54
58 41 24 07 46 55 02 27

179 C 00oDR

Mackay et al 2003

11 20 47 42 23 14 53 50
46 41 12 21 52 49 24 15
19 10 43 48 13 22 51 54
40 45 18 01 60 55 16 25
09 64 39 44 17 02 29 56
38 35 06 61 32 59 26 03
63 08 33 36 05 28 57 30
34 37 62 07 58 31 04 27

180 C 27fR

Reuss 1880

11 22 37 60 13 18 35 64
38 59 12 21 36 63 14 17
23 10 39 58 61 16 19 34
40 57 24 09 20 33 62 15
25 08 41 56 45 50 03 32
42 55 26 07 04 31 46 49
27 06 53 44 29 48 51 02
54 43 28 05 52 01 30 47

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 181 - 200

181 C 27eR Reuss 1880 11 22 37 60 13 18 35 64 38 59 12 21 36 63 14 17 23 10 39 58 61 16 19 34 56 41 08 25 20 33 62 15 09 24 57 40 45 50 03 32 42 55 26 07 04 31 46 49 27 06 53 44 29 48 51 02 54 43 28 05 52 01 30 47	182 B 27aR Beverley 1848 11 22 37 60 13 18 35 64 38 59 12 21 36 63 14 17 23 10 57 40 61 16 19 34 58 39 24 09 20 33 62 15 07 26 41 56 45 50 03 32 42 55 08 25 04 31 46 49 27 06 53 44 29 48 51 02 54 43 28 05 52 01 30 47	183 B 27dR Jaenisch 1862 11 22 37 60 13 18 35 64 58 39 12 21 36 63 14 17 23 10 59 38 61 16 19 34 40 57 24 09 20 33 62 15 25 08 41 56 45 50 03 32 42 55 06 27 04 31 46 49 07 26 53 44 29 48 51 02 54 43 28 05 52 01 30 47	184 A 23a Francony 1882 11 22 47 50 03 26 63 38 46 49 10 23 64 39 02 27 21 12 51 48 25 04 37 62 52 45 24 09 40 61 28 01 13 20 41 56 29 36 05 60 44 53 14 17 08 57 32 35 19 16 55 42 33 30 59 06 54 43 18 15 58 07 34 31
185 A 23aR Francony 1882 11 22 47 50 07 58 31 34 46 49 10 23 32 35 06 59 21 12 51 48 57 08 33 30 52 45 24 09 36 29 60 05 13 20 41 56 25 04 37 64 44 53 14 17 40 61 28 03 19 16 55 42 01 26 63 38 54 43 18 15 62 39 02 27	186 A 00bCR Mysore 1852 11 22 47 50 35 26 07 62 46 49 10 23 08 63 34 27 21 12 51 48 25 36 61 06 52 45 24 09 64 05 28 33 13 20 41 56 29 60 37 04 44 53 14 17 40 01 32 59 19 16 55 42 57 30 03 38 54 43 18 15 02 39 58 31	187 B 23fR Ligondes 1883 11 26 35 50 13 24 39 62 34 51 12 25 38 63 22 15 27 10 49 36 23 14 61 40 52 33 28 09 64 37 16 21 29 08 53 48 17 04 41 60 54 47 32 05 44 57 20 01 07 30 45 56 03 18 59 42 46 55 06 31 58 43 02 19	188 B 23hR Ligondes 1883 11 26 35 50 13 24 39 62 34 51 12 25 38 63 22 15 27 10 49 36 23 14 61 40 52 33 28 09 64 37 16 21 29 08 53 48 17 04 41 60 54 47 32 05 58 43 20 01 07 30 45 56 03 18 59 42 46 55 06 31 44 57 02 19
189 B 25aR Ligondes 1883 11 26 35 50 13 64 39 22 34 51 12 25 38 23 62 15 27 10 49 36 63 14 21 40 52 33 28 09 24 37 16 61 29 08 53 48 57 04 41 20 54 47 32 05 44 17 60 01 07 30 45 56 03 58 19 42 46 55 06 31 18 43 02 59	190 A 23gR Ligondes 1883 11 26 35 50 23 14 39 62 34 51 12 25 38 63 22 15 27 10 49 36 13 24 61 40 52 33 28 09 64 37 16 21 29 08 53 48 17 04 41 60 54 47 32 05 44 57 20 01 07 30 45 56 03 18 59 42 46 55 06 31 58 43 02 19	191 B 23eR Ligondes 1883 11 26 35 50 23 14 39 62 34 51 12 25 38 63 22 15 27 10 49 36 13 24 61 40 52 33 28 09 64 37 16 21 29 08 53 48 17 04 41 60 54 47 32 05 58 43 20 01 07 30 45 56 03 18 59 42 46 55 06 31 44 57 02 19	192 A 23dR Ligondes 1883 11 26 35 50 63 14 39 22 34 51 12 25 38 23 62 15 27 10 49 36 13 64 21 40 52 33 28 09 24 37 16 61 29 08 53 48 57 04 41 20 54 47 32 05 44 17 60 01 07 30 45 56 03 58 19 42 46 55 06 31 18 43 02 59
193 C 12gR Jolivald 1882 11 34 13 60 07 62 15 58 50 23 10 35 14 59 06 63 33 12 51 22 61 08 57 16 24 49 36 09 52 21 64 05 37 32 53 20 41 04 17 56 48 25 40 29 54 19 44 01 31 38 27 46 03 42 55 18 26 47 30 39 28 45 02 43	194 B 12e Wenzelides 1858 11 34 13 60 07 62 23 50 58 15 10 35 22 51 06 63 33 12 59 14 61 08 49 24 16 57 36 09 52 21 64 05 37 32 53 20 41 04 25 48 56 17 40 29 46 27 44 01 31 38 19 54 03 42 47 26 18 55 30 39 28 45 02 43	195 B 00hB Ligondes 1906 11 34 13 60 07 62 47 26 58 15 10 35 46 27 06 63 33 12 59 14 61 08 25 48 16 57 36 09 28 45 64 05 37 32 53 20 41 04 49 24 56 17 40 29 52 21 44 01 31 38 19 54 03 42 23 50 18 55 30 39 22 51 02 43	196 B 01fR Murray 1939 11 38 57 32 13 36 55 18 58 31 12 37 56 17 14 35 29 10 39 60 33 54 19 16 40 59 30 09 20 15 34 53 07 28 41 48 61 52 01 22 42 45 08 25 04 21 64 51 27 06 47 44 49 62 23 02 46 43 26 05 24 03 50 63
197 C 00nB Mackay et al 2003 11 46 51 40 09 38 31 34 52 41 10 45 32 35 08 37 47 12 43 50 39 06 33 30 42 53 48 01 44 29 36 07 55 20 13 28 49 64 05 26 14 17 54 23 02 27 60 63 21 56 19 16 61 58 25 04 18 15 22 57 24 03 62 59	198 C 12cR Béligne 1880 14 09 58 33 52 07 56 31 59 34 13 08 57 32 51 06 10 15 36 53 12 49 30 55 35 60 11 16 29 54 05 50 18 37 22 61 48 43 28 03 23 62 17 44 21 04 47 42 38 19 64 25 40 45 02 27 63 24 39 20 01 26 41 46	199 C 23oR Ligondes 1910 14 11 34 59 30 07 54 51 35 60 13 10 55 52 29 06 12 15 58 33 08 31 50 53 61 36 09 20 45 56 05 28 16 21 40 57 32 01 44 49 37 62 19 24 41 46 27 04 22 17 64 39 02 25 48 43 63 38 23 18 47 42 03 26	200 C 12dR Jolivald 1882 14 11 58 33 50 07 56 31 59 34 13 10 57 32 49 06 12 15 36 53 08 51 30 55 35 60 09 20 29 54 05 48 16 37 22 61 52 41 28 03 23 62 19 40 21 04 47 44 38 17 64 25 42 45 02 27 63 24 39 18 01 26 43 46

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 201 - 220

<p>201 A 34eR Exner 1876 14 11 62 41 60 07 26 39 63 42 13 10 27 40 59 06 12 15 44 61 08 57 38 25 43 64 09 16 37 28 05 58 18 45 36 29 56 49 24 03 33 30 17 48 21 04 55 52 46 19 32 35 50 53 02 23 31 34 47 20 01 22 51 54</p>	<p>202 C 07a Mackay et al 2003 14 11 64 59 16 39 30 27 63 60 15 12 29 26 17 38 10 13 58 61 40 19 28 31 57 62 09 20 25 32 37 18 08 03 56 45 36 41 24 47 55 52 07 04 21 46 33 42 02 05 50 53 44 35 48 23 51 54 01 06 49 22 43 34</p>	<p>203 C 07aR Mackay et al 2003 14 11 64 59 16 43 22 32 63 60 15 12 21 30 17 42 10 13 58 61 44 19 32 23 57 62 09 20 29 24 41 18 08 03 56 45 40 33 28 47 55 52 07 04 25 46 37 34 02 05 50 53 36 39 48 27 51 54 01 06 49 26 35 38</p>	<p>204 B 00dAR Reuss 1880 14 23 42 51 40 25 02 63 43 50 15 24 01 62 27 38 22 13 52 41 26 39 64 03 49 44 21 16 61 04 37 28 20 53 12 45 36 29 60 05 11 48 17 56 07 58 33 30 54 19 46 09 32 35 06 59 47 10 55 18 57 08 31 34</p>
<p>205 A 00cBR Francony 1881 14 23 42 51 40 25 02 63 43 50 15 24 01 62 39 26 22 13 52 41 28 37 64 03 49 44 21 16 61 04 27 38 20 53 12 45 36 29 06 59 11 48 17 56 05 60 33 30 54 19 46 09 32 35 58 07 47 10 55 18 57 08 31 34</p>	<p>206 A 05fR Francony 1881 14 23 62 39 50 03 42 27 63 38 15 24 41 26 51 02 22 13 40 61 04 49 28 43 37 64 21 16 25 44 01 52 12 17 60 33 56 05 48 29 59 36 09 20 45 32 53 06 18 11 34 57 08 55 30 47 35 58 19 10 31 46 07 54</p>	<p>207 B 12eR Wenzelides 1858 15 02 41 60 17 64 39 22 42 59 16 01 40 21 18 63 03 14 57 44 61 38 23 20 58 43 04 13 24 19 62 37 05 30 51 56 45 36 11 26 52 55 06 29 12 25 46 35 31 50 53 08 33 48 27 10 54 07 32 49 28 09 34 47</p>	<p>208 C 00oD Mackay et al 2003 15 12 51 42 23 18 45 54 50 41 16 13 44 53 24 19 11 14 43 52 17 22 55 46 40 49 10 05 64 47 20 25 09 36 63 48 21 26 01 56 62 39 06 33 04 59 30 27 35 08 37 60 29 32 57 02 38 61 34 07 58 03 28 31</p>
<p>209 C 00jAR Reuss 1880 15 18 41 56 03 58 39 30 42 55 14 17 40 31 02 59 19 16 43 54 57 04 29 38 44 53 20 13 32 37 60 01 21 12 45 52 61 28 05 36 46 49 22 11 08 33 64 27 23 10 51 48 25 62 35 06 50 47 24 09 34 07 26 63</p>	<p>210 A 00cAR Francony 1881 15 18 41 56 03 58 39 30 42 55 14 17 40 31 02 59 19 16 53 44 57 04 29 38 54 43 20 13 32 37 60 01 11 22 45 52 61 28 05 36 46 49 12 21 08 33 64 27 23 10 51 48 25 62 35 06 50 47 24 09 34 07 26 63</p>	<p>211 B 00dD Reuss 1880 15 18 41 56 03 58 39 30 54 43 14 17 40 31 02 59 19 16 55 42 57 04 29 38 44 53 20 13 32 37 60 01 21 12 45 52 61 28 05 36 46 49 10 23 08 33 64 27 11 22 51 48 25 62 35 06 50 47 24 09 34 07 26 63</p>	<p>212 B 03cR Ligondes 1884 15 18 51 42 03 30 39 62 50 43 14 17 40 63 02 31 19 16 41 52 29 04 61 38 44 49 20 13 64 37 32 01 21 12 53 48 33 28 05 60 54 45 10 23 08 57 36 27 11 22 47 56 25 34 59 06 46 55 24 09 58 07 26 35</p>
<p>213 B 03dR Ligondes 1884 15 18 51 42 03 30 39 62 50 43 14 17 40 63 02 31 19 16 41 52 29 04 61 38 44 49 20 13 64 37 32 01 21 12 53 48 33 28 05 60 54 45 24 09 58 07 36 27 11 22 47 56 25 34 59 06 46 55 10 23 08 57 26 35</p>	<p>214 B 23bR Ligondes 1885 15 18 51 42 03 30 63 38 50 43 14 17 64 39 02 31 19 16 41 52 29 04 37 62 44 49 20 13 40 61 32 01 21 12 53 48 57 28 05 36 54 45 10 23 08 33 60 27 11 22 47 56 25 58 35 06 46 55 24 09 34 07 26 59</p>	<p>215 C 16aR Lehmann 1933 15 20 39 24 45 50 41 26 38 23 16 19 40 25 48 51 17 14 21 44 49 46 27 42 22 37 18 13 28 43 52 47 59 12 63 36 53 06 29 02 64 35 60 05 32 03 54 07 11 58 33 62 09 56 01 30 34 61 10 57 04 31 08 55</p>	<p>216 B 00gB Ligondes 1883 15 22 39 64 17 02 59 42 38 63 16 21 60 41 18 03 23 14 61 40 01 20 43 58 62 37 24 13 44 57 04 19 25 12 35 52 29 06 45 56 36 51 26 09 48 55 30 05 11 34 49 28 53 32 07 46 50 27 10 33 08 47 54 31</p>
<p>217 A 00bBR Mysore 1852 15 22 43 52 17 54 11 46 42 51 16 21 12 45 18 55 23 14 49 44 53 20 47 10 50 41 24 13 48 09 56 19 63 26 01 40 57 32 07 34 02 39 62 25 08 35 58 31 27 64 37 04 29 60 33 06 38 03 28 61 36 05 30 59</p>	<p>218 A 03bR Marlow 1988 15 26 41 48 17 24 39 50 42 45 16 25 40 49 20 23 27 14 47 44 21 18 51 38 46 43 28 13 52 37 22 19 03 12 53 64 29 08 33 58 54 63 04 09 36 57 30 07 11 02 61 56 05 32 59 34 62 55 10 01 60 35 06 31</p>	<p>219 B 01b Ligondes 1883 15 30 53 40 03 42 51 26 38 55 14 29 52 27 02 43 31 16 39 54 41 04 25 50 56 37 32 13 28 49 44 01 17 12 57 36 45 24 05 64 58 33 10 19 08 61 48 23 11 18 35 60 21 46 63 06 34 59 20 09 62 07 22 47</p>	<p>220 B 01c Ligondes 1883 15 30 53 40 03 42 51 26 38 55 14 29 52 27 02 43 31 16 39 54 41 04 25 50 56 37 32 13 28 49 44 01 17 12 57 36 45 24 05 64 58 33 20 09 62 07 48 23 11 18 35 60 21 46 63 06 34 59 10 19 08 61 22 47</p>

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 221 - 240

221 B 00gC Ligondes 1883 15 30 53 40 03 42 51 26 38 55 14 29 52 27 02 43 31 16 39 54 41 04 25 50 56 37 32 13 28 49 44 01 17 12 57 36 61 24 05 48 58 33 10 19 08 45 64 23 11 18 35 60 21 62 47 06 34 59 20 09 46 07 22 63	222 B 00mBR Wenzelides 1858 15 30 53 40 03 42 51 26 38 55 14 29 52 27 02 43 31 16 39 54 41 04 25 50 56 37 32 13 28 49 44 01 33 12 17 60 45 64 05 24 18 57 36 09 22 07 48 63 11 34 59 20 61 46 23 06 58 19 10 35 08 21 62 47	223 A 00iC Unknown 1879 15 38 55 32 17 42 27 34 54 31 16 37 28 33 18 43 39 14 29 56 41 20 35 26 30 53 40 13 36 25 44 19 03 12 57 52 61 08 21 46 58 51 04 09 24 45 62 07 11 02 49 60 05 64 47 22 50 59 10 01 48 23 06 63	224 B 00gBR Ligondes 1883 15 38 55 32 57 18 11 34 54 31 16 37 12 33 58 19 29 14 39 56 17 10 35 60 40 53 30 13 36 59 20 09 03 28 41 52 21 08 61 46 42 51 04 25 64 45 22 07 27 02 49 44 05 24 47 62 50 43 26 01 48 63 06 23
225 A 12oR Jaenisch 1859 15 42 55 04 17 06 59 62 54 03 16 43 58 61 18 07 41 14 01 56 05 20 63 60 02 53 44 13 64 57 08 19 51 40 25 32 45 12 21 34 28 31 52 37 24 33 46 09 39 50 29 26 11 48 35 22 30 27 38 49 36 23 10 47	226 A 00aB Jaenisch 1862 18 03 32 47 06 61 34 59 31 46 19 04 33 58 07 62 02 17 48 29 64 05 60 35 45 30 01 20 57 36 63 08 16 55 44 49 28 09 22 37 43 52 13 56 21 40 25 10 54 15 50 41 12 27 38 23 51 42 53 14 39 24 11 26	227 A 23nR Ligondes 1883 18 03 42 59 22 15 38 63 43 58 17 02 39 64 23 14 04 19 60 41 16 21 62 37 57 44 01 20 61 40 13 24 46 05 56 29 52 25 36 11 55 30 45 08 33 12 51 26 06 47 32 53 28 49 10 35 31 54 07 48 09 34 27 50	228 B 00eAR Jaenisch 1862 18 03 44 57 06 55 46 31 59 42 19 04 45 30 07 54 02 17 58 43 56 05 32 47 41 60 01 20 29 48 53 08 16 21 40 61 52 09 28 33 39 64 13 24 35 26 49 10 22 15 62 37 12 51 34 27 63 38 23 14 25 36 11 50
229 B 00fD Ligondes 1883 18 03 44 57 06 55 46 31 59 42 19 04 45 30 07 54 02 17 58 43 56 05 32 47 41 60 01 20 29 48 53 08 16 21 40 61 52 09 28 33 39 64 23 14 25 36 49 10 22 15 62 37 12 51 34 27 63 38 13 24 35 26 11 50	230 B 00fAR Ligondes 1883 18 03 58 43 56 05 46 31 59 42 19 04 45 30 07 54 02 17 44 57 06 55 32 47 41 60 01 20 29 48 53 08 16 21 40 61 52 09 28 33 39 64 13 24 35 26 49 10 22 15 62 37 12 51 34 27 63 38 23 14 25 36 11 50	231 B 00eD Jaenisch 1862 18 03 58 43 56 05 46 31 59 42 19 04 45 30 07 54 02 17 44 57 06 55 32 47 41 60 01 20 29 48 53 08 16 21 40 61 52 09 28 33 39 64 23 14 25 36 49 10 22 15 62 37 12 51 34 27 63 38 13 24 35 26 11 50	232 A 00bD Mysore 1852 18 11 54 45 16 43 22 51 55 46 17 12 21 52 15 42 10 19 48 53 44 13 50 23 47 56 09 20 49 24 41 14 34 07 32 57 40 01 26 63 31 58 35 08 25 62 39 02 06 33 60 29 04 37 64 27 59 30 05 36 61 28 03 38
233 B 14cR Ligondes 1883 18 15 42 55 40 01 58 31 43 54 19 16 57 30 03 38 14 17 56 41 02 39 32 59 53 44 13 20 29 60 37 04 12 21 52 45 64 05 28 33 51 48 23 10 25 36 61 06 22 11 46 49 08 63 34 27 47 50 09 24 35 26 07 62	234 C 05aR Bouvier 1876 18 15 54 13 52 11 50 47 55 42 17 44 21 48 23 10 16 19 14 53 12 51 46 49 41 56 43 20 45 22 09 24 30 01 40 57 36 25 64 07 39 58 29 04 61 08 35 26 02 31 60 37 28 33 06 63 59 38 03 32 05 62 27 34	235 A 05bR Caldwell 1879 18 15 54 43 22 11 50 47 55 42 17 14 51 48 23 10 16 19 44 53 12 21 46 49 41 56 13 20 45 52 09 24 30 01 40 57 36 25 64 07 39 58 29 04 61 08 35 26 02 31 60 37 28 33 06 63 59 38 03 32 05 62 27 34	236 B 00dBR Reuss 1880 18 15 56 41 02 39 58 31 43 54 19 16 57 30 03 38 14 17 42 55 40 01 32 59 53 44 13 20 29 60 37 04 12 21 52 45 64 05 28 33 51 48 23 10 25 36 61 06 22 11 46 49 08 63 34 27 47 50 09 24 35 26 07 62
237 A 00cCR Francony 1881 18 15 56 41 02 39 58 31 55 42 19 16 57 30 03 38 14 17 44 53 40 01 32 59 43 54 13 20 29 60 37 04 22 11 52 45 64 05 28 33 51 48 21 12 25 36 61 06 10 23 46 49 08 63 34 27 47 50 09 24 35 26 07 62	238 C 00jCR Reuss 1880 18 15 56 41 02 39 58 31 55 42 19 16 57 30 03 38 14 17 54 43 40 01 32 59 53 44 13 20 29 60 37 04 12 21 52 45 64 05 28 33 51 48 11 22 25 36 61 06 10 23 46 49 08 63 34 27 47 50 09 24 35 26 07 62	239 A 12pR Murray 1939 18 15 58 09 56 07 50 47 59 12 19 16 49 46 53 06 14 17 10 57 08 55 48 51 11 60 13 20 45 52 05 54 30 21 44 61 40 25 36 03 43 62 29 24 33 04 39 26 22 31 64 41 28 37 02 35 63 42 23 32 01 34 27 38	240 A 01d Murray 1939 18 15 58 09 56 07 50 47 59 12 19 16 49 46 53 06 14 17 10 57 08 55 48 51 11 60 13 20 45 52 05 54 62 21 44 29 40 25 36 03 43 30 61 24 33 04 39 26 22 63 32 41 28 37 02 35 31 42 23 64 01 34 27 38

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 241 - 260

<p>241 A 01dR Murray 1939</p> <p>18 15 58 09 56 07 50 47 59 12 19 16 49 46 53 06 14 17 10 57 08 55 48 51 11 60 13 20 45 52 05 54 62 29 40 25 36 21 44 03 39 26 61 32 41 04 35 22 30 63 28 37 24 33 02 43 27 38 31 64 01 42 23 34</p>	<p>242 A 12bR Wenzelides 1850</p> <p>18 27 42 33 16 55 38 31 43 34 17 28 37 32 15 54 26 19 36 41 56 13 30 39 35 44 25 20 29 40 53 14 46 21 08 61 52 57 12 03 07 62 45 24 09 04 51 58 22 47 64 05 60 49 02 11 63 06 23 48 01 10 59 50</p>	<p>243 B 00fCR Ligondes 1883</p> <p>18 43 04 57 46 55 06 31 59 02 19 44 05 30 47 54 42 17 58 03 56 45 32 07 01 60 41 20 29 08 53 48 40 21 16 61 52 33 28 09 15 64 23 38 25 12 49 34 22 39 62 13 36 51 10 27 63 14 37 24 11 26 35 50</p>	<p>244 B 00eCR Jaenisch 1862</p> <p>18 43 04 57 46 55 06 31 59 02 19 44 05 30 47 54 42 17 58 03 56 45 32 07 01 60 41 20 29 08 53 48 40 21 16 61 52 33 28 09 15 64 37 24 11 26 49 34 22 39 62 13 36 51 10 27 63 14 23 38 25 12 35 50</p>
<p>245 B 01cR Ligondes 1883</p> <p>18 43 04 57 46 55 06 31 59 02 19 44 05 30 47 54 42 17 58 03 56 45 32 07 01 60 41 20 29 08 53 48 64 21 16 37 52 33 28 09 15 40 61 24 11 26 49 34 22 63 38 13 36 51 10 27 39 14 23 62 25 12 35 50</p>	<p>246 A 12mR Wenzelides 1858</p> <p>18 43 54 03 46 39 30 27 55 02 19 44 29 26 47 38 42 17 04 53 40 45 28 31 01 56 41 20 25 32 37 48 16 05 64 57 52 09 24 33 63 60 13 08 21 36 49 10 06 15 58 61 12 51 34 23 59 62 07 14 35 22 11 50</p>	<p>247 B 05eR Francony 1882</p> <p>18 43 54 13 52 11 22 47 55 14 17 44 21 48 51 10 16 19 42 53 12 23 46 49 41 56 15 20 45 50 09 24 30 01 40 57 36 25 64 07 39 58 29 04 61 08 35 26 02 31 60 37 28 33 06 63 59 38 03 32 05 62 27 34</p>	<p>248 A 00iAR Unknown 1879</p> <p>18 43 58 03 30 39 22 47 59 02 19 44 21 46 31 38 42 17 04 57 40 29 48 23 01 60 41 20 45 24 37 32 16 05 56 61 36 09 28 49 55 64 13 08 25 52 33 10 06 15 62 53 12 35 50 27 63 54 07 14 51 26 11 34</p>
<p>249 A 12aR Wenzelides 1849</p> <p>18 43 58 03 46 39 22 31 59 02 19 44 21 30 47 38 42 17 04 57 40 45 32 23 01 60 41 20 29 24 37 48 16 05 56 61 52 09 28 33 55 64 13 08 25 36 49 10 06 15 62 53 12 51 34 27 63 54 07 14 35 26 11 50</p>	<p>250 B 00eB Jaenisch 1862</p> <p>18 43 58 03 56 45 06 31 59 02 19 44 05 30 47 54 42 17 04 57 46 55 32 07 01 60 41 20 29 08 53 48 40 21 16 61 52 33 28 09 15 64 23 38 25 12 49 34 22 39 62 13 36 51 10 27 63 14 37 24 11 26 35 50</p>	<p>251 B 00fB Ligondes 1883</p> <p>18 43 58 03 56 45 06 31 59 02 19 44 05 30 47 54 42 17 04 57 46 55 32 07 01 60 41 20 29 08 53 48 40 21 16 61 52 33 28 09 15 64 37 24 11 26 49 34 22 39 62 13 36 51 10 27 63 14 23 38 25 12 35 50</p>	<p>252 B 01bR Ligondes 1883</p> <p>18 43 58 03 56 45 06 31 59 02 19 44 05 30 47 54 42 17 04 57 46 55 32 07 01 60 41 20 29 08 53 48 64 21 16 37 52 33 28 09 15 40 61 24 11 26 49 34 22 63 38 13 36 51 10 27 39 14 23 62 25 12 35 50</p>
<p>253 A 00aD Jaenisch 1862</p> <p>19 10 21 46 07 56 43 58 22 47 18 09 44 59 06 55 11 20 45 24 53 08 57 42 48 23 12 17 60 41 54 05 13 62 33 52 25 04 31 40 34 49 16 61 32 37 28 03 63 14 51 36 01 26 39 30 50 35 64 15 38 29 02 27</p>	<p>254 A 34d Ligondes 1883</p> <p>19 10 39 62 07 22 59 42 38 63 20 09 60 41 06 23 11 18 61 40 21 08 43 58 64 37 12 17 44 57 24 05 13 32 49 36 25 04 45 56 50 35 16 29 48 53 26 03 31 14 33 52 01 28 55 46 34 51 30 15 54 47 02 27</p>	<p>255 B 34a Exner 1876</p> <p>19 10 39 62 21 08 59 42 38 63 20 09 60 41 06 23 11 18 61 40 07 22 43 58 64 37 12 17 44 57 24 05 13 32 49 36 25 04 45 56 50 35 16 29 48 53 26 03 31 14 33 52 01 28 55 46 34 51 30 15 54 47 02 27</p>	<p>256 B 00dC Reuss 1880</p> <p>19 10 55 46 57 08 31 34 54 47 18 09 32 35 06 59 11 20 45 56 07 58 33 30 48 53 12 17 36 29 60 05 13 44 21 52 61 04 37 28 22 49 16 41 26 39 64 03 43 14 51 24 01 62 27 38 50 23 42 15 40 25 02 63</p>
<p>257 A 00cD Francony 1881</p> <p>19 10 55 46 57 08 31 34 54 47 18 09 32 35 58 07 11 20 45 56 05 60 33 30 48 53 12 17 36 29 06 59 13 44 21 52 61 04 27 38 22 49 16 41 28 37 64 03 43 14 51 24 01 62 39 26 50 23 42 15 40 25 02 63</p>	<p>258 B 34b Ligondes 1883</p> <p>19 10 61 40 07 22 59 42 38 63 20 09 60 41 06 23 11 18 39 62 21 08 43 58 64 37 12 17 44 57 24 05 13 32 49 36 25 04 45 56 50 35 16 29 48 53 26 03 31 14 33 52 01 28 55 46 34 51 30 15 54 47 02 27</p>	<p>259 B 34c Ligondes 1883</p> <p>19 10 61 40 21 08 59 42 38 63 20 09 60 41 06 23 11 18 39 62 07 22 43 58 64 37 12 17 44 57 24 05 13 32 49 36 25 04 45 56 50 35 16 29 48 53 26 03 31 14 33 52 01 28 55 46 34 51 30 15 54 47 02 27</p>	<p>260 A 00bAR Mysore 1852</p> <p>22 11 50 47 34 07 26 63 51 48 23 10 25 62 35 06 12 21 46 49 08 33 64 27 45 52 09 24 61 28 05 36 20 13 56 41 32 37 60 01 53 44 19 16 57 04 29 38 14 17 42 55 40 31 02 59 43 54 15 18 03 58 39 30</p>

KNIGHT'S TOUR NOTES

List of Magic Knight Tours 261 - 280

261 B 00hBR

Ligondes 1906

22 15 64 41 60 17 02 39
63 42 21 16 01 40 59 18
14 23 44 61 20 57 38 03
43 62 13 24 37 04 19 58
26 11 36 45 56 51 30 05
35 46 25 12 29 06 55 52
10 27 48 33 08 53 50 31
47 34 09 28 49 32 07 54

262 C 23pR

Ligondes 1911

22 19 38 63 26 03 46 43
37 62 23 20 45 42 27 04
18 21 64 39 02 25 44 47
61 36 13 24 41 52 05 28
12 17 40 57 32 01 48 53
35 60 09 14 51 56 29 06
16 11 58 33 08 31 54 49
59 34 15 10 55 50 07 30

263 A 00aAR

Jaenisch 1862

22 19 46 09 44 07 58 55
47 10 21 18 59 56 43 06
20 23 12 45 08 41 54 57
11 48 17 24 53 60 05 42
34 13 52 61 32 25 40 03
49 62 33 16 37 04 31 28
14 35 64 51 26 29 02 39
63 50 15 36 01 38 27 30

264 A 27iR

Exner 1876

22 39 20 15 50 45 26 43
17 14 23 40 25 42 51 48
38 21 16 19 46 49 44 27
13 18 37 24 41 28 47 52
36 63 12 57 32 53 06 01
11 60 33 62 07 04 29 54
64 35 58 09 56 31 02 05
59 10 61 34 03 08 55 30

265 A 34dR

Ligondes 1883

23 06 43 58 03 26 55 46
42 59 24 05 56 45 02 27
07 22 57 44 25 04 47 54
60 41 08 21 48 53 28 01
09 20 61 40 29 16 33 52
62 39 12 17 36 49 30 15
19 10 37 64 13 32 51 34
38 63 18 11 50 35 14 31

266 B 34bR

Ligondes 1883

23 06 43 58 25 04 55 46
42 59 24 05 56 45 02 27
07 22 57 44 03 26 47 54
60 41 08 21 48 53 28 01
09 20 61 40 29 16 33 52
62 39 12 17 36 49 30 15
19 10 37 64 13 32 51 34
38 63 18 11 50 35 14 31

267 B 34aR

Exner 1876

23 06 57 44 03 26 55 46
42 59 24 05 56 45 02 27
07 22 43 58 25 04 47 54
60 41 08 21 48 53 28 01
09 20 61 40 29 16 33 52
62 39 12 17 36 49 30 15
19 10 37 64 13 32 51 34
38 63 18 11 50 35 14 31

268 B 34cR

Ligondes 1883

23 06 57 44 25 04 55 46
42 59 24 05 56 45 02 27
07 22 43 58 03 26 47 54
60 41 08 21 48 53 28 01
09 20 61 40 29 16 33 52
62 39 12 17 36 49 30 15
19 10 37 64 13 32 51 34
38 63 18 11 50 35 14 31

269 A 05cR

Francony 1882

23 10 47 50 07 26 63 34
46 49 24 09 64 35 06 27
11 22 51 48 25 08 33 62
52 45 12 21 36 61 28 05
13 20 53 44 57 04 37 32
54 43 14 17 40 29 60 03
19 16 41 56 01 58 31 38
42 55 18 15 30 39 02 59

270 B 05dR

Bouvier 1882

23 10 47 50 25 08 63 34
46 49 24 09 64 35 06 27
11 22 51 48 07 26 33 62
52 45 12 21 36 61 28 05
13 20 53 44 57 04 37 32
54 43 14 17 40 29 60 03
19 16 41 56 01 58 31 38
42 55 18 15 30 39 02 59

271 A 00aCR

Jaenisch 1862

23 26 39 12 41 14 51 54
38 11 24 27 50 53 42 15
25 22 09 40 13 44 55 52
10 37 28 21 56 49 16 43
35 08 57 64 29 20 45 02
60 63 36 05 48 01 30 17
07 34 61 58 19 32 03 46
62 59 06 33 04 47 18 31

272 C 01iR

Roberts 2003

26 07 40 59 10 15 42 61
39 58 27 08 41 60 11 16
06 25 64 37 14 09 62 43
57 38 01 28 63 44 17 12
24 05 56 51 36 13 30 45
55 50 21 02 29 52 33 18
04 23 48 53 20 35 46 31
49 54 03 22 47 32 19 34

273 B 27bR

Wenzelides 1850

26 07 54 43 52 01 30 47
55 42 27 06 29 48 51 02
08 25 44 53 04 31 46 49
41 56 05 28 45 50 03 32
24 09 60 37 20 33 62 15
57 40 21 12 61 16 19 34
10 23 38 59 36 63 14 17
39 58 11 22 13 18 35 64

274 B 00hC

Ligondes 1906

27 10 49 32 53 08 47 34
50 31 28 09 48 33 54 07
29 26 11 52 05 56 35 46
12 51 30 25 36 45 06 55
23 62 13 44 57 04 19 38
14 43 24 61 20 37 58 03
63 22 41 16 01 60 39 18
42 15 64 21 40 17 02 59

275 C 27oR

Murray 1936

30 01 52 47 28 03 54 45
51 48 29 02 53 46 27 04
08 31 42 49 06 25 44 55
41 50 07 32 43 56 05 26
62 09 20 37 60 33 24 15
19 40 61 12 21 16 57 34
10 63 38 17 36 59 14 23
39 18 11 64 13 22 35 58

276 C 27nR

Murray 1936

30 01 52 47 28 03 54 45
51 48 29 02 53 46 27 04
08 31 42 49 06 55 44 25
41 50 07 32 43 26 05 56
62 09 20 37 60 33 24 15
19 40 61 12 21 16 57 34
10 63 38 17 36 59 14 23
39 18 11 64 13 22 35 58

277 B 27rR

Reuss 1883

30 43 52 05 28 01 54 47
51 06 29 44 53 48 27 02
08 31 42 49 04 25 46 55
41 50 07 32 45 56 03 26
62 09 20 37 60 33 24 15
19 40 61 12 21 16 57 34
10 63 38 17 36 59 14 23
39 18 11 64 13 22 35 58

278 B 27sR

Reuss 1883

30 43 52 05 28 01 54 47
51 06 29 44 53 48 27 02
08 31 42 49 04 55 46 25
41 50 07 32 45 26 03 56
62 09 20 37 60 33 24 15
19 40 61 12 21 16 57 34
10 63 38 17 36 59 14 23
39 18 11 64 13 22 35 58

279 A 27pR

Reuss 1883

30 43 52 05 28 01 54 47
51 06 29 44 53 48 27 02
42 31 08 49 04 25 46 55
07 50 41 32 45 56 03 26
62 09 20 37 60 33 24 15
19 40 61 12 21 16 57 34
10 63 38 17 36 59 14 23
39 18 11 64 13 22 35 58

280 B 27qR

Reuss 1883

30 43 52 05 28 01 54 47
51 06 29 44 53 48 27 02
42 31 08 49 04 55 46 25
07 50 41 32 45 26 03 56
62 09 20 37 60 33 24 15
19 40 61 12 21 16 57 34
10 63 38 17 36 59 14 23
39 18 11 64 13 22 35 58

Magic Knight Tours on Larger Boards

12×12 Magic Tours

For reasons of space, and to avoid small print, we show the tours graphically, leaving the reader to put in the numbers and check the magic totals if desired. The end cells are marked by dots, and the links between quarters (36-37, 72-73, 108-109 and 144-1) in reentrant tours) by thicker grey-shaded lines. These usually help to indicate the structure of the tour.

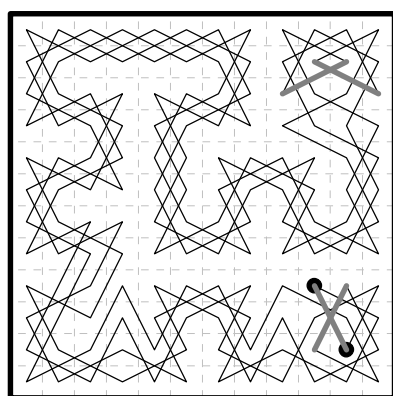
The magic constant for the 12×12 board is 870. Reversing the numbering will also give a magic array, though other numerical features such as diagonal sums may thereby be altered; an entry E becomes 145-E, and a sum of both diagonals D becomes 1740-D. Only four diagonally magic knight's tours 12×12 have been constructed, by Awani Kumar 2003 (shown at the end of this section since the results are shown mostly in chronological order). Examples with one diagonal magic or with both diagonals together adding to twice the magic constant (thus differing from the constant by plus and minus the same amount) were achieved before that.

The first magic knight tour on a board larger than 8×8 is this 12×12 tour due to **Krishnaraja Wodeyar III** (Maharaja of Mysore). It appears as #3 in the Harikrishna ms (1871) as reproduced in Iyer (1982). It is formed, like the Rajah's smaller magic tour, on the squares and diamonds principle.

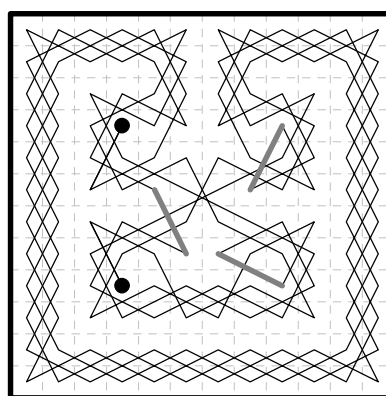
This tour was not known to Murray. The Iyer book was discovered by David Pritchard and the tour was publicised on the cover of *The Games and Puzzles Journal* (vol.2 #14 Dec 1996 p.225).

It will be seen that it includes a long braid component. The snake-like pattern of this tour is similar to that of tour (11) by von Schinnern (1825).

The diagonals add to 608 and 980 (or 760 and 1132 in the reversed numbering). Numbers in the six 4×4 areas that use the simple braid add to 290 in every rank and file, but the other three have excesses and deficits that cancel out across the board.



Mysore (1871)

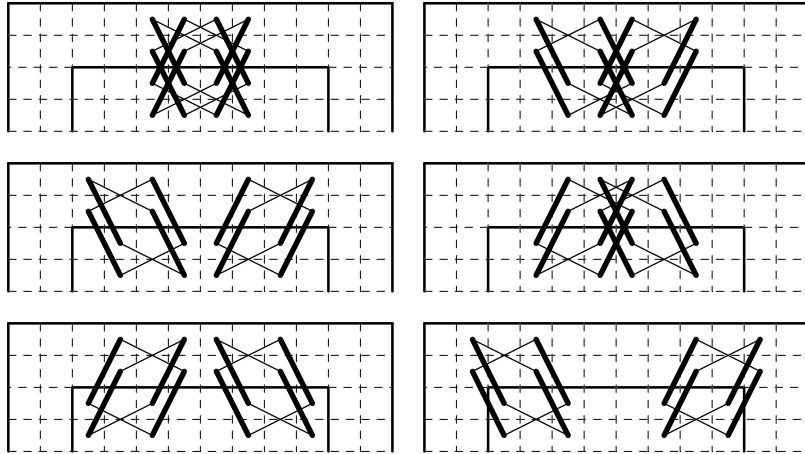


Lange (1932)

Several large magic tours appear in M. B. Lehmann's *Neue Mathematische Spiele: Der Geometrische Aufbau Gleichsummiger Zahlenfiguren* 1932, they are due to **E. Lange** of Hamburg. Lange introduces the device of extending the braid in some 8×8 tours to form a border round the square thus making a 12×12 square. In the diagram shown above the diagonals add to 872 and 1012 (reverse 728 and 868). One diagonal is only 2 away from the magic constant 870. This was formed by adding a border braid to one of the 8×8 magic tours (05g).

Lange indicated three ways in which the links to be deleted in the parent tour can be arranged, Murray (1951) indicated two ways in which each of these can be connected to the border braid. The dark lines are the inserted moves and the light lines the deleted moves. Murray counted 180 magic tours formed by this method from the 8×8 magic tours then known. See also later examples by T. H. Willcocks (1993) for some other methods.

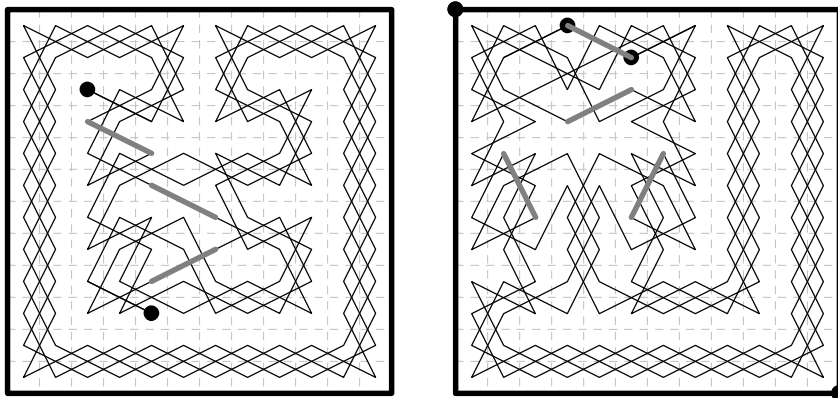
KNIGHT'S TOUR NOTES



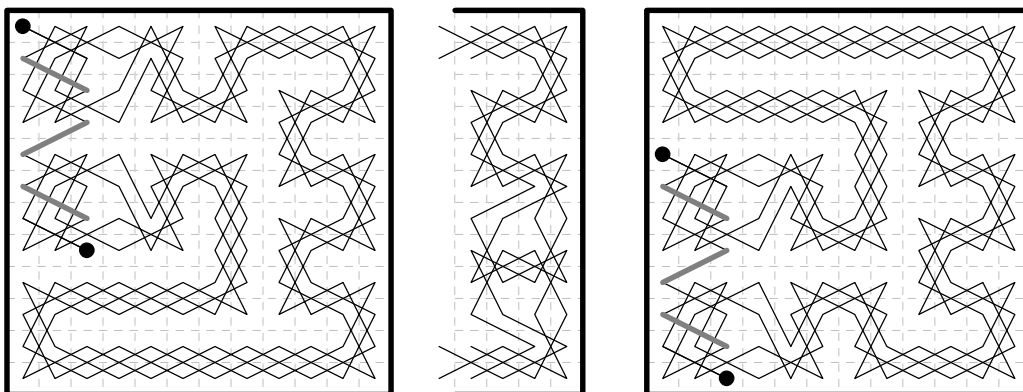
Here are two tours constructed by this braid method by **H. J. R. Murray** in his unpublished mss of 1942 and 1951 which are now in the Bodleian Library, Oxford.

The first tour is formed by adding a border braid to one of the 8×8 magic tours (27g) as in the Lange (1932) example (Murray 1942 ms Fig.282). Diagonals 786 and 932, numbered from $c_{10} = 1$.

The second tour, dated 1947, is the first to have one diagonal magic, although Murray may not have realised this, since he does not mention the property in his 1951 manuscript. He constructed the tour as one of 130 tours of 'gnomon' type, formed analogously to the Lange bordered tours. He points out that the tour remains magic when renumbered from 37, 73 and 109. This cyclic property it inherits from its parent tour 00d. Diagonals 870 and 1018 (722 and 870).



Based on Wihnyk's 16×16 example (1885) Murray gave the following two 12×12 tours, dated 1941 and 1947, which extend and modify the braid part of Beverley's tour.

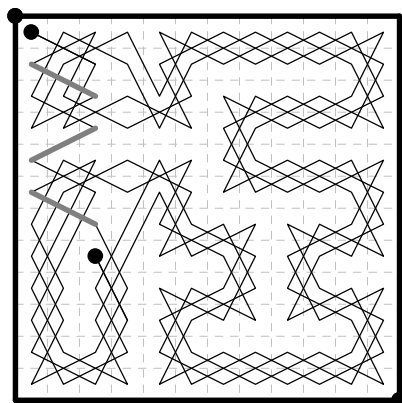


KNIGHT'S TOUR NOTES

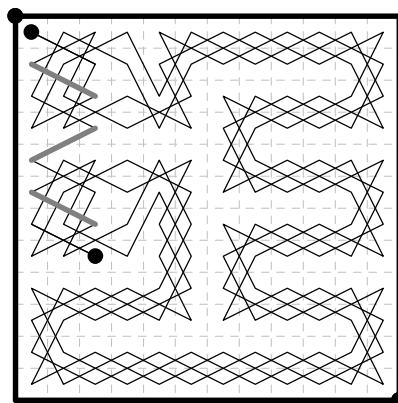
They differ only by the inversion of the Beverley component. By design the 4×4 quads in these are all magic, i.e. they add to 290 in each row and column. Diagonals: 916 and 950 (790 and 824); 722 and 824 (916 and 1018).

Murray noted that a pair of quads in his 1941 example shown above could be replaced by alternative symmetric formations, as can be done in the case of the 8×8 Beverley tour. I show one example of this, inset. Diagonals now 916, 944 (796, 824).

The first published 12×12 magic tour in which one diagonal added to the magic constant, 870 was given by **T. H. Willcocks** in *Fairy Chess Review* (Dec 1955). It distorts the lower edge of the Beverley half-board into a short braid. Diagonal sums 708 870 (reverse 870 1032). The other example is from his 1962 *Recreational Mathematics Magazine* article. Diagonals: 852 870 (reverse 870 888).

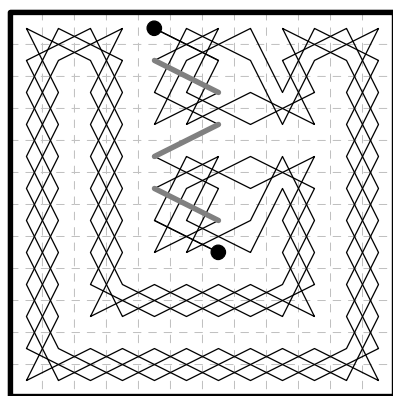


Willcocks (1955)

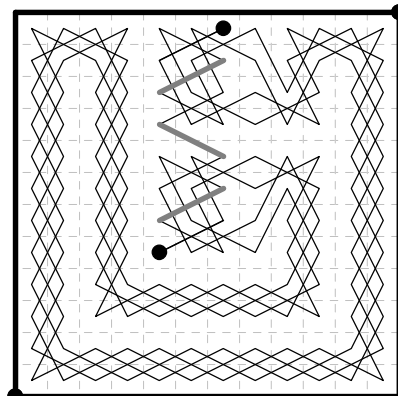


Willcocks (1962)

Another 12×12 tour with one diagonal magic appears in the article by Willcocks in *Journal of Recreational Mathematics* (1968). It is almost identical to one of Murray's gnomon examples (dated 1950) shown on the left here for comparison.



Murray 1942



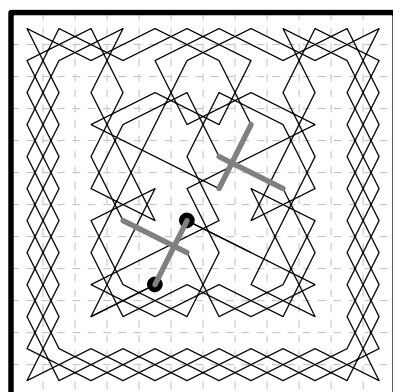
Willcocks 1968

The diagonal sums are Murray: 734 and 802 when numbered from $e_{12} = 1$ (reverse 938 and 1006) Willcocks: 870 and 938 when numbered from $e_5 = 1$ (reverse 602 and 870). The only difference is that the Beverley quartes are inverted (or if the diagram is inverted the gnomon braid goes over the top instead of round the bottom).

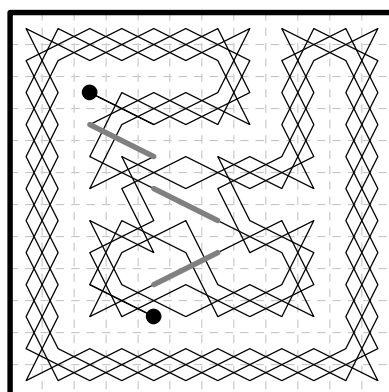
There are 6 geometrically distinct positions of the Beverley quads that allow tours. I have a note that all the ways of connecting them by braids to cover the 12×12 board give rise to 165 magic tours (each asymmetric case gives two tours, each symmetric case one tour).

KNIGHT'S TOUR NOTES

In a letter to me dated 17 June 1993 THW provided two examples of 12×12 bordered magic tours using other connection schemes than those described by Lange and Murray. The parent magic squares used are 12j and 27g. (a) Diagonals 956 and 1040 (reverse 700 and 784). (b) Diagonals 800 and 896 (reverse 844 and 940).



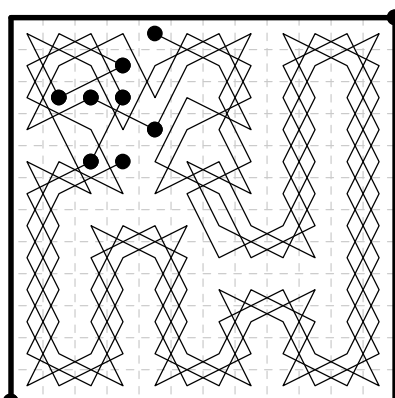
Willcocks (1993)



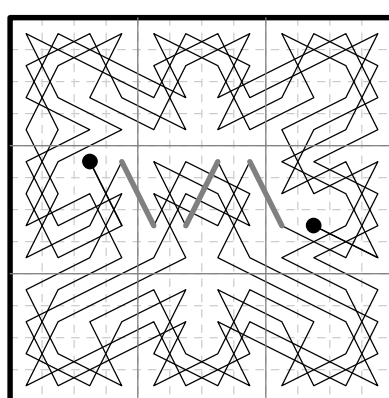
Willcocks (1993)

My correspondence with T. H. Willcocks inspired some work of my own on 12×12 magic tours, trying unsuccessfully to find a method to make one with both diagonals magic. Here is a magic tour which has one diagonal (bottom left to top right) adding to the magic constant 870. It is derived from the 8×8 magic tour 14a. The braid that forms a significant part of the parent tour is simply extended to cover the 12×12 board. Jelliss (1993) diagonals 870 and 890 (reverse 850).

Three others can be derived from this by replacing the braid in the bottom right corner, a-d, 1-8 (which is the same as the right-hand side of the Beverley tour 27a) by alternative symmetric patterns (as in the 8×8 tours 27e, 27d, 27c). In these the non-magic diagonal sum varies.



Jelliss (1993)



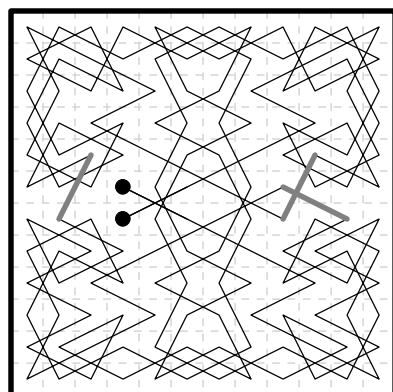
Jelliss (1993)

In his account of 8×8 tours of the Beverley type, analysed in terms of contiguous contraparallel chains, Murray mentions that the method is applicable to larger boards, but strangely he does not give any example on the 12×12 board. The second diagram above is one constructed (Jelliss Aug 1993) to show that it is possible. The two pairs of contraparallel chains are of course 1-36||108-73 and 37-72||144-109. The first two quads in the middle files are of Beverley type, the other three mid-edge quads are of the braid type, and the corner quads are all alike and arranged in biaxial symmetry to each other so that their variances are guaranteed to cancel out (they vary only by ± 2 in the files). Diagonals 626, 1190. I sent this out on my New Years card for 1995.

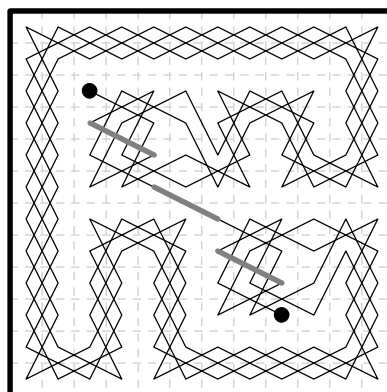
T. H. Willcocks sent me the following magic tour constructed by the method of biaxial symmetry, which depends on finding a way of converting one or both of the rook moves into a knight move. This was in *Chessics* #26 (Summer 1986 p.128). The two diagonals add to 1740. Note that it uses three cells in each rank. Diagonals 870 ± 74 .

KNIGHT'S TOUR NOTES

The second diagram (Jelliss 2000) shows one way that the Beverley quads can be used, placing them corner-to-corner instead of edge-to-edge. The single link joining the two quads limits the movement possible. Diagonals 780, 908 (832, 960).

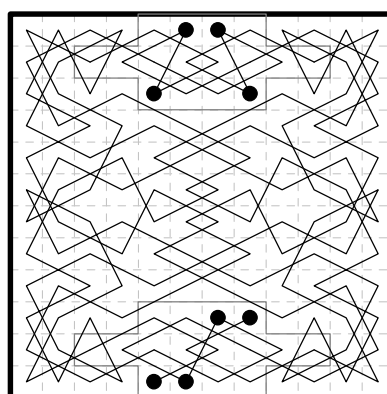
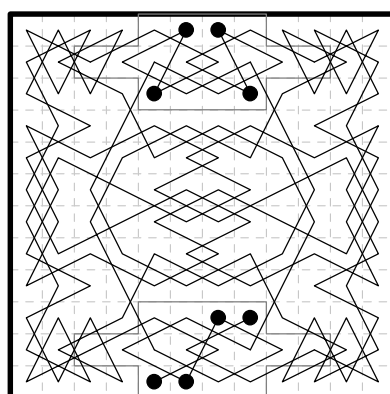
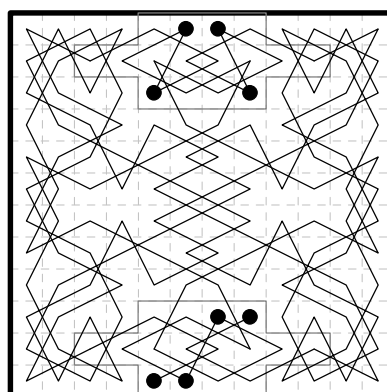
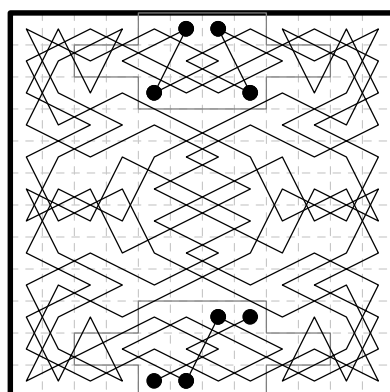


Willcocks (1986)



Jelliss (2000)

On my *New Year and Millennium Card* for 2000/1 I showed 'Three new magic tours 12x12' chosen from 25, constructed by a new transposition method using just two transpositions 1-3 and 73-75 to convert them from biaxial tours with rook-move connections. In these 25 tours the two diagonals together add to 1740 and all have the moves shown in the 16-cell lozenge (or rolling-pin?) shaped areas marked in the following diagrams (Jelliss 2000).

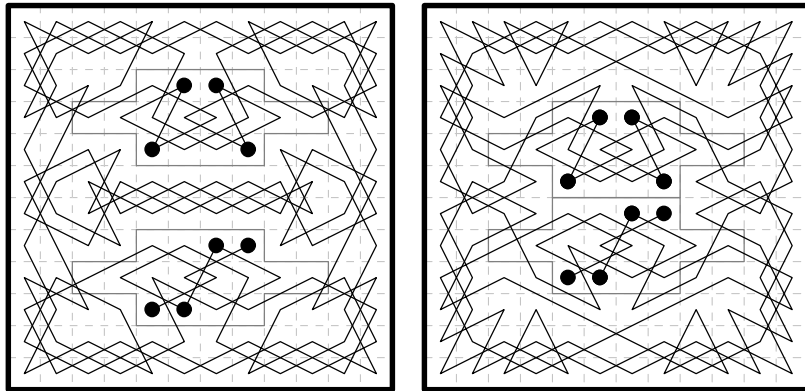


The only moves that deviate from quaternary symmetry are within these areas. From each of these 25 magic tours three others can be derived by inverting one or both of these lozenges (thus making 100 in all). This enumeration was done by hand and has not been independently checked.

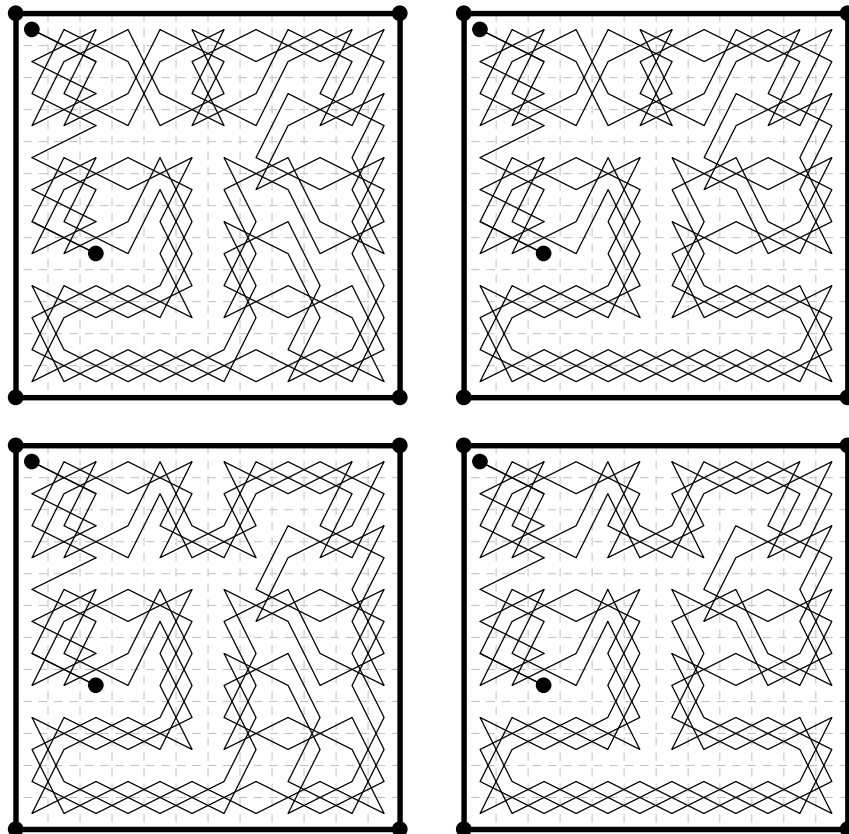
The four diagrams shown are those in which the deviation from 870 is less than 50. Diagonals: ± 22 (type 3-3), ± 46 (type (3-3)), ± 34 (type 2-4), ± 42 (type 3-3). See the website for the full list.

KNIGHT'S TOUR NOTES

Similar tours can be constructed on this plan with the lozenge areas moved inwards, but this results in one of the transposed numbers within the lozenges occurring on a diagonal, so that the sum of the two diagonals is no longer 1740 but is increased or decreased by 2. Here are two examples (Jelliss 2000, diagonals 842 900 and 840 898):



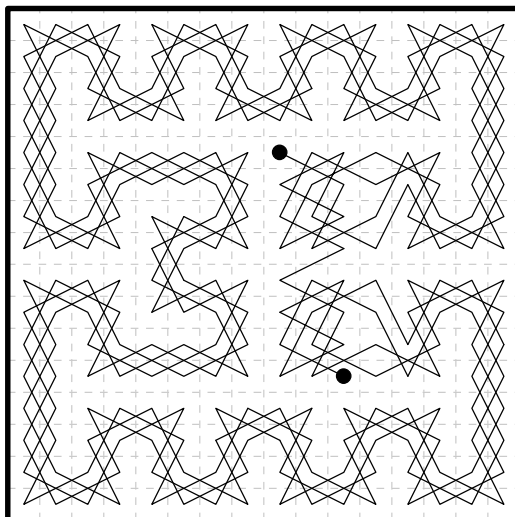
The next to take up the challenge of 12x12 magic knight tours was **Awani Kumar** who found these four diagonally magic tours. His work is reported in a series of studies published in *The Games and Puzzles Journal* (online). His article 'Four Perfect Magic Tours 12x12' in #26 (Mar-Apr 2003) is a fine achievement, solving a long-standing problem.



These are still the only examples with both diagonals magic known. It will be seen that the tours differ in the 4x4 area in the middle of the top four ranks or in the 6x6 area at the bottom right. A question still unanswered is: can diagonal magic be achieved in a 12x12 closed tour?

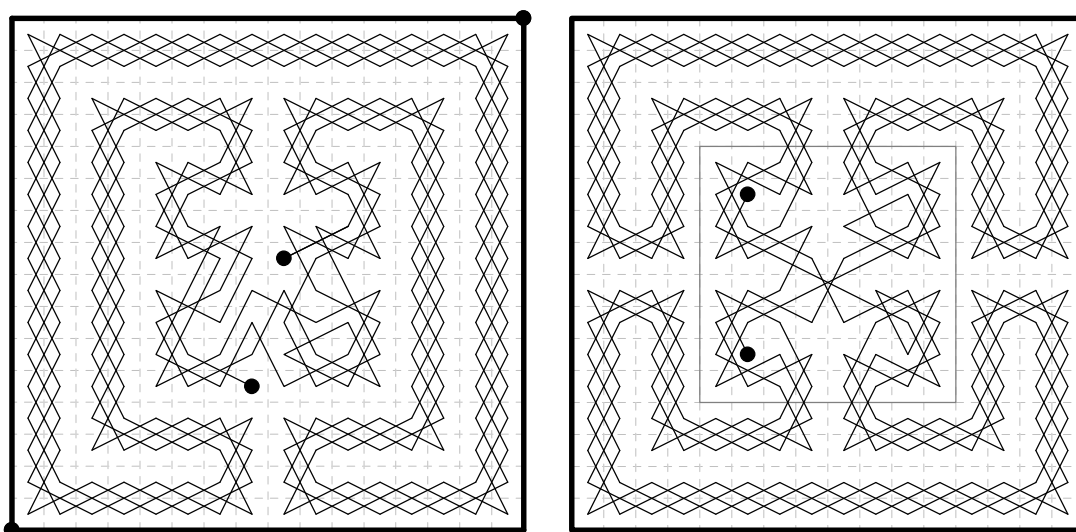
16×16 Board

Braid Method. The first 16×16 magic knight tour occurs in an article by **Maxwell Wihnyk** in *Deutsche Schachzeitung* (1885) showing how the braid in Beverley's tour could be extended to fill extra 4×4 areas to give magic tours on all square boards of side $8 \cdot n$ with $(n > 1)$. The magic constant for the 16×16 board is 2056. The tour is oriented here so that the Beverley quads are in the same relative position as in the original 8×8 Beverley tour.



Using Murray's terminology, the two pairs of contiguous contraparallel chains in the 16×16 tour are 1-64||192-129 and 65-128||256-193. Every quad (4×4 grid component) is magic. (Diagonals add to 1898 and 2090, numbered from $i12 = 1$, reverse 2022 and 2214). Apart from within the two Beverley quads the symmetry about the horizontal axis ensures that the numbers in the second half of the tour are arranged so that each number x and its complement $(257-x)$ lie in the same file.

The next 16×16 tour, from Murray (1951), applies the method of Lange of extending an 8×8 tour onto larger boards by adding successive braids in the form of a border round the tour. The one shown here (using the 8×8 rhombic tour 14b) numbered from h5, has the even diagonal adding to the magic constant 2056, though Murray did not draw attention to this property (the other diagonal is 2104 or 2008 in the reverse numbering).



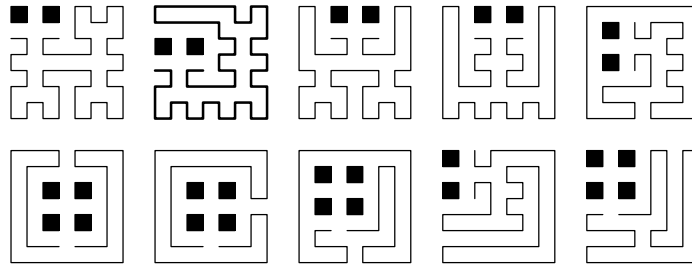
KNIGHT'S TOUR NOTES

The other tour above is a 16×16 magic tour of my own construction (unpublished) using the braid method. The parent tour is (05g). The diagonals sum to 2056 ± 168 . This and the irregular tour (12i) are the only 8×8 magic tours with two type 0 braids in the middle of opposite sides.

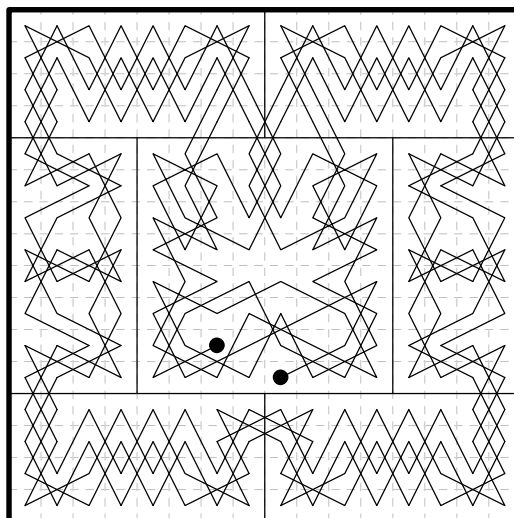
Many other tours of braid type can be formed, in which the arrangement of the braids can be much more free-form, but none of them as far as I know show both diagonals magic. Surprisingly Murray seemed more interested in trying to work out how many tours of these types could be constructed than in finding special cases with interesting properties, though he did place emphasis on the property of each 4×4 block itself being magic, as in Beverley's original tour.

It was while drawing the diagrams for these tours that I realised that in this type of tour the restriction to all the quadrants being magic was unnecessary and that the braids covering all the board except for the two Beverley quads could in fact be arranged with almost complete freedom without affecting the magic property.

The first four of the following diagrams are the braid patterns of magic tours formed on the same plan as Wihnyk's tour, as given by H. J. R. Murray in his 1951 manuscript, and dated 1947-8. The next four are non-diagonal magic tours given by Murray formed by applying the E. Lange border method to 12×12 tours (which may in turn have been formed by braid extension of suitable 8×8 tours or Beverley quads). The ninth and tenth are similarly formed by Murray's gnomon method.

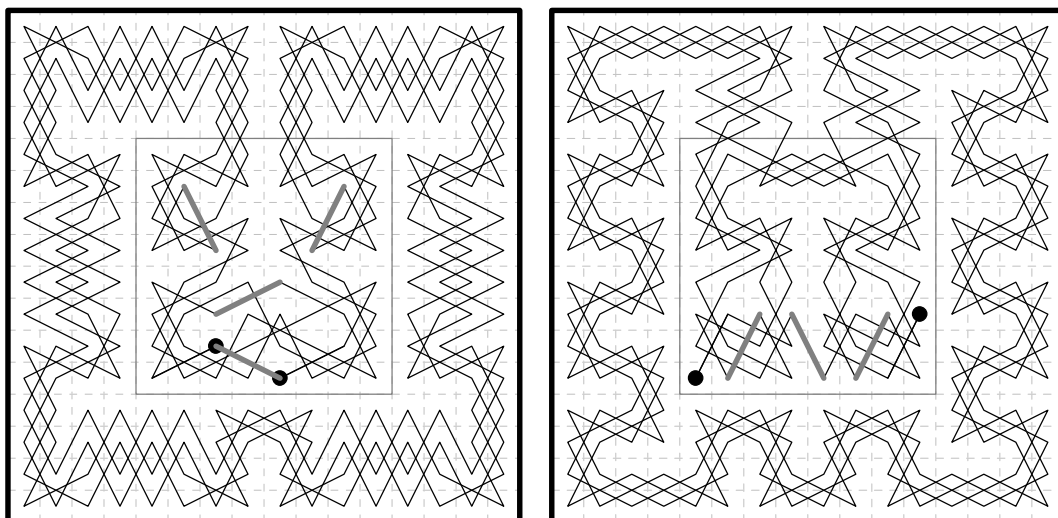


Surround Method. Another way of deriving a 16×16 magic tour from an 8×8 example was devised by **E. Lange**, published in Lehmann (1932), in which a central 8×8 tour is joined to six blocks each 4×8. The patterns of moves within these added panels take the forms typified by the alternative right-hand sides that can be substituted in Beverley's 27a to form the related tours 27b, c, d, e, f. In this 16×16 example the diagonals add to 2056 ± 158 .



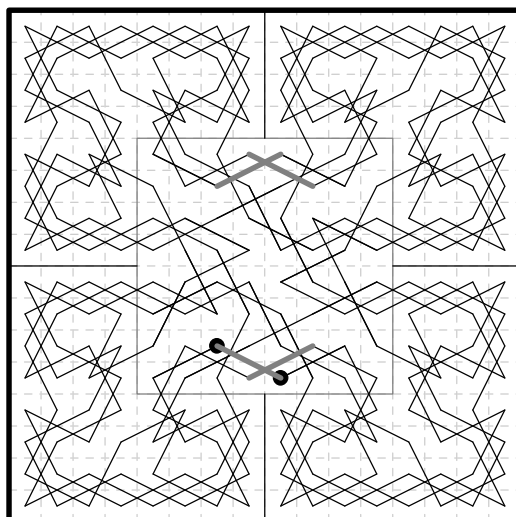
KNIGHT'S TOUR NOTES

Here are two 16×16 magic tours formed on the Lange panelled surround method, from Murray (1942). He indicates that further borders can be added to the second.



Extended Quartes Method. Many 16×16 magic knight tours were constructed by this new scheme described by **H. J. R. Murray** beginning in *Fairy Chess Review* (Aug 1942), where he wrote: “The principle of construction ... is really an extension of ... squares and diamonds on the 8×8. Take any magic tour composed of squares and diamonds entirely, and insert the moves connecting their terminals on the central 8×8 of a 16×16 board. Then replace the squares and diamonds by circuits of squares and diamonds round the quarterboard of the 16×16 in which the terminals happen to lie. One pair of these is taken clockwise and the other anticlockwise, but all these chains have the same division of moves in the central 8×8, either 2 and 2, or 3 and 1 moves.”

In this example (which is also Fig.255 of his 1942 ms) the parent tour used is 12b. (Diagonals 1680 and 2432, numbered from $i5 = 1$, i.e. 2056 ± 376).

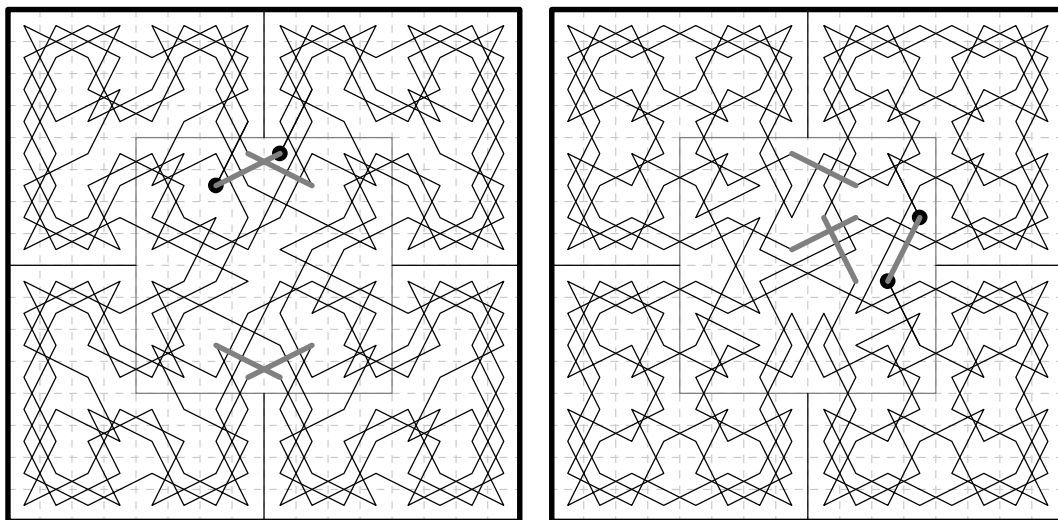


In his 1942 ms he noted with regard to the extended quartes method: “There are 12 matrices containing only single-indexed quartes [i.e. not Beverley type] giving 31 magic tours on the chessboard. Each tour gives two magic tours in the 16×16 board, 62 in all. If the parent tour on the chessboard has more than one starting point, so has the tour on the 16×16 board.”

KNIGHT'S TOUR NOTES

Here are two other examples of this method by Murray (1942).

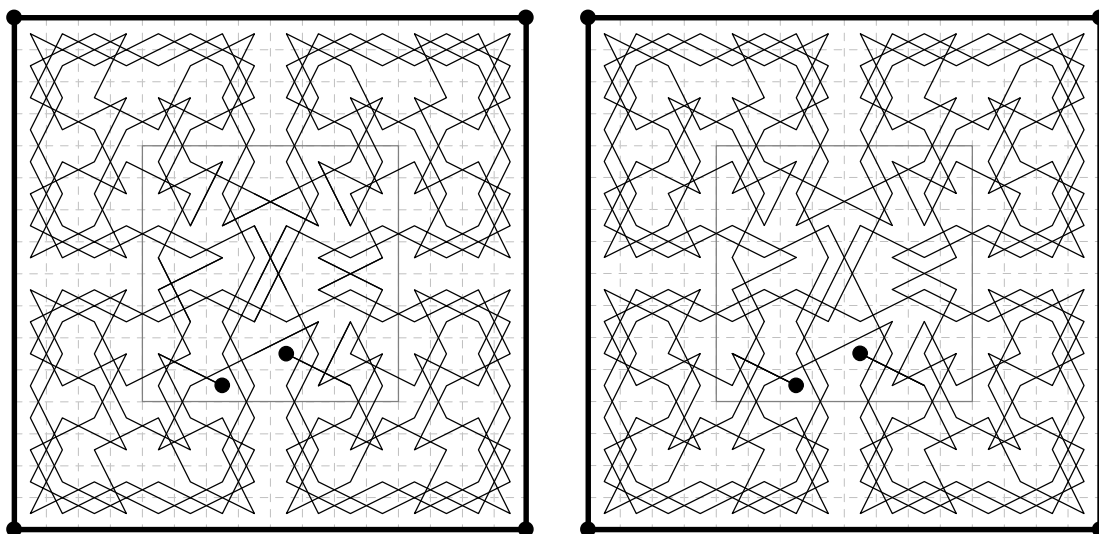
These are Fig.256 (Diagonals E = 1712, O = 2400, numbered from i12 = 1) and Fig.257 (Diagonals E = 2064 and O = 2048, numbered from k8 = 1). The latter, based on Jaenisch 00a, is magic when numbered from five points.



In his 1951 manuscript Murray explained that this method could also be applied to tours containing Beverley or irregular quartes, provided that certain elaborate conditions are satisfied. The main ones being that the parent tour must have $X = 60$ and $N = 20$ for every line, and the quartes must all be broken in the same way, either 2 and 2 or 3 and 1.

Diagonal Magic on the 16×16 Board

Extended Quartes Method. The first diagonally magic knight tours were constructed in correspondence between **H. E. de Vasa** and **T. H. Willcocks** in the 1950s at which time de Vasa was an invalid living in Paris. They applied Murray's Extended Quartes to construct diagonally magic tours on boards of side 16, 24, 32, 48 and 64. These results were reported in the issue of *Fairy Chess Review* following the obituary for Murray (vol.9 no.6 Oct 1955 p.46-47), though space prevented publication of diagrams.



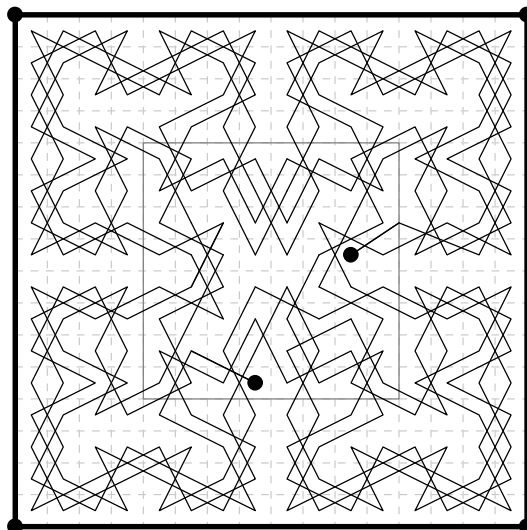
These two very similar de Vasa 16×16 solutions were published separately.

KNIGHT'S TOUR NOTES

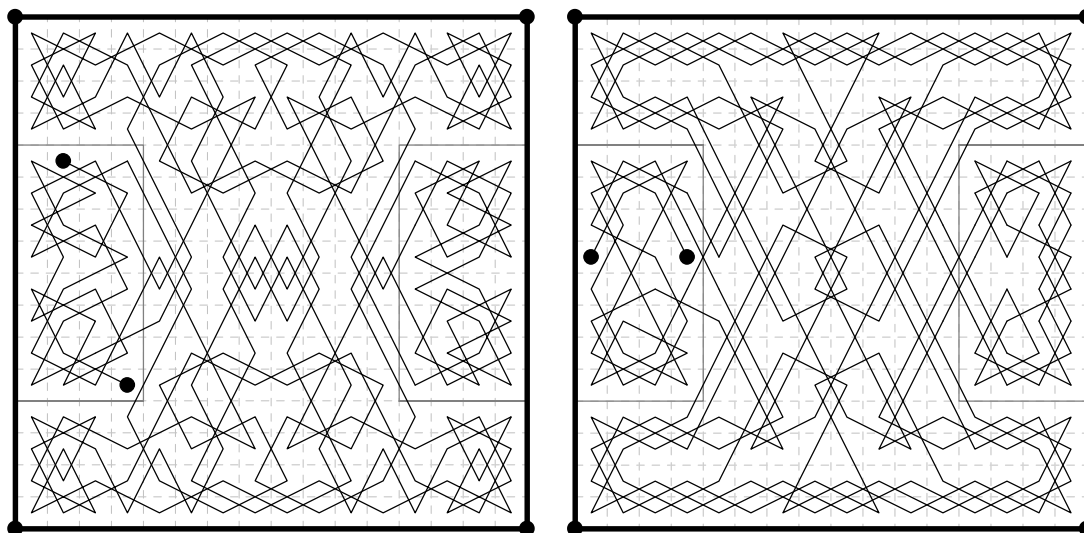
One in G. d'Hooghe *Les Secrets du Cavalier* (1962). The other in an article by T. H. Willcocks in *Recreational Mathematics Magazine* (1962).

This tour is also quoted in numerical form by J. S. Madachy in *Mathematics on Vacation* (1966) also republished by Dover Publications as *Madachy's Mathematical Recreations* (1979) but without acknowledgement to the composer. The tours are based on the Jaenisch 00a tour on the 8×8 . Both diagonals 2056.

The T. H. Willcocks 16×16 diagonally magic tour mentioned in *FCR* was at last published in *Journal of Recreational Mathematics* (1968). It is based on 34d. It includes a graphic W.



Split and Fix Method. In a Greetings Card for New Year 1991/2 I published three 16×16 diagonally magic knight tours by a new 'Split and Fix' method which involves cutting an existing 8×8 magic tour in half, placing the halves at opposite edges of the 16×16 and joining up the loose ends in a biaxially symmetric pattern, which also meets certain other criteria.

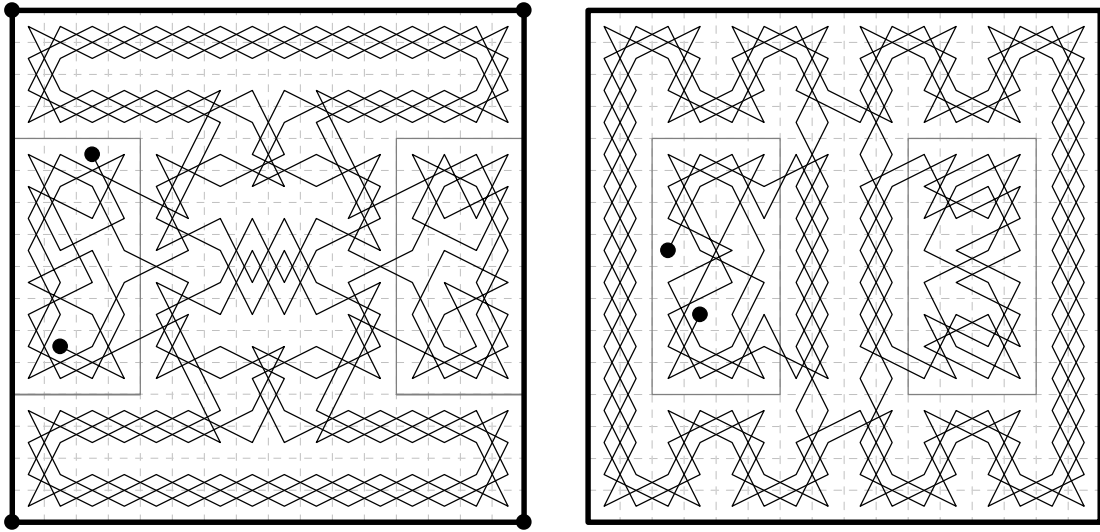


The biaxial property means that the numbers in the files add in complementary pairs to 257 while in the ranks half add to 193 and half to 321 (adding to twice 257). In the first two tours derived from parent tours 27i and 03b the pairs adding to 257 are symmetric with respect to the horizontal axis.

However, in the next tour derived from parent tour 16a the pairs adding to 257 occupy pairs of adjacent squares. In this tour each connecting strand has a strange type of symmetry, in that if the r th move takes the knight to the m th file then the r th move from the other end of the strand takes the

KNIGHT'S TOUR NOTES

knight to the $(16 - m)$ th file. Thus the middle move crosses the vertical median. This ensures that each file that contains k also contains $257 - k$ (in fact they form domino pairs).



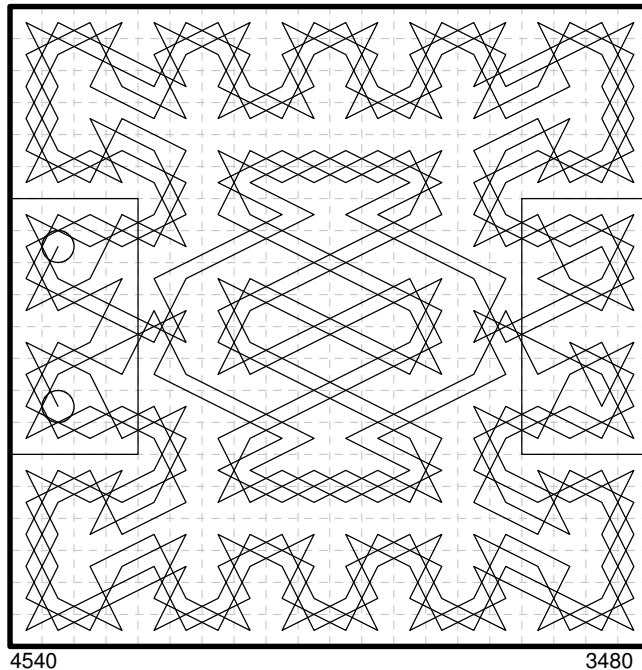
In these tours the fact that the sections of the split tour do not overlap the diagonals makes it easier to ensure that the diagonals add to the magic constant. Others that I tried with the components differently placed could not be made magic in the diagonals. Above is one I composed 3 Nov 1991. The diagonals add to 2056 ± 170 . The parent tour is 00d (00j can also be substituted):

It may be noted that the Wihnyk tour can be regarded as formed by splitting the Beverley tour and placing the two parts back to back in the centre of the larger board.

20×20 Board

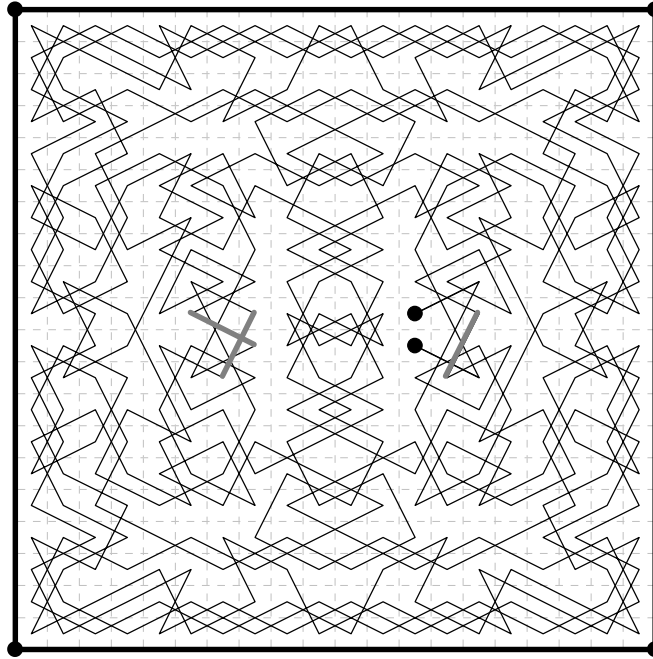
It is simple to cover a 20×20 square with 16 tours 5×5 linked, as shown by Kraitchik 1927, or indeed 4 tours 10×10.

Magic tour, non-diagonal, Jelliss (1991) split and fix method.



KNIGHT'S TOUR NOTES

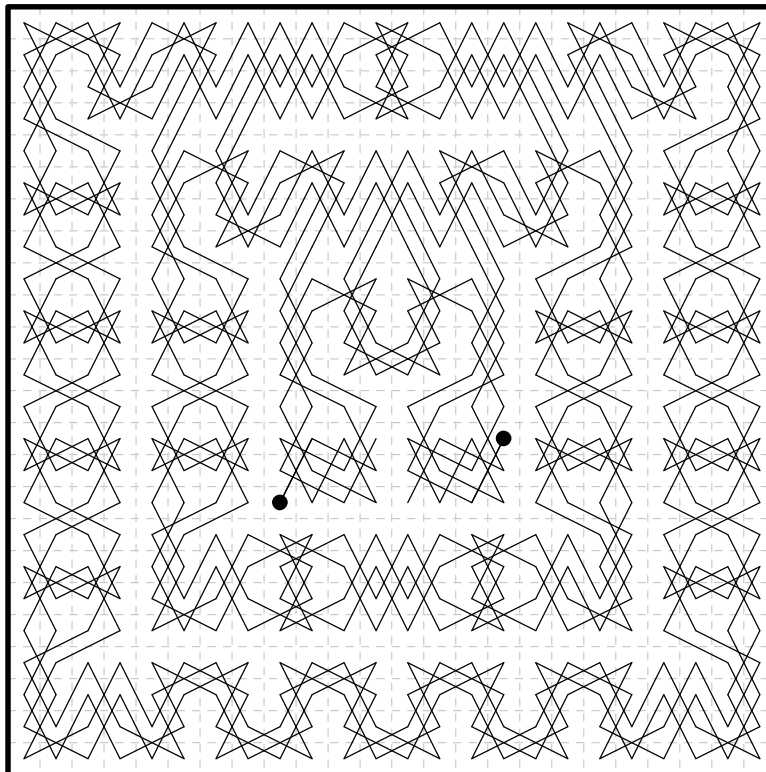
Here is a diagonally magic 20×20 knight tour by **T. H. Willcocks** which concludes his article 'The Construction of Magic Knight Tours' *Journal of Recreational Mathematics* (vol.1 #4 Oct 1968 p.225-233). This is the first example of this size, and in fact the only one I know of.



The magic constant is 4010. The central 4×4 is a double Beverley quad. The emphasised grey lines mark the links between the quarters (100-101, 200-201, 300-301).

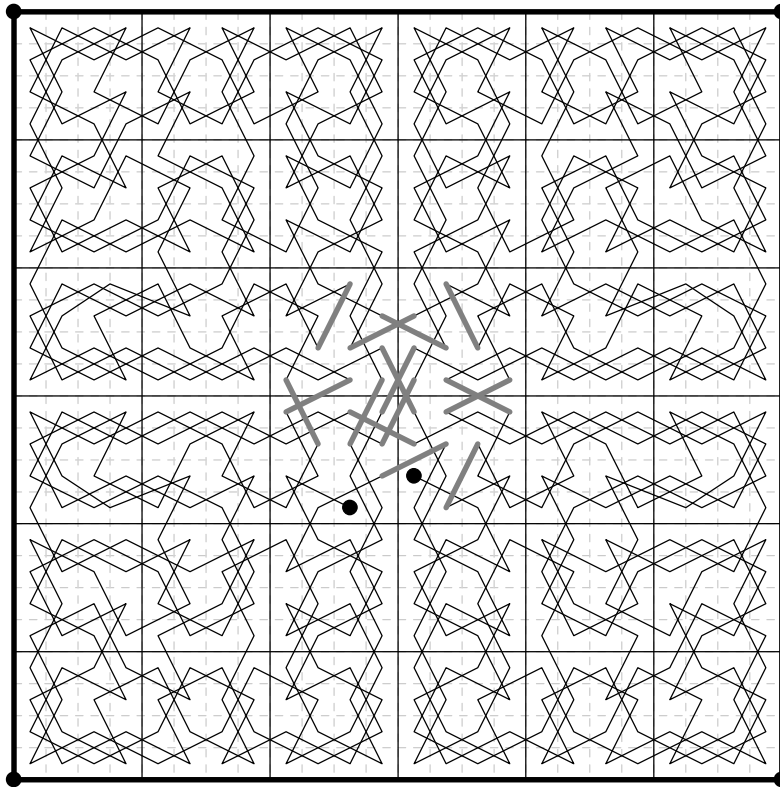
24x24 Board

Magic Tour by E. Lange in Lehmann (1932) by the surround method.



KNIGHT'S TOUR NOTES

Diagonally Magic Tour by T. H. Willcocks (1956) by the Extended Quartes method.

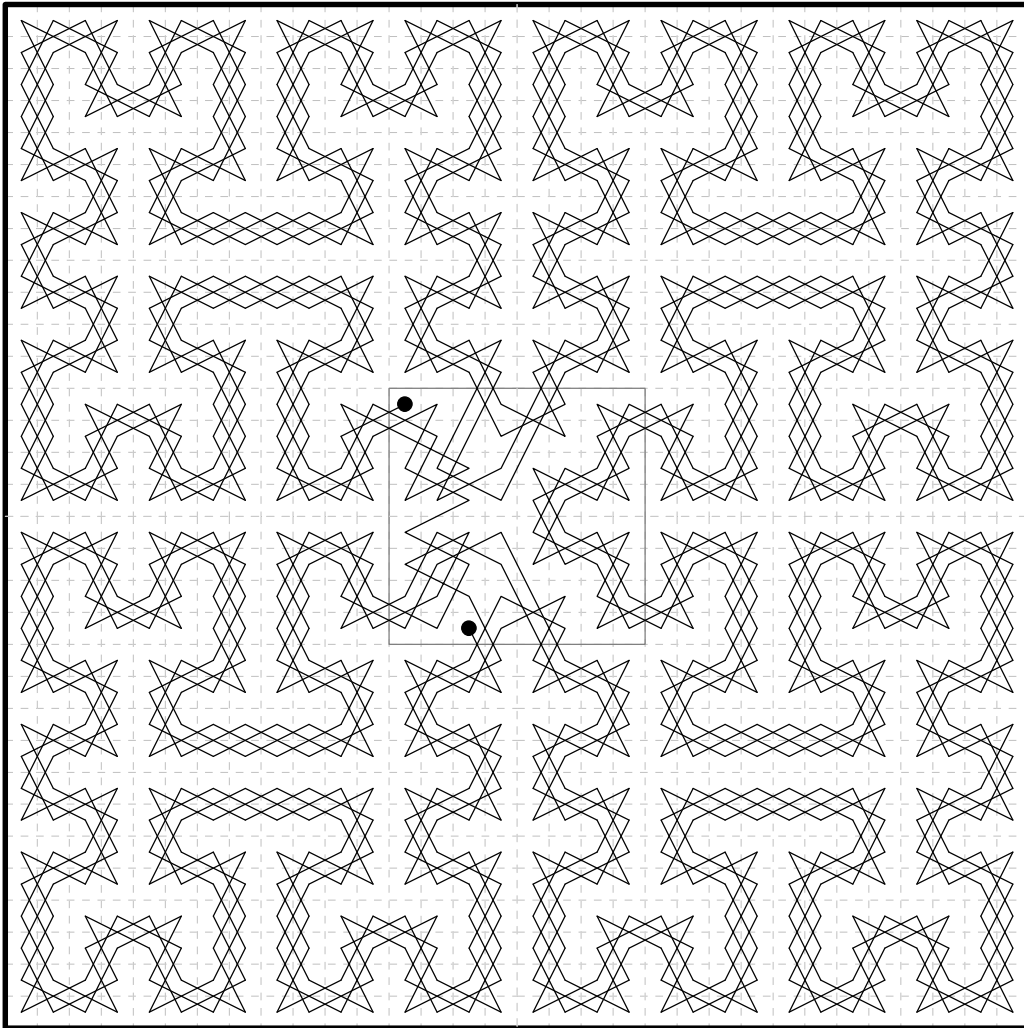


Details of the above 24×24 tours. The first by E. Lange from Lehmann (1932) is constructed by placing a border of 4×8 components round a 16×16 magic tour, which in turn is a bordered form of Beverley's tour. Diagonals 6924 – 222 and + 38.

The second is a diagonally magic tour by T. H. Willcocks, composed 16 Nov 1956. This was a product of his work with H. E. de Vasa mentioned in *Fairy Chess Review* 1955, but was not published at the time. It is based on the (00a) tour by Jaenisch (1862). It was sent to me in a handwritten letter from T. H. Willcocks at the time I produced the 1986 issue of *Chessics* devoted to magic tours.

32×32 Board

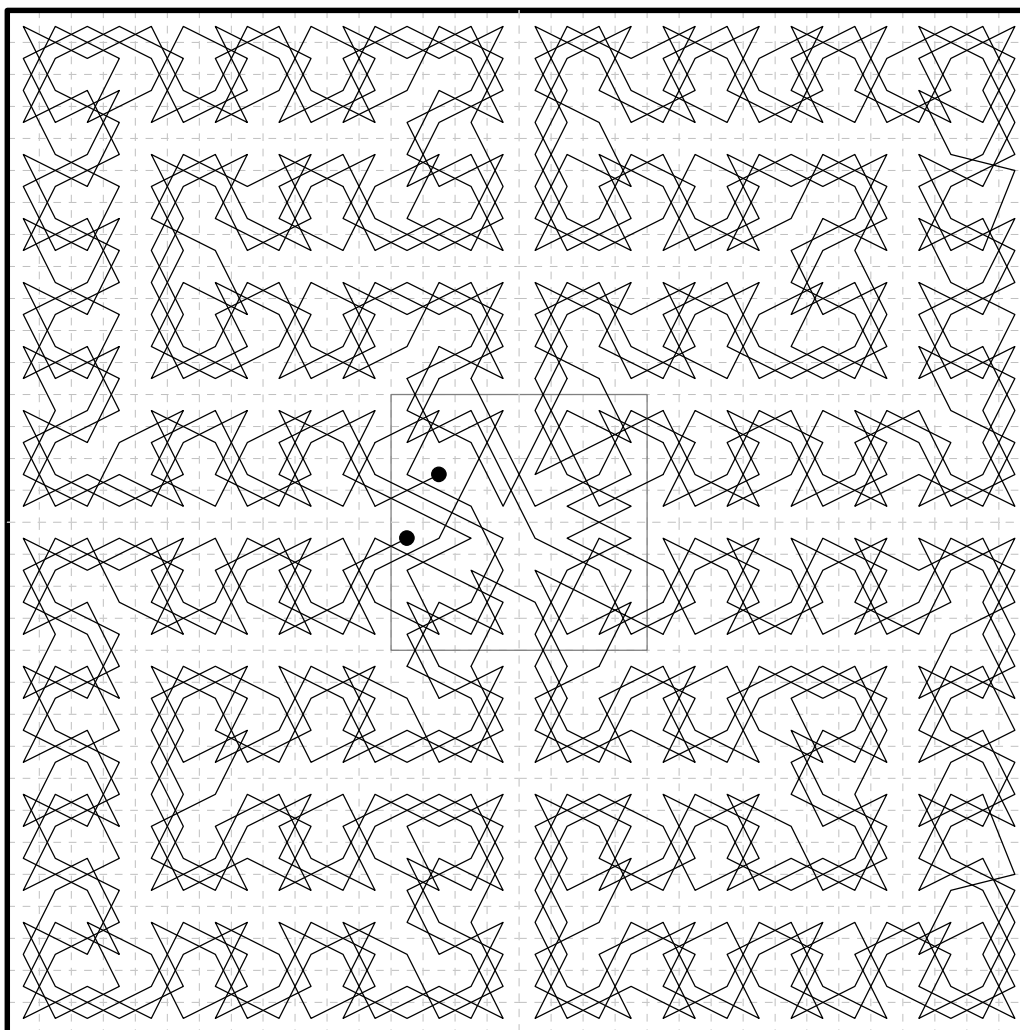
Extended Quartes Method. In this 32×32 open magic tour by H. J. R. Murray, using his method of Extended Quartes, every quarterboard and 4×4 quad is also magic. In fact any square combination of quads is magic. The parent 8×8 magic square is Beverley's (27a). The magic constant is 16400 .



I toyed with the idea of constructing a tour of this type in which the braids delineate letters, but the letters possible are rather limited This is an idea that others might like to try.

KNIGHT'S TOUR NOTES

In the following 32×32 reentrant magic tour by H. J. R. Murray the diagonals add to 8368 and 23920 (reverse 8880 and 24432).



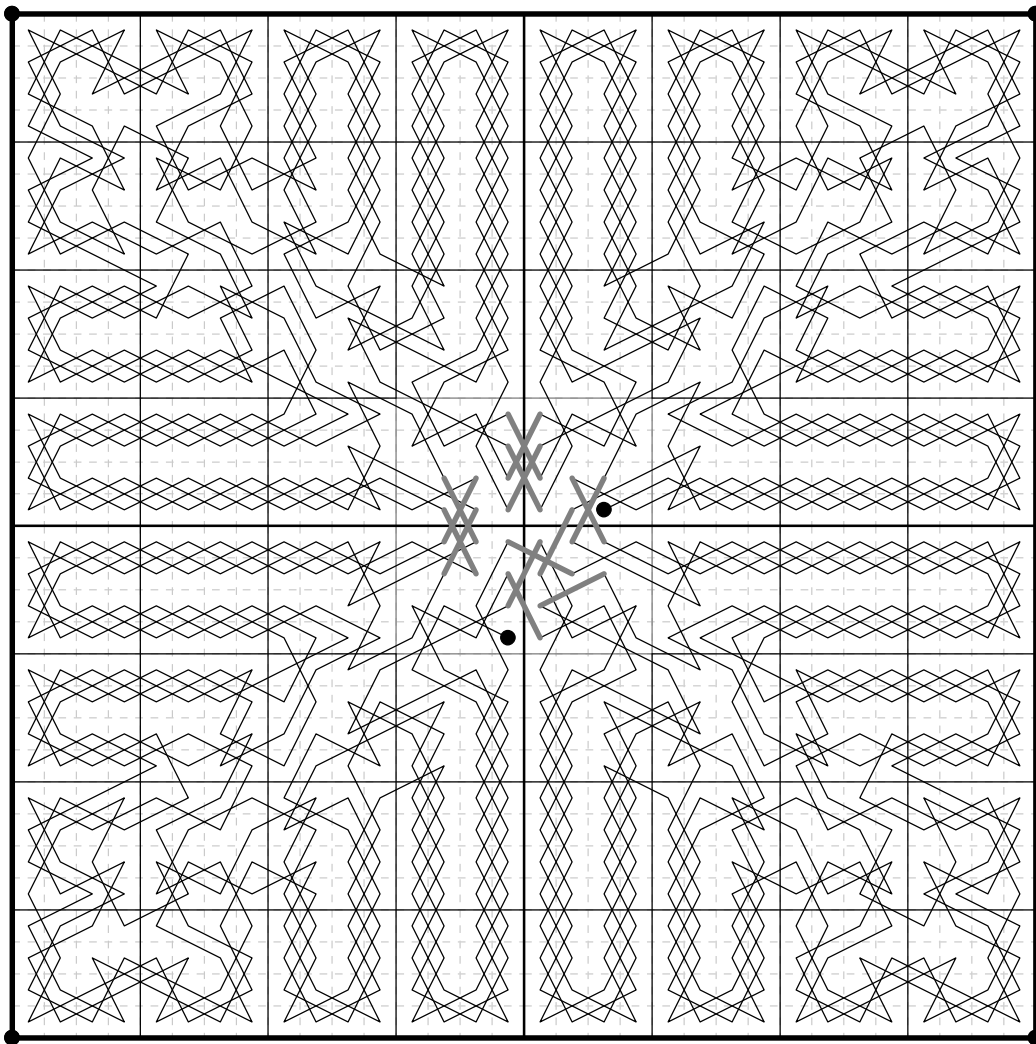
This is the last tour in the Murray 1951 ms. It is based on one of his own 8×8 magic tours (12p), providing the connections in the central square.

Our title page shows the first diagonally magic closed knight tour constructed on the board of 32×32 squares by H. E. de Vasa. It was sent to the editors of *Fairy Chess Review*, Dennison Nixon and C. E. Kemp, in 1956 and was mentioned in *FCR* though not diagrammed. The details have been obtained from de Vasa's hand-written letters, provided by T. H. Willcocks, and subsequently deposited in the Murray collection at the Bodleian Library, Oxford, in 1991

The lines marking out the four corners are as shown in the original manuscript. The central square outlines the 8×8 parent tour from which the larger tour is derived by a modification of Murray's Extended Quartes method. The darker lines mark the links between the sixteen extended quartes, each of covering 64 cells with a non-crossing knight path.

KNIGHT'S TOUR NOTES

Here is a diagonally magic 32×32 open tour composed by T. H. Willcocks at the same time as the above de Vasa tour (November 1956) as reported in *FCR*. The details of this were sent to me by Mr Willcocks in a handwritten letter.

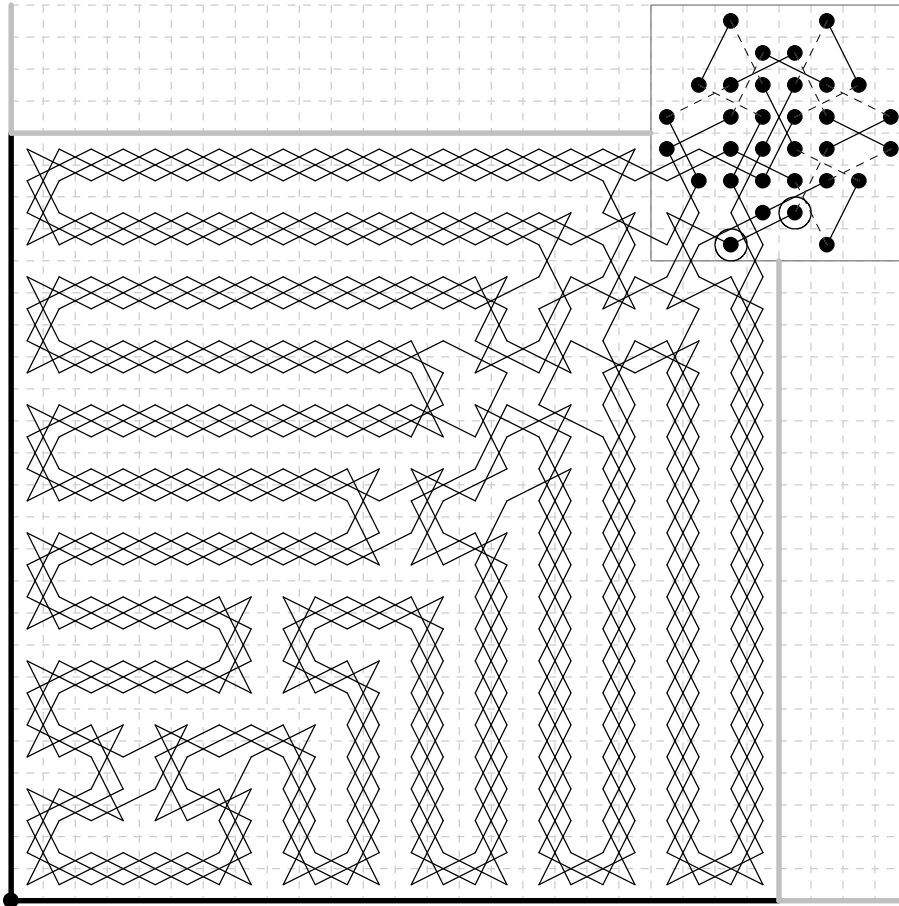


The 8×8 parent tour is (34d) the same as in the 16×16 tour composed at the same time. The four quarters, apart from the linking moves shown by bolder lines, are all identical apart from orientation, being arranged in direct quaternary symmetry.

48×48 Board

As noted earlier the *Fairy Chess Review* (vol.9 no.6 October 1955 p.46-47) reported work by H. E. de Vasa who had applied Murray's 16×16 method to construct diagonally magic tours on boards of side 24, 32, 48 and 64, though space prevented publication of diagrams.

This diagram shows one quarter of a diagonally magic tour 48×48 by **Helge Emanuel de Vasa**, composed Feb 1957. The other three quarters are reflections of this one.



Numbering, 1 to $48^2 = 2304$, starts at one circled dot and ends at the other. Magic constant 55320. The connections in the central 8×8 are the same as for tour 00a.

This diagram was used as the cover illustration on *Chessics* #26 (1986) which was the special issue on Magic Tours. A full chart of the tour, in numerical form covering four pages, is in the papers now in the Bodleian Library.

This is very similar to the 32×32 tour with the outer braids extended.